

Tutorial letter 201/2/2016

MATHEMATICS AND MATHEMATICS TEACHING

PST201F

Semester 2

Department Mathematics Education

IMPORTANT INFORMATION:

This tutorial letter contains the memorandum for Assignment 02

IMPORTANT INFORMATION:

At the end of this letter, you will find exam guidance.

BAR CODE

ASSIGNMENT 02

SOLUTIONS AND COMMENTS

Dear student

This tutorial letter contains solutions to Assignment 02. You will need to compare the solutions to your own attempts. This feedback is very important. Work through the solutions, and use the examples to prepare for the exam.

REMEMBER

NOT every question will be marked. That does not mean that the questions not marked are not important (Interpret the three NOTS in this sentence!!*). Look carefully at the marking schedule and the marks you obtained.

We are committed to help you to pass this module. Please be committed from your side to pass.

Success will not come without effort.

*It means everything is important!

The solutions are placed in boxes for your convenience

PART B

ACTIVITY 1.1

Write a short paragraph on your experiences as a learner in a Mathematics class when you were at school. Write at least one good experience and one bad experience.

(5)

Own ideas

ACTIVITY 1.2

Write a paragraph to describe what you think **each** of the pictures (photos) represent. Do not rush through the activity. Write **as much as you can** about the situation depicted in each picture.

(6)

Own ideas:

Some students' opinions (students give their own impressions)

- 1 The teacher stands behind her table, all children listen passively to the teacher
- 2 Learners are engaged in a problem, where they have manipulatives (concrete apparatus which they can use to solve a problem)
- 3 Looks like a very old classroom set-up. One can almost smell the fear of the children – sitting on their hands!
- 4 Teacher listening attentively to learners
- 5 Learners sitting alone, grappling with mathematics – not a happy picture
- 6 I can feel the enthusiasm with which learners are listening to teachers. Learners actively engaged in the lesson

ACTIVITY 1.3

- 2 Look at the verbs related to the “doing” of mathematics in a mathematics classroom as mentioned in the text.

Use each of them in a sentence to relate them to the doing of mathematics.

(4)

Here are some verbs:

Compare	explain	predict	conjecture	explore	represent	construct
Formulate	solve	describe	investigate	verify	develop	justify

Own sentences that relate to the doing of Mathematics

ACTIVITY 1.5

Turn to page 18 of your prescribed textbook.
Read the instructions and answer the question:

- 1 *Who has the best chance of getting a red and a blue?*

Now answer the following questions:

- 2 *Was there a rule that you could follow to answer the question?*

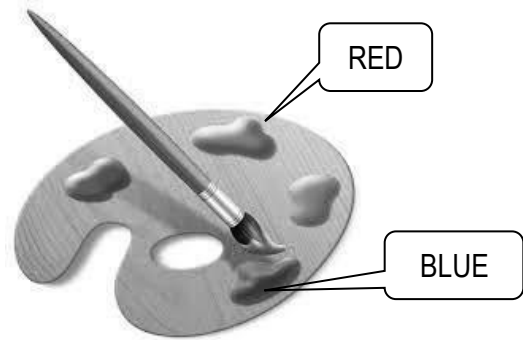
There was no "rule" to be followed

- 3 *Did you make up your own strategy?*

Own answer

- 4 *How do you feel about solving a problem without someone*

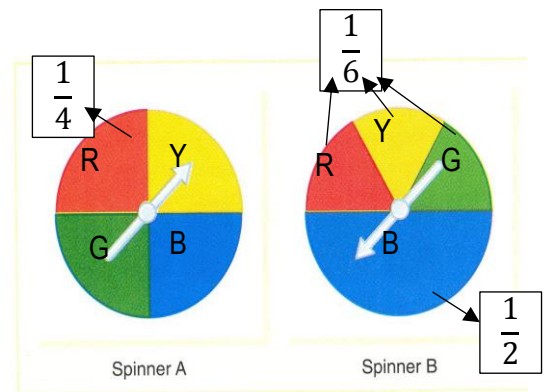
Own answer



If you mix blue paint with red paint, you will have purple.

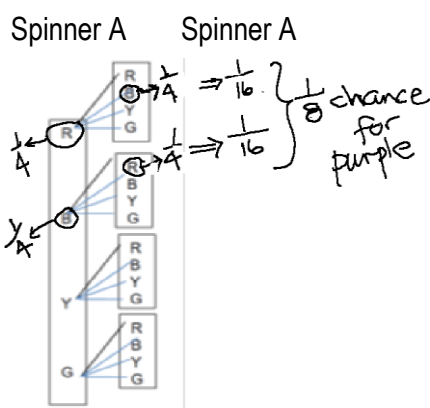
Here is the problem and some solutions

Three students are spinning to get purple with two spinners, either by spinning first red and then blue, or first blue and then red. They may choose to spin each spinner once, or one of the spinners twice. Mary chooses to spin twice on spinner A, John chooses to spin twice on spinner B, and Susan chooses to spin first on spinner A and then on spinner B. Who has the best chance of getting a red and a blue?

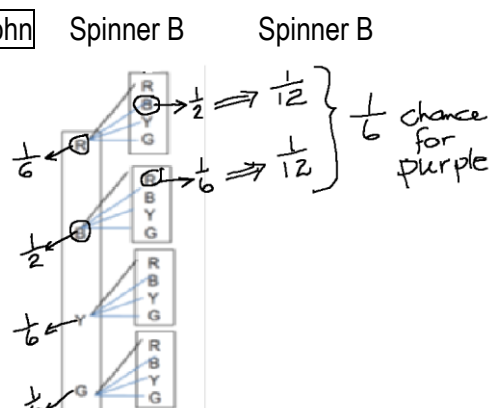


A solution

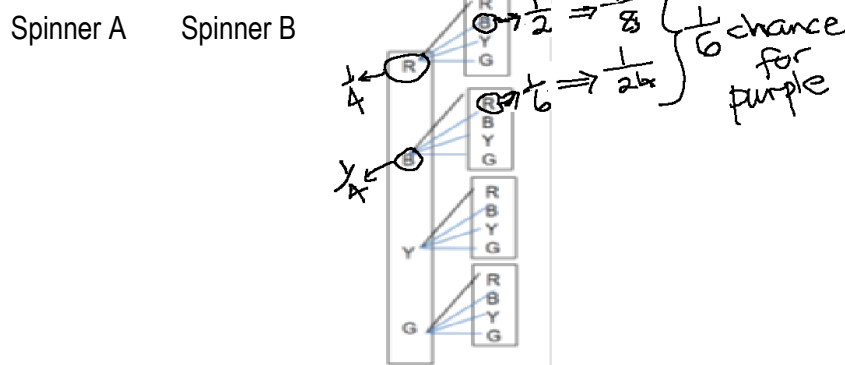
Mary



John



Susan



$$\begin{aligned} \frac{1}{8} + \frac{1}{24} \\ &= \frac{3+1}{24} \\ &= \frac{1}{6} \end{aligned}$$

NB: Another strategy is explained on page 19 of the text book.

If you use a square to indicate each of the possibilities as a fraction part, then it will look as follows:

MARY

SPINNER A

RED
BLUE
GREEN
YELLOW

SPINNER A

RED
BLUE
GREEN
YELLOW

SPINNER A overlaps with itself

RR	RB	RG	RY
BR	BB	BG	BY
GR	GB	GG	GY
YR	YB	YG	YY

The square is divided into 16 parts of equal size.

Two of the parts are both red and blue.

$$\frac{2}{16} = \frac{1}{8}$$

There is one-eighth chance of getting purple.

If you superimpose the one square (SPINNER A) over itself, you can see which parts overlap. Can you see the parts that are "purple"?

SUSAN

SPINNER A

RED
BLUE
GREEN
YELLOW

SPINNER B

BLUE
BLUE
BLUE
YELLOW
GREEN
RED

SPINNERS A AND B OVERLAPPING

BR	BB	BG	BY
BR	BB	BG	BY
BR	BB	BG	BY
YR	YB	YG	YY
GR	GB	GG	GY
RR	RB	RG	RY

The square is divided into 24 parts of equal size. Four of the parts are both red and blue.

$$\frac{4}{24} = \frac{1}{6}$$

There is one-sixth chance of getting purple.

If you superimpose the one square (SPINNER A) over the other (SPINNER B) you can see which parts overlap. Can you see the overlapping parts?

JOHN

If you superimpose the one square (SPINNER B) over a duplicate of itself you can see which parts overlap. Let us see which parts will be blue as well as red (BR):

SPINNER B

BLUE
BLUE
BLUE
YELLOW
GREEN
RED

SPINNER B

BLUE
BLUE
BLUE
YELLOW
GREEN
RED

SPINNER B overlaps with itself

BB	BB	BB	BY	BG	BR
BB	BB	BB	BY	BG	BR
BB	BB	BB	BY	BG	BR
YB	YB	YB	YY	YG	YR
GB	GB	GB	GY	GG	GR
RB	RB	RB	RY	RG	RR

The square is divided into 36 parts of equal size. Six of the parts are both red and blue.

$$\frac{6}{36} = \frac{1}{6}$$

There is one-sixth chance of getting purple.

ACTIVITY 1.6

(4)

1. **Assimilation:** It occurs when new concepts fits into an existing network of ideas. The new information expands an existing network.
2. **Accommodation:** It takes place when the new concept does not fit into an existing network of ideas. The brain has to revamp or reorganise the network
3. **Disequilibrium:** When new knowledge and existing knowledge do not match and the learner needs to modify the rearranged concepts and connections.
4. **Reflective thought:** to sift through existing ideas to find those that seem related to the new knowledge.

ACTIVITY 1.8

(3)

1. The learner borrowed a “ten” from the “hundreds” column
2. The learner borrowed 100 from the 600 and carried over as a 10 to the units
3. No. The learners does not understand the value of the ONE that he/she is borrowing.

ACTIVITY 1.9

(3)

Weaknesses of rote learning-your own words for the following:

- Learners do not always understand what they are learning. They will not enjoy the subject. They may perform poorly
- They forget what is taught, often because there are too many rules and algorithms to memorise. They are not able to apply their knowledge to a new situation.
- They do not know how this knowledge applies or links to the real world.
- The teacher who uses this method is more concerned about getting through the work than how and if the students are really learning.
- It is an instrumentalist form of learning. Learners are not encourage to link the new knowledge to existing knowledge.
- It is considered to be weak construction. The learner is not really building a useful network of ideas.

ACTIVITY 1.11

- 2 Explain why **relational understanding** has a far greater potential for promoting reflective thinking than **instrumental understanding**. (2)

Bookwork. See textbook and tut letter 501

Relational understanding is an understanding of what to do AND WHY. All concepts are in a connected network and when a learner wants to recall a concept, all the other related concepts are understood and can easily be recalled.

Instrumental understanding is on the opposite continuum of understanding. This is also called "doing without real understanding". The ideas are isolated, and cannot be connected (related) to other ideas. Instrumental understanding is related to the rote learning of concepts.

Give own conclusion why relational understanding promotes reflective thought.

ACTIVITY 1.13

(2)

- 2 A model for a mathematical concept is any object, picture or drawing that presents the concept or onto which the relationship for that concepts can be imposed. Examples can be blocks, bottle tops, graph, explanation etc.

ACTIVITY 1.14

(5)

1. Define each of the five behaviours and dispositions mentioned

Conceptual Understanding: Understanding mathematical concepts operations and relations

Procedural fluency-being able to carry out mathematical procedures flexibly, accurately and efficiently

Productive disposition- to see mathematics as sensible, useful and worthwhile.

Strategic competence-able to formulate, represent and solve mathematical problems

Adaptive reasoning- logical thought, reflection, explanation and justification

2. Benefits of Mathematical Proficiency

Learners learn new concepts more effectively

There is less to remember

Increased retention and recall

Enhanced problem solving abilities

Improved attitudes and beliefs

ACTIVITY 2.1

(3)

Distinguish between the following three approaches:

- **Teaching for problem-solving:** Teaching a skill that a student can later solve a problem using this skill
- **Teaching about Problem-solving:** Teaching students how to solve problems. Giving them general strategies (draw a picture) for solving problems

- **Teaching through problem solving** : Learners will learn mathematical concepts through solving problems

ACTIVITY 2.6

- 1 What does it mean to say that a task has multiple entry points?

A problem where different strategies can be used to find a solution, is called a problem with multiple entry points

(1)

ACTIVITY 2.7

- 1 Give reasons why there should be a shift in the thinking about mathematics teaching.

Think about:

Mathematics should not been seen as learning a set of rules and procedures. A variety of methods to solve the same problem should be encouraged. Learners should be given good problems and not just repetitive exercises of the same kind.

- We want to move away from procedural knowledge to conceptual understanding of mathematics.
- Learners should do less rote learning of rules and become more involved in making sense of concepts
- There should be more classroom discussion to enhance understanding
- Learners have to be given the opportunity to voice their opinions about how they understand and solve problems
- Teacher must LISTEN to learners to understand how they think and reason
Learners should not be criticized for making mistakes.

- 2 Use your own words to describe the teacher's actions in the *before*, *during* and *after* phases of a problem-solving lesson. USE OWN WORDS

(6)

Before ⇒ Getting ready

- Activate prior knowledge
- Be sure the problem is understood
- Establish clear expectations

During ⇒ Students work

Let go!

- Notice students' mathematical thinking
- Provide appropriate support
- Provide worthwhile extensions

After ⇒ Class discussion

- Promote math community
- Listen actively
- Summarize main ideas

PART C

These questions relate to Chapter 11 - 13 of the prescribed book:

- 1 Name the three ways one can count a set of objects, and explain how these methods of counting can be used to combine concepts and written names for numbers. (6)

See page 206 van de Walle

- 2 How can learners learn to write two- digit numbers in a way that it is connected to the base 10 meaning of ones and tens? (4)

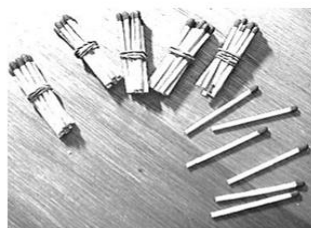
Learners can use a variety of base ten apparatus (manipulatives). They will only need units and tens concrete models. They can use the beans as illustrated in your tutorial letter 501, or matches or Dienes blocks. You can make your own Dienes blocks (see end of tutorial letter 501 for templates to cut out). The idea is that learners understand that ten ones make 10, and then the place value of the digit in the tens place means tens (or ten ones)

See tutorial letter 102 page 7

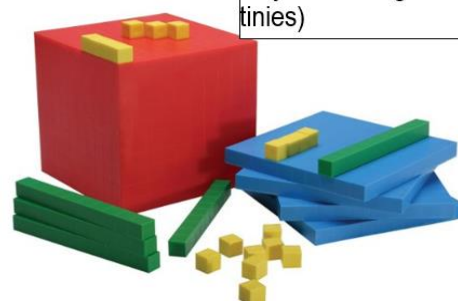
Beans – see packaging



Match sticks – bundles of 10



Dienes blocks



(For 2 digit numbers only use longs and tinies)

Describe in your own words

NB: You get groupable models and pre-grouped models. With groupable models, children can arrange the counters to suit their own counting but with pre-grouped models children cannot take pieces apart or group them

- 3 Show a way in which a hundreds chart can aid the teaching of addition of two digit numbers (2)

100s Chart									
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- In the rows, the unit digits grow by 1 as you move from left to right.
- The last column has all unit digits as zero, but the tens digits increase by one as you move down.
- In the columns, the tens digit grows by one as you go down.
- The last number in the chart is 100

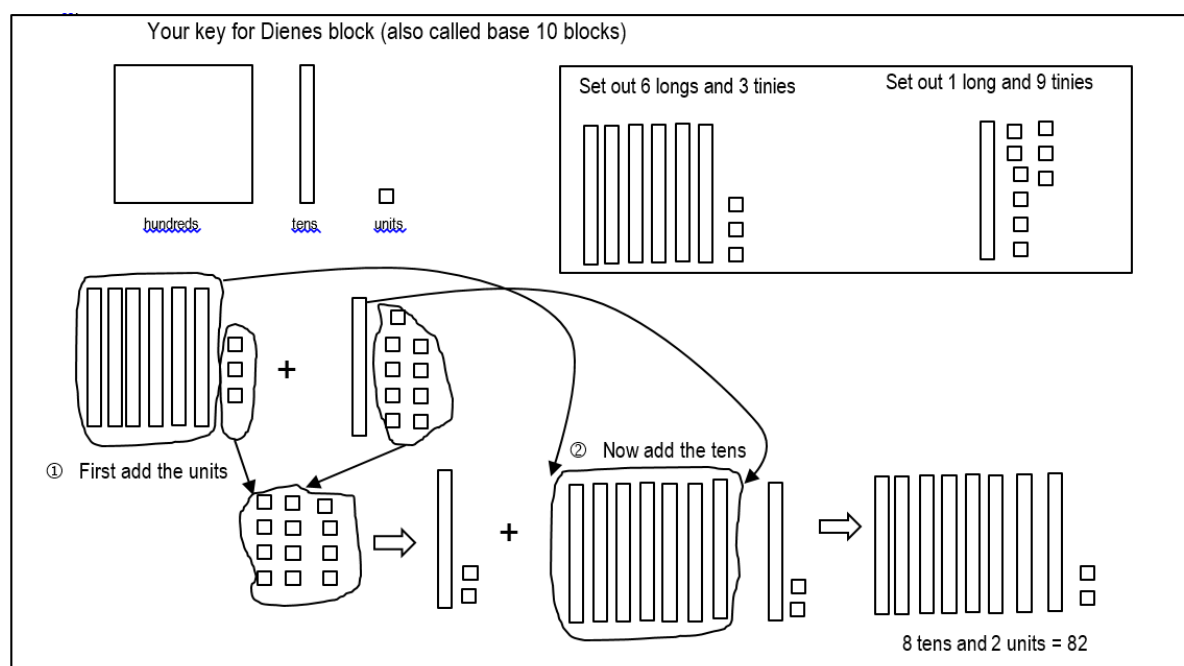
H	T	U
1	0	0

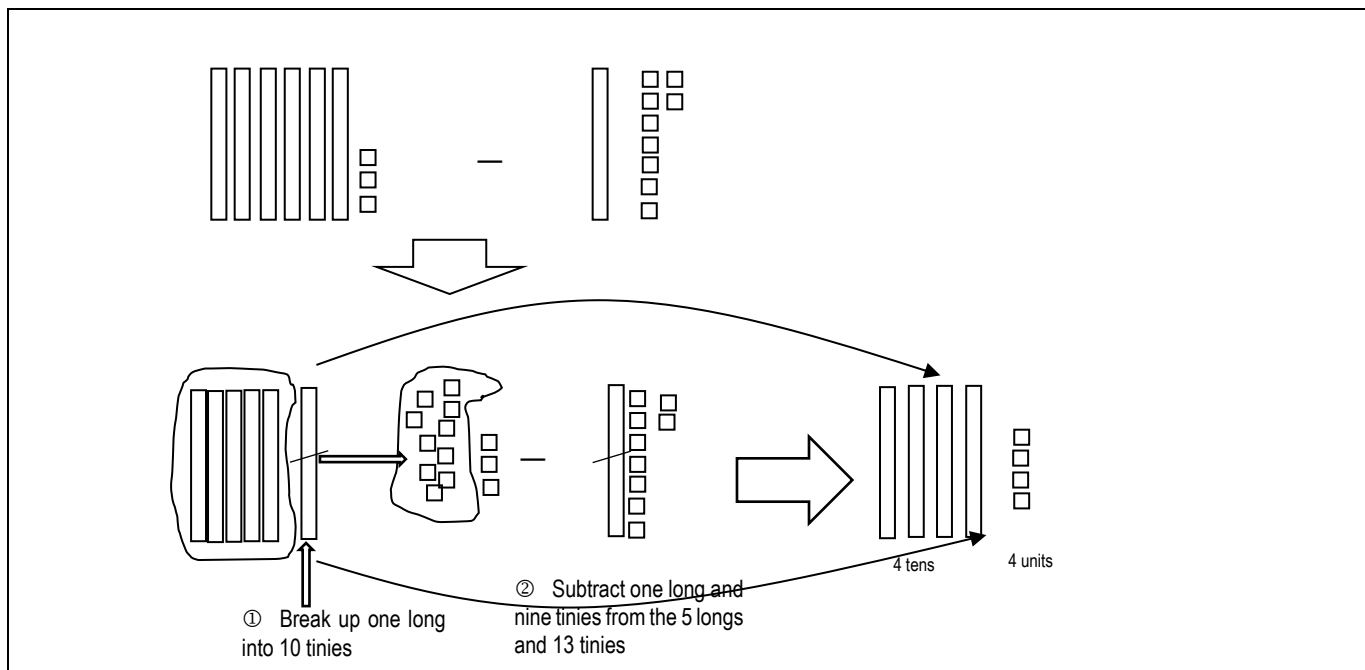
Example: $27 + 40 + 2 = 69$

Start from 27 and move four steps downwards (adding 4 tens) and two steps to the right (adding two units (ones)). You will therefore land in 69 which is the answer

4 Draw Dienes block to show how to find the solution to:

(a) $63 + 19$



b) $63 - 19$ 

5 Test the following numbers for divisibility by 8, 9, and 11. (Do not factorise)

a) 904 904 904

For 8: We look at the last 3 digits $904 \div 8 = 113$. So 8 will go into 904 904 904For 9: Add the digits: $9+4+9+4+9+4 = 39$, which is not divisible by 9, so 904 904 904 is not divisible by 9 (but it is divisible by 3)For 11:① Add every second digit: $9+4+0+9+4 = 26$ ② Add the other digits $0+9+4+0 = 13$ ③ Subtract: $26 - 13 = 13$ (not 11 or 0)904 904 904 is not divisible by 11

b) 13 666 653

Not divisible by 8 (it is an odd number)

Divisible by both 9 and 11 (test for both)

 $13\,666\,653 = 3 \times 3 \times 7 \times 11 \times 13 \times 37 \times 41$ (seven prime factors)

(6)

6 Select the prime numbers from the list below:

(2)

a) 103 Prime

b) 3367 $7 \times 13 \times 37$ Not Prime

c) 97 Prime

d) 187 11×17 Not Prime

- 7 Use the vertical and horizontal algorithms to find the sum or difference of: (Explain the "borrow and carry" concepts)

(4)

a) $784 + 847$

② This small 1 here is 1 hundred

① Add the units: $4 + 7 = 11$. This small 1 here is 1 ten

17	8	4
+	8	4
+	7	
1	6	3

$$\begin{aligned}
 784 + 847 &\Rightarrow 700 + 80 + 4 + 800 + 40 + 7 \\
 &= 700 + 800 + 80 + 40 + 4 + 7 \\
 &= 1\,500 + 120 + 11 \\
 &= 1\,000 + 500 + 100 + 20 + 10 + 1 \\
 &= 1\,000 + 600 + 30 + 1 = 1\,631
 \end{aligned}$$

Some steps may be left out

b) $847 - 784$

① Subtract 4 from seven

② Now borrow 100 from the 800. The 40 now becomes 140

~~7~~ 8	~~14~~ 4	7
-	7	8
		4
	6	3

- 8 How are standard algorithms different from student-invented strategies? Explain the benefits of invented strategies over standard algorithms (give at least two valid points).

(2)

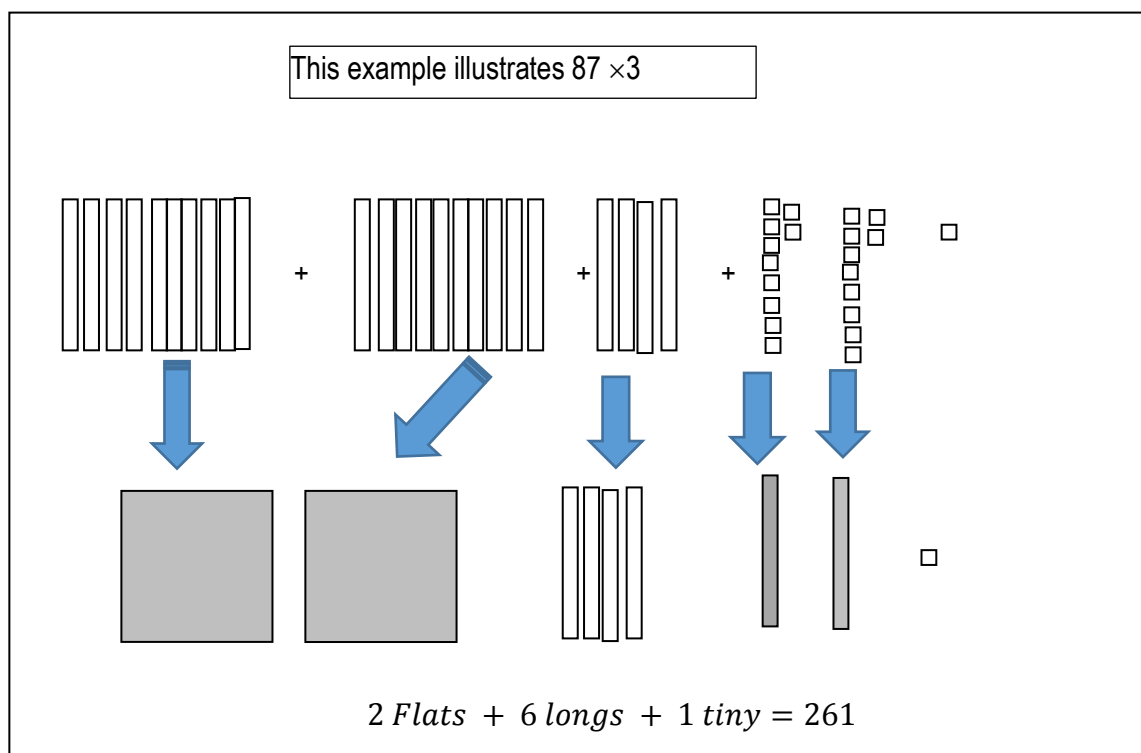
An algorithm is a rule that can be used; a procedure that is carried out which is usually learned by rote, with little understanding. Invented strategies are methods which learners produce themselves. It is usually built on concepts that the learners knows already. One often finds that learners invent their own strategies in the way that they understand, and not necessarily the way in which the teacher explains.

- 9 Draw pictures showing how 87×3 could be modelled: with counters, with base – ten blocks, or with areas on base 10 grids. (Use at least two of the methods)

(5)

Base 10 blocks

Set out 8 longs and 7 tinies (3 times)



- 10 What would you do if your learners persist on using repeated subtraction for division /repeated addition for multiplication problems without really doing any division or multiplication?

(3)

Let learners use their preferred method of calculation for as long as they are comfortable with it. It is not wrong to use repeated addition/subtraction when large numbers are involved. However, when learners use repeated addition/subtraction to calculate big numbers like 247×7 or $1729 \div 7$ then it becomes tedious, and learners could be guided to use a different strategy.

Example on multiplication:

$$\begin{array}{r} 200 \times 7 = 1\,400 \\ 40 \times 7 = 280 \\ 7 \times 7 = 49 \\ \hline 1\,729 \text{ (add the products)} \end{array}$$

NB: Think of a strategy that you could use for calculating problems on division

[40]

PART D

These questions relate to Chapter 15 – 17 of the prescribed book:

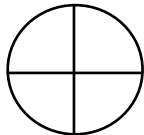
- 1 Sharing and “equal sharing” (fare sharing): what is the difference? (2)

Own work/explanation

- 2 Give examples of each of three categories of fractions: [Illustrate with diagrams]

2.1 Area model

Continuous whole (area model)



Example" A pizza divided into four equal parts

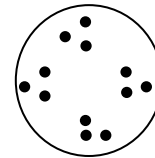


2.2 Set model

Discrete whole (set model)

Veronica shares 12 sweets between her and 3 friends (4 people). Each friend gets one-quarter of the 12 sweets (3 sweets)

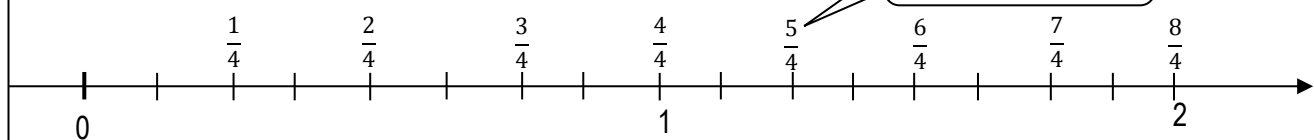
Twelve sweets divided by four people – loose objects



2.3 Length model

Length model (number line) Paper strips can also be used as length models

Example



- 3 In the context of choosing a “whole”, explain when a “fifth” is not always equal to a “fifth”.

Give an example.

(3)

When thinking of a fraction as a part of a whole, the part will depend on the size of the whole. For example, one fifth of an apple is not the same as one fifth of a paw-paw. So when we contextualise fractions as a part of "something", this "something" has to be exactly the same. In the world of numbers, when we talk about a third, we always mean a third of ONE. (One unit)

4 Use the correct language to explain the difference between the expressions:

"It is the third of March"

This means the third day of the month of March (ordinal number).

"You are third in the row"

This means if there is a row of people, you are the third person from the beginning (ordinal number).

"I want a third of the pizza"

This means if you divide a pizza in three equal parts, each part is one third of the whole (fraction).

What types of numbers are we dealing with in the above expressions?

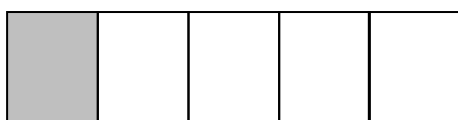
(4)

SOME NUMBERS TO NOTE

- A cardinal number tells "how many." Cardinal numbers are also known as "counting numbers," because they show quantity. Cardinal numbers are whole numbers, so fractions are not cardinal numbers.
- Ordinal numbers tell the order of things in a set—first, second, third, etc. Ordinal numbers do not show quantity. They only show rank or position.
- A nominal number names something. Nominal numbers do not show quantity or rank. They are used only to identify something (e.g. a postal code).

5.1 If the given rectangle represents one whole

- Show one fifth of the whole



- Show two thirds of the whole



- Show five thirds of the whole

(3)

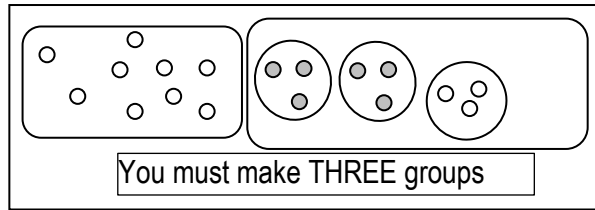


5.2 If nine counters are a whole set, how many are there in two -thirds of the set?

Answer: ____6

(2)

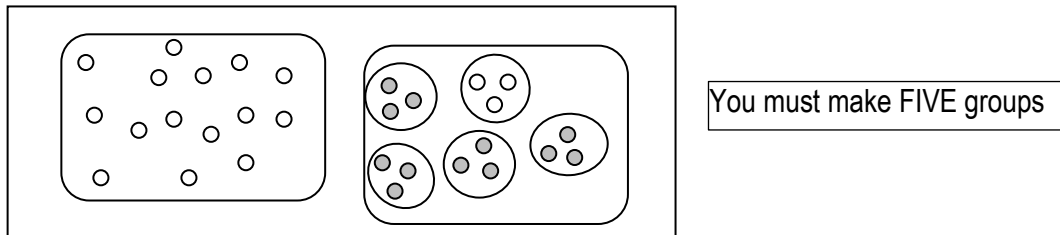
SOLUTION



5.3 If 15 counters are a whole set, how many counters would make four-fifths?

Answer: ____ 12

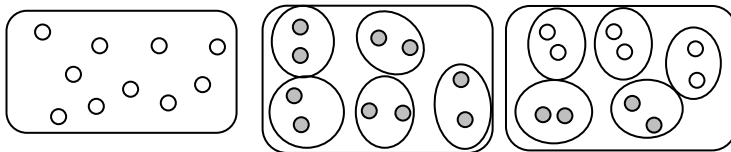
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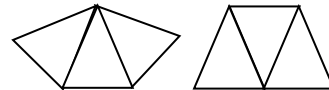
5.4 If 10 counters are a whole, how many are in seven-fifths of the set?

Answer: ____ 14

(2)

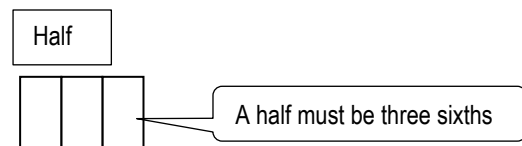
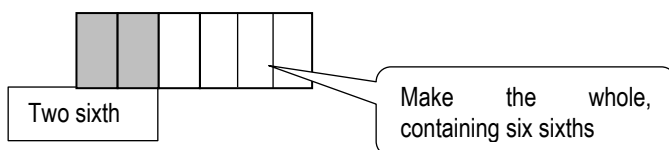


5.5 If this triangle is one third of the whole, what would the whole look like?



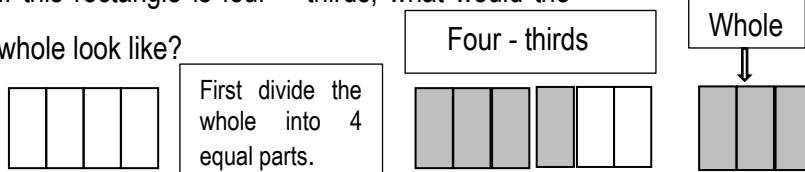
(2)

5.6 If this rectangle is two-sixths of the whole, draw a shape that could be the half.



(2)

5.7 If this rectangle is four – thirds, what would the whole look like?



(2)

6 Use the following models to show the equivalence of the fractions $\frac{2}{5}$ and $\frac{4}{10}$

6.1 a set model

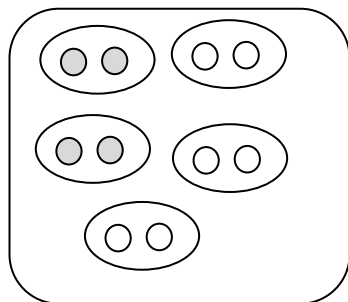
6.2 an area model

6.3 the number line

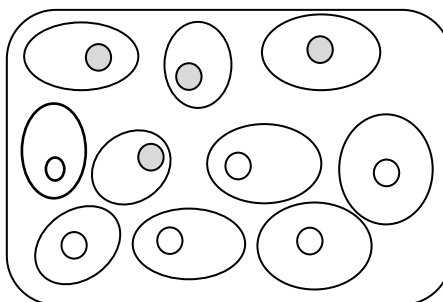
(9)

6.1 A set model

Choose the whole to be 10 marbles (loose objects) Can you choose another whole?



The 5 in the $\frac{2}{5}$ tells you into how many parts the whole must be divided



The 10 in the $\frac{4}{10}$ tells you into how many parts the whole must be divided

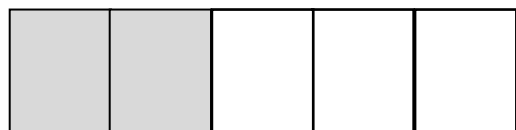
The 2 in the $\frac{2}{5}$ tells you how many parts you must choose:
Choose 2 parts of 2 marbles each,
which is 4 marbles

The 4 in the $\frac{4}{10}$ tells you how many parts you must choose:
Choose 4 parts of 1 marble each,
which is 4 marbles

Remember to use
the same whole

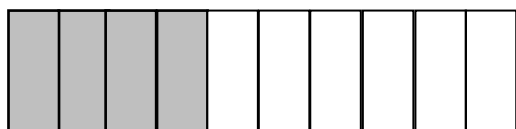
6.2 Area model

Let us choose a rectangle as the whole



For $\frac{2}{5}$

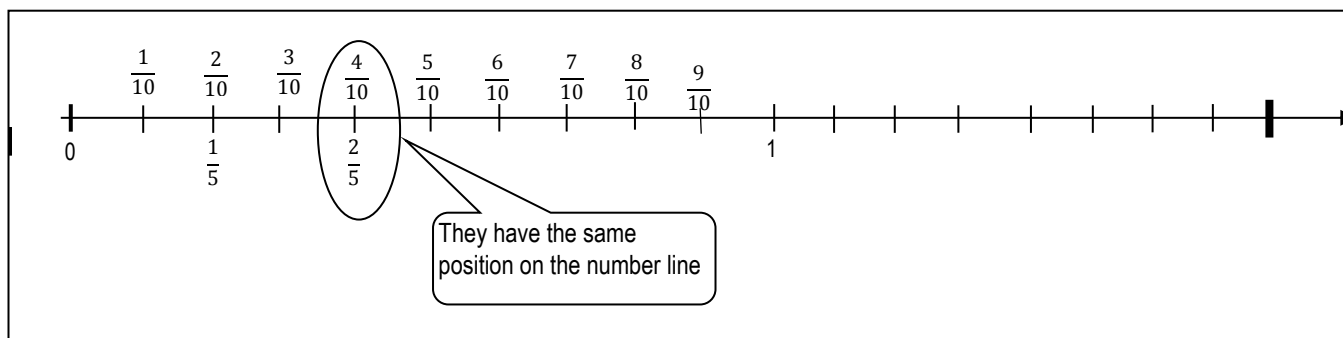
Divide the whole into 5 equal parts
and shade 2



For $\frac{4}{10}$

Divide the whole into 10 equal parts
and shade 4

6.3 The number line



- 7 Compare the fractions $\frac{3}{4}$; $\frac{2}{3}$ and $\frac{5}{8}$ by making use of the area model. (You should be able to arrange the fractions from small to big). Make sure that you choose the whole correctly, and make accurate drawings.

(6)

You will have to choose a whole into which all the denominators can divide, so that you can divide the whole into 3, 4 as well as 8 equal parts.

I am choosing a rectangle with length of 12 cm. (The number that 4, 3 and 8 can divide is 24, but that will be too long for the page, so I choose a length of 12cm. You must use a ruler to divide the parts accurately.

Remember to use the same whole

$\frac{3}{4}$

$\frac{2}{3}$

$\frac{5}{8}$

$$\frac{5}{8} < \frac{2}{3} < \frac{3}{4}$$

- 8 Use a standard algorithm to calculate: $2 - \frac{1}{3} - \frac{1}{6}$ (2)

- ① Make the denominators the same. Choose a number that 3 and 6 can divide into (6)
- ② Now write all the fractions with denominators of 6 $2 = \frac{12}{6}$ $\frac{1}{3} = \frac{2}{6}$ $\frac{1}{6} = \frac{1}{6}$
- ③ $2 - \frac{2}{6} - \frac{1}{6}$
- $$= \frac{12}{6} - \frac{2}{6} - \frac{1}{6}$$
- $$= \frac{12-2-1}{6} \quad \text{Put over the same denominator}$$
- $$= \frac{9}{6} \text{ or } \frac{3}{2} \text{ or } 1\frac{1}{2}$$

9. Mama Matebane bought 3 Bar-Ones for her two children and their two friends to share fairly between them. What fraction of a whole Bar-One will each child get? Illustrate the solution with drawings.



(5)

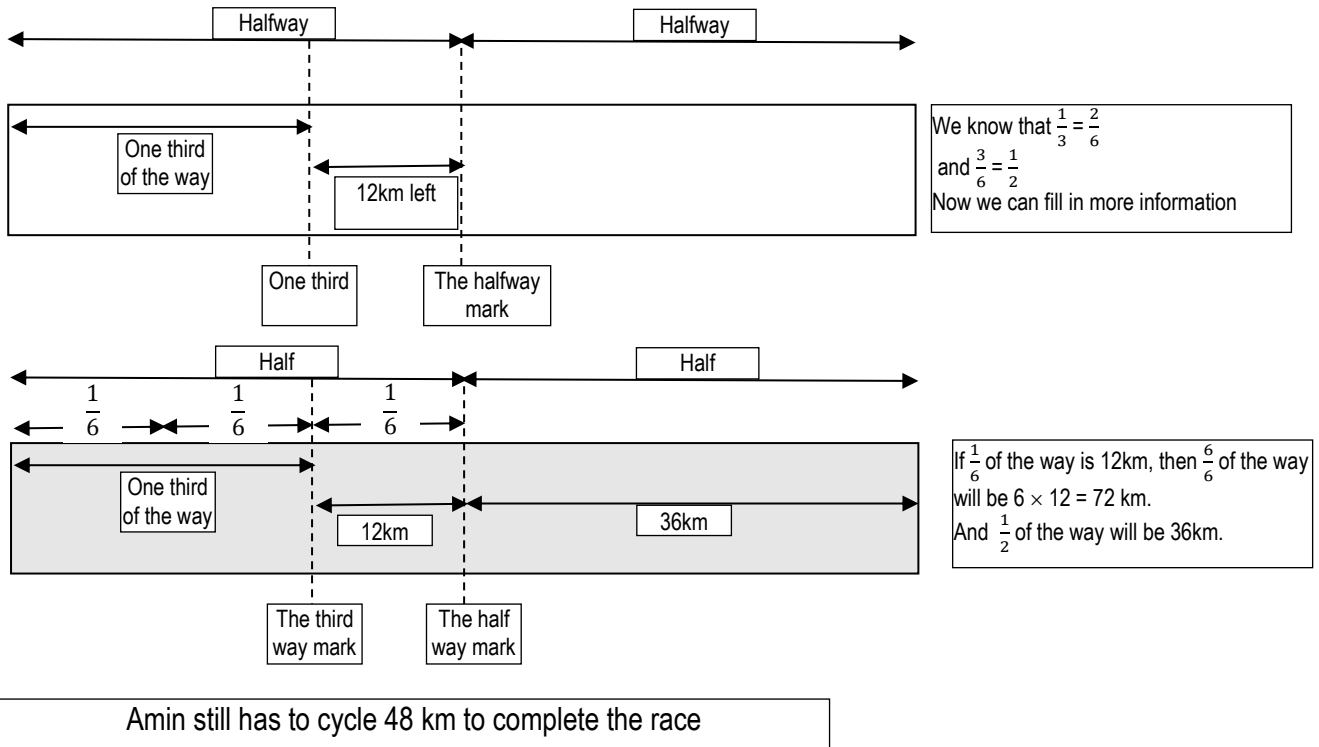
THREE Bar-ones must be divided for FOUR children

Divide three bar ones into four parts each 	Each child gets a quarter (one fourth) of three Bar ones 	⇒	So each child will get three quarters of a whole 
$\frac{1}{4} \text{ of } 3 = \frac{3}{4} \text{ of } 1$			

- 10 Amin has completed one-third of his cycling race. He is still has 12km to go to reach the halfway mark. How may kilometres he still has to cycle to finish? [Use a diagram to help you] (6)

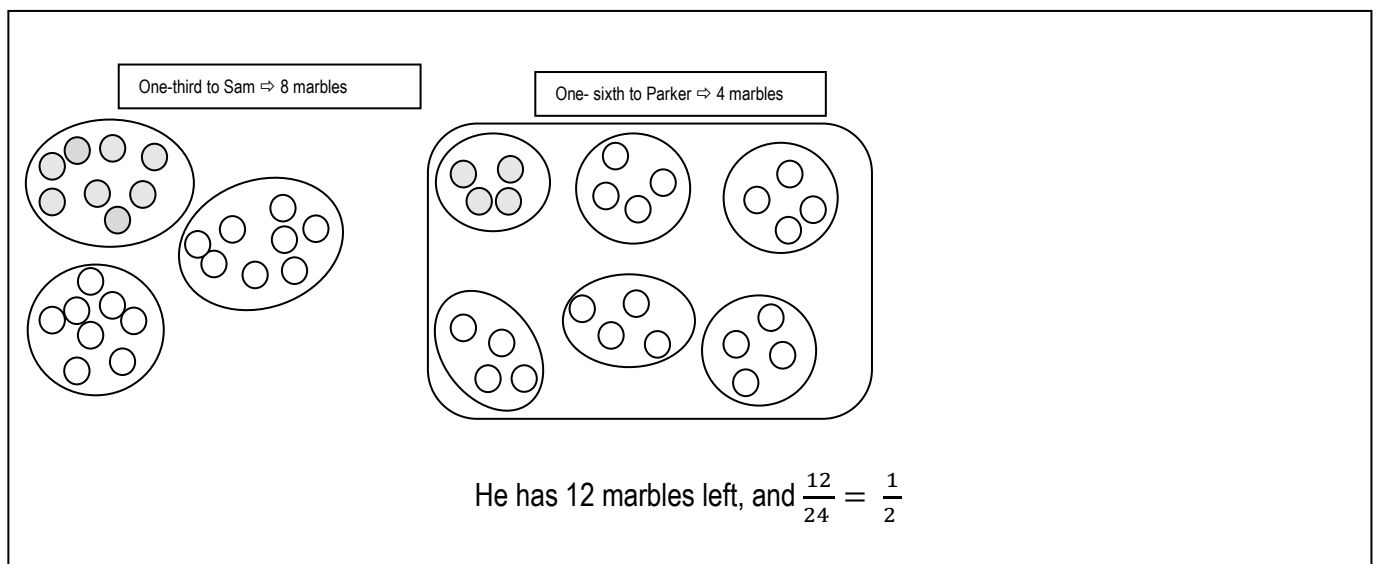
Let us use a rectangle to model the entire race.

We know that:



11 Use diagrams to illustrate and explain how you will solve the problem:

11.1 Sally has a bag of 24 marbles. She gave one –third of them away to Sam and one-sixth of them to Parker. What fraction of her marbles does she has left? How many marbles it that? (4)

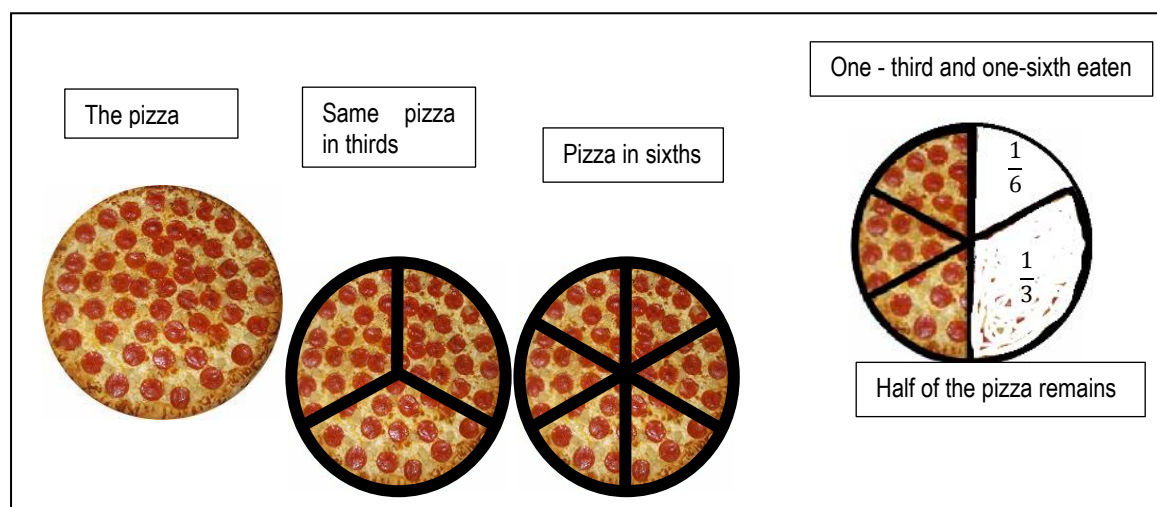


Using an algorithm to find what fraction he has left:

$$\begin{aligned}
 1 - \frac{1}{3} - \frac{1}{6} \\
 &= \frac{12 - 4 - 2}{12} \\
 &= \frac{6}{12} \\
 &= \frac{1}{2}
 \end{aligned}$$

You can also use 6 as the denominator

- 11.2 Vuyi got a huge pizza to share with her two brothers. Jamie, the youngest got one-sixth of the pizza, and Mpo, the oldest, got one-third. What part of the original pizza is left for herself? (4)

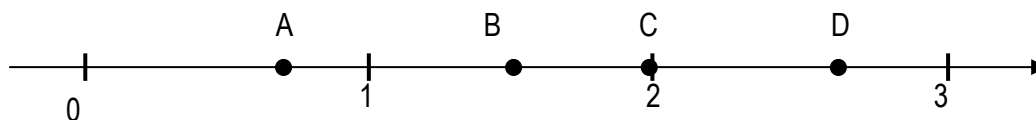


The algorithm: $1 - \frac{1}{3} - \frac{1}{6} = \frac{6 - 2 - 1}{6} = \frac{3}{6} = \frac{1}{2}$

- 12 The following question was given to Grade 4 learners:

Which point on the number line shows $\frac{2}{3}$?

Draw a circle around the correct letter.



Nine percent of the learners chose A as the answer, 14% chose B, 43% chose C and 34% chose D.

- 12.1 What is the correct answer? (1)

Correct position of $\frac{2}{3}$ is at A (only 9% of learners had it correct)

12.2 Give an explanation about the misconceptions learners had when they chose the incorrect answers?

(3)

Possible misconception if learners chose B

Learner could have thought that the 2 indicated the whole, and saw $\frac{2}{3}$ as a part of that length

Possible misconception if learners chose C

This was the most popular incorrect response. Learners mistakenly took the whole number line (from 1 to 3) as the whole, and then saw $\frac{2}{3}$ as the position of C

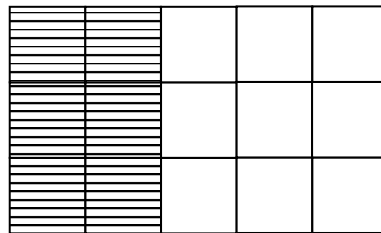
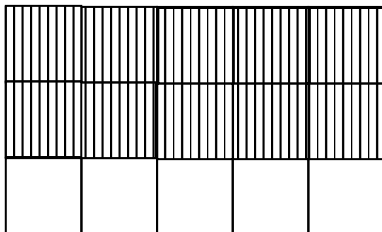
Learners who chose D, possibly thought of $\frac{2}{3}$ as a number between 2 and 3.

13 Use the grid below to illustrate: $\frac{2}{5}$ of $\frac{2}{3}$

(4)

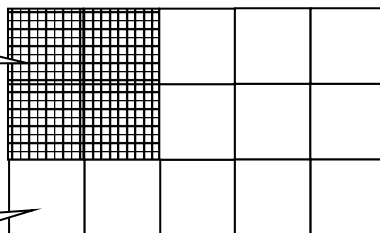
① Shade $\frac{2}{3}$ of the whole like this: 

② Shade $\frac{2}{5}$ of the whole like this: 



③ The section that overlaps is shaded like this:

$\frac{4}{15}$ of the whole is double shaded



This shows $\frac{2}{5}$ of $\frac{2}{3}$ of the whole

Each little block is $\frac{1}{15}$ of the whole

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PART E

These questions relate to Chapter 20 of the prescribed book:

- 1 Describe in your own words the first three *van Hiele* levels of Geometric thought (level 0, 1 and 2). How do the activities that you will give learners on the three levels differ? (6)

Own words See also tut letter 501 page 91

LEVEL 0 Visualization

Learners can classify shapes according to what they look like. They will see a rectangle as the same as a "door". They can distinguish between a rectangle and a square, but only by what the shape looks like and not according to their properties.

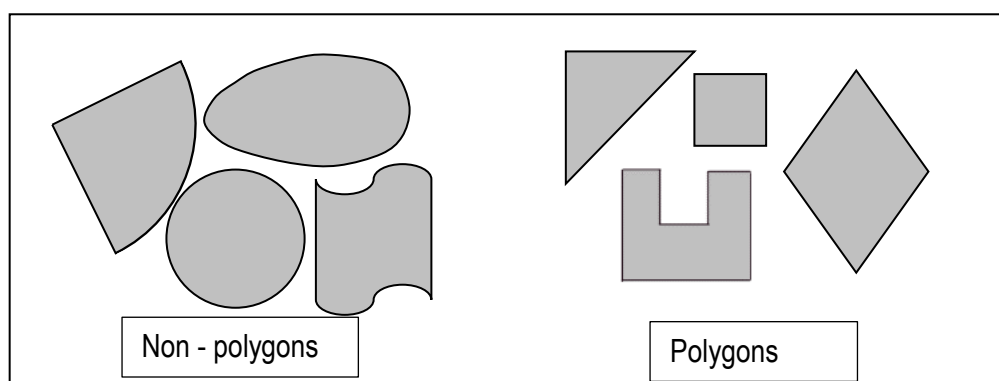
LEVEL 1 Descriptive/Analysis

Learners can describe shapes according to their properties, for example, a rectangle has equal opposite sides, and all right angles. They can group shapes according to characteristics, but do not yet know how to identify a shape if a set of properties are given. They would usually describe a shape by naming ALL the properties of that shape. For example, they will say a rectangle has equal and parallel opposite sides, equal diagonals, equal angles etc. In other words they do not yet see the minimum requirements for a shape to be identified.

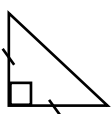
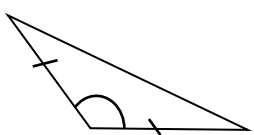
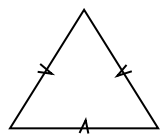
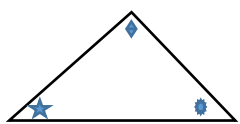
LEVEL 2 Abstract/Relational/Informal Deduction

Learners at this level understand the relationships between shapes. They can now identify the minimum requirements for a shape to be identified. So they know that a rectangle is a parallelogram with one right angle, because they understand that all the other properties of a rectangle are included in that definition. They can also identify a square as a kite, because a square has all the properties of a kite.

- 2 Draw an assortment of different types of flat shapes that include polygons and non-polygons (at least three of each kind). Explain your classification. Why is this type of exercise necessary for learners on *van Hiele* level 0? [own explanation] (8)

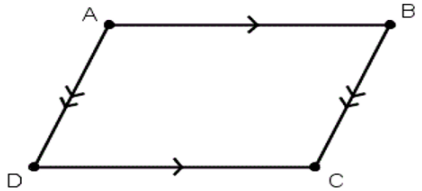


- 3 Draw the following triangles. Your sketches must be done with a ruler and very neatly drawn. (4)

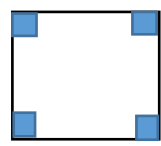
3.1 Isosceles, right angled triangle	
3.2 Isosceles, obtuse angled triangle	
3.3 Equilateral triangle All sides equal	
3.4 Acute angled triangle Each angle is less than a right angle	

- 4 Which quadrilaterals are described by the following characteristics? Make a neat drawing of each. (Do not assume properties that are not given)

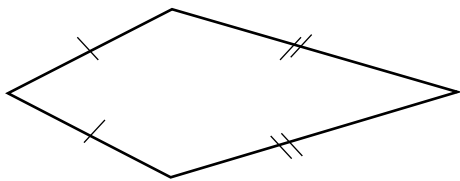
- 4.1 Two pairs of parallel sides

Parallelogram	
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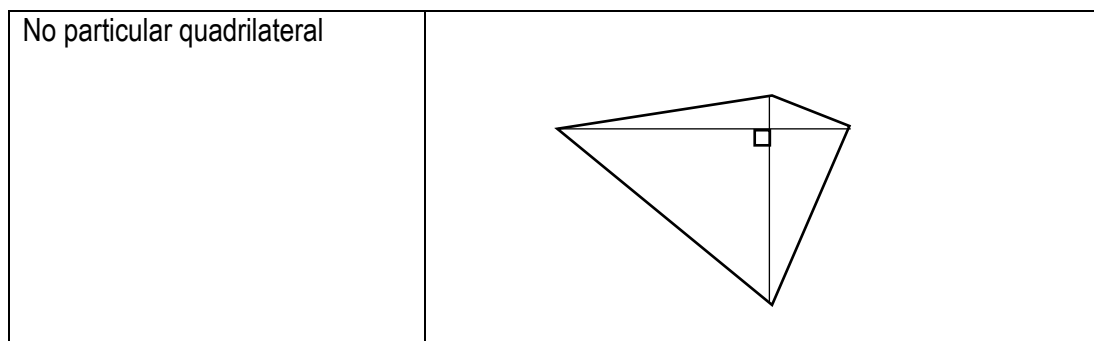
- 4.2 A rhombus with all angles equal

Square	
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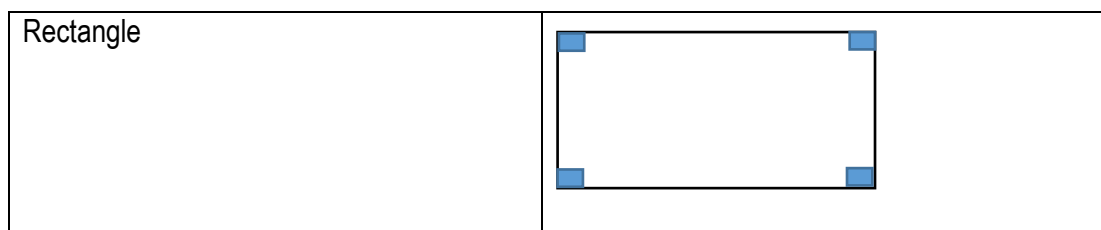
- 4.3 Two pairs of equal adjacent sides

Kite	
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4.4 Diagonals perpendicular



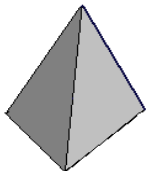
4.5 All angles equal

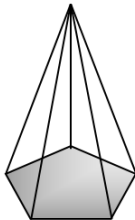
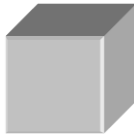


(10)

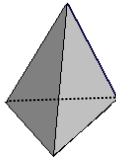
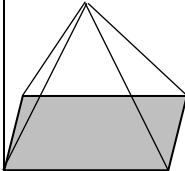
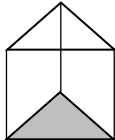
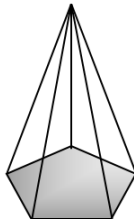
5 Classify the following shapes: polyhedrons or non-polyhedrons. Name each of the objects (a mathematical name)

(11)

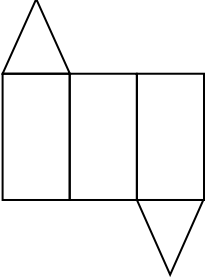
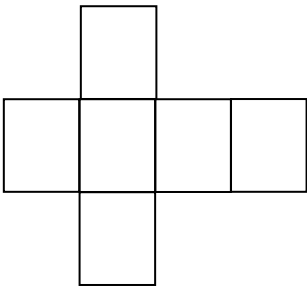
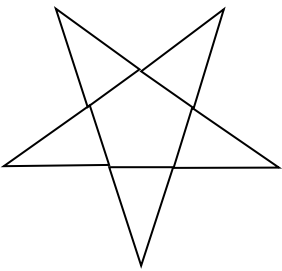
1  P: Tetrahedron (triangular pyramid)	6  P: Hexagonal prism
2  NP: Ellipsoid	7  NP: Cone
3  NP: Sphere	8  P: Square pyramid
4  NP: Cylinder	9  NP: Torus

5  P: Pentagonal pyramid	10  P: Cube
	11  NP: Ovoid

6 Draw neat diagrams of the following polyhedra, made up of: (Name the polyhedra) (8)

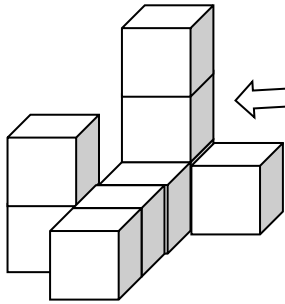
6.1 Four triangular faces	
6.2 One rectangular and four triangular faces	
6.3 Two triangles and rectangles	
6.4 A hexahedron which is a pyramid	

7 Draw the nets of the following polyhedra:

7.1	Triangular prism	7.2	Cube	7.3	Pentagonal pyramid
					

(6)

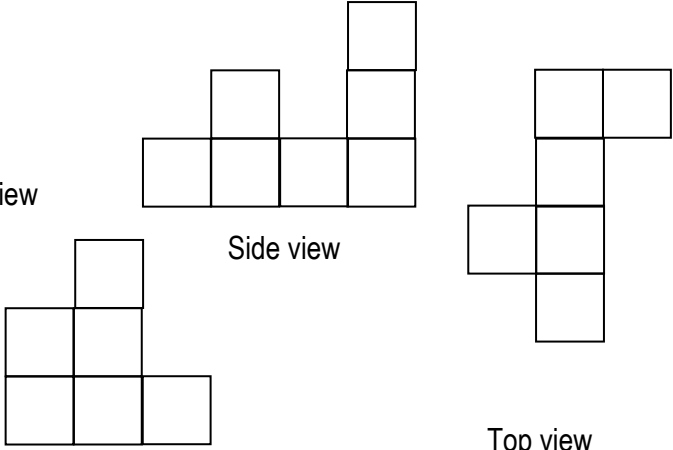
8 Draw the front, top and side view of the following structure



Top view

Side view

Front view



Side view

Top view

(3)

[56]