NATIONAL EDUCATIONAL ASSESSMENT AND EXAMINATIONS AGENCY
(NEAEA)
ETHIOPIAN SECONDARY SCHOOL LEAVING CERTIFICATE EXAMINATION
(ESSLCE)
MATHEMATICS FOR NATURAL SCIENCE STREAM
TIKIMT 2013/14 NOVEMBER 2021

BOOKLET CODE: 29 **SUBJECT CODE: 82**
Number of Items: 65 **Time Allowed: 3 hours**

D 1. Given two vectors $u = (1, 3)$ and $v = (-3, 5)$ in the plane. Then $6u + 2v$ is equal to

A. $(0, 18)$ C. $(2, 28)$

B. $(3, 3)$ D. $(0, 28)$

Soln: $6u+2v = (6,18)+(-6,10) = (6-6, 18+10) = (0,28)$

B 2. Which one of the following is the standard form of the equation of the circle centered at $(2, -3)$ and radius 5?

A. $(x-2)^2+(y-3)^2=25$ C. $(x-2)^2+(y+3)^2=5$

B. $(x-2)^2+(y+3)^2=25$ D. $(x+2)^2+(y-3)^2=25$

Soln: $(x-2)^2+(y-(-3))^2 = 5^2$
$\Rightarrow (x-2)^2+(y+3)^2 = 25$

A 3. If f is a continuous function on $[a, b]$ and F is its anti-derivative, then which of the following is true?

A. $\int_a^b F'(x)dx = f(b)-f(a)$ C. $\int_a^b f(x)dx = F(b)-F(a)$

B. $\int_a^b F(x)dx = f(b)-f(a)$ D. $\int_a^b f'(x)dx = F(b)-F(a)$

Soln: $f(x) = F'(x) \Rightarrow \int_a^b f(x)\,dx = \int_a^b F'(x)dx \Rightarrow \int_a^b f(x)dx = F(x)\Big|_a^b = F(b)-F(a)$

A 4. Which of the following defines the equation of a sphere whose center is at $(0, -1, 2)$ and radius 6 units?

A. $x^2+(y+1)^2+(z-2)^2=36$ C. $x^2+(y+1)^2+(z-2)^2=12$

B. $x^2+(y-1)^2+(z-2)^2=36$ D. $x^2+(y+1)^2+(z+2)^2=36$

Soln: $(x-0)^2+(y-(-1))^2+(z-2)^2 = 6^2$
$\Rightarrow x^2+(y+1)^2+(z-2)^2 = 36$

5. Which of the following represents the vector (3, –5, 7) in the space using the standard unit vectors i, j and k?

A. $3i - 5j - 7k$ C. $3i - 5j + 7k$

B. $3i + 5j + 7k$ D. $-3i - 5j - 7k$

Soln:- $3i-5j+7k$ in its Unit Vector form.

6. Which one of the following is the scalar (dot) product of the vectors (2, –4, 6) and (–2, –1, 0)?

A. 0 C. 8

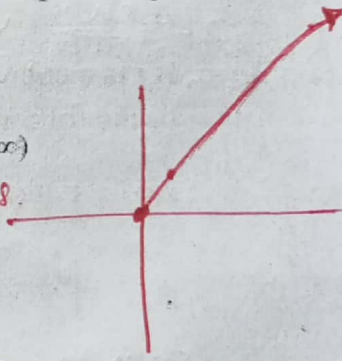
B. –8 D. 6

Soln $U \cdot V = x_1x_2 + y_1y_2 + z_1z_2 = 2(-2) + -4(-1) + 6(0)$.
$= -4+4+0 = 0$

7. What are the domain and range of the function $f(x) = 2x^{\frac{3}{4}}$ respectively?

A. $\mathbb{R}$ and $[0,\infty)$ C. $[0,\infty)$ and $\mathbb{R}$

B. $[0,\infty)$ and $(0,\infty)$ D. $[0,\infty)$ and $[0,\infty)$



Soln:- $f(x) = 2x^{3/4}$, when $x \geq 0$, $y \geq 0$, thus.
domain $= [0,\infty)$.
Range $= [0,\infty)$

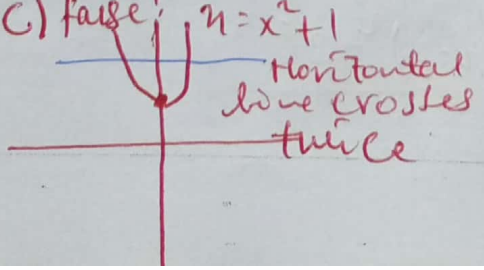
8. Which one of the following defines a one-to-one function?

A. $f = \{(x, y): x \text{ is a sister of } y\}$ C. $f = \{(x, y): y = x^2 + 1\}$

B. $f = \{(x, y): y = 3x - 1)\}$ D. $f = \{(x, y): y \text{ is a father of } x\}$

Soln: A) false: because some one may have more than one sister
B) True: $y = 3x - 1$ is a linear function its graph is straight line and any horizontal line touches the graph more than once.

C) false: $y = x^2 + 1$



D) false: A father has more than one child.

9. The equation of a circle is given by $(x-3)^2+(y+2)^2=9$. What are the center and the radius of the circle respectively?

A. (3, –2) and 3 C. (3, –2) and 9

B. (3, 2) and 3 D. (3, 2) and 9

Soln: Center = (3,-2) and radius = r = $\sqrt{9}$ = 3

10. Which one of the following sentences has a truth value?

A. Man is selfish. C. Man is mortal.

B. We shall win! D. Tomorrow is a beautiful day.

Soln: man is mortal is a statement because it is a true sentence but A, B and D are not a statement.

11. What is the determinant of the matrix $\begin{bmatrix} 1 & -7 \\ 6 & 5 \end{bmatrix}$?

A. –37 C. 47

B. –47 D. 37

Soln: $\begin{vmatrix} 1 & -7 \\ 6 & 5 \end{vmatrix} = 1\cdot 5 - 6\cdot(-7) = 5+42 = 47$

12. The additive inverse of the complex number $3-4i$ is

A. $3+4i$ C. $3-4i$

B. $-3-4i$ D. $-3+4i$

Soln: additive inverse of $a+bi$ is $-(a+bi)$ thus additive inverse of $3-4i$ is $-(3-4i) = -3+4i$

13. What is the modulus of the complex number $-6+8i$?

A. 5 C. 10

B. 2 D. 8

Soln: modulus of $z = a+bi$ is $|z| = \sqrt{a^2+b^2}$, thus modulus of $-6+8i$ is $|-6+8i| = \sqrt{(-6)^2+8^2}$

$= \sqrt{36+64}$

$= \sqrt{100}$

$= 10$

14. What are the modulus and argument of the complex number $2 + 2i$ respectively?

A. $2\sqrt{2}$ and $\frac{\pi}{4}$

C. $2\sqrt{2}$ and $-\frac{\pi}{4}$

B. 2 and $\frac{\pi}{4}$

D. 4 and $\frac{\pi}{4}$

Soln: modulus $= |2+2i| = \sqrt{2^2+2^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

argument $= \theta = \tan^{-1}\left(\frac{2}{2}\right) = \tan^{-1}(1) = \pi/4$

15. Consider the following frequency distribution table.

Value	5	3	7	6	8
Frequency	3	4	5	3	5

Table 1

$n = 3+4+5+3+5 = 20$

x	3	5	6	7	8
f	4	3	3	5	5

What is the sixth decile?

A. 6.5

C. 7.5

B. 5

D. 7

Soln: for D6, $\frac{in}{10} = \frac{6 \cdot 20}{10} = 12^{th}$ item thus.

$D6 = \left(\frac{in}{10}^{th} + \left(\frac{in}{10}+1\right)^{th}\text{item}\right)\Big/2 = \left(\frac{6\cdot 20}{10}^{th} + \left(\frac{6\cdot 20}{10}+1\right)^{th}\text{item}\right)\Big/2 = \left(\frac{25}{2}\right)^{th} = 12.5^{th}$

$= 12^{th} + 0.5(13^{th} - 12^{th}) = 7 + 0.5(7-7) = 7$

16. In a certain health center there are 3 doctors, 8 nurses and 2 physicians. In how many ways one can form a group of 5 members consisting of 1 doctor, 3 nurses and 1 physician?

A. 168

C. 336

B. 112

D. 330

No of ways $= \binom{3}{1}\binom{8}{3}\binom{2}{1} = \frac{3!}{2!1!} \cdot \frac{8!}{3!5!} \cdot \frac{2!}{1!1!} = \frac{3\times 2!}{2!} \cdot \frac{8\times 7\times 6\times 5!}{3\times 2\times 5!} \cdot \frac{2\times 1!}{1!}$

$= 3 \cdot 56 \cdot 2$

$= 6 \cdot 56$

$= 336$

17. Which one of the following forms represents an exhaustive set of events if a die is thrown once?

A. {1, 2}, {4}, {4, 5, 6}

B. {1, 2, 3}, {4, 5}, {6}

C. {1, 2}, {2}, {4, 5}, {5, 6}

D. {1, 2, 3}, {3, 4}, {6}

Soln: An exhaustive set is a set which contains all the elements in a set i.e B = {1, 2, 3, 4, 5, 6}

18. A bag contains 2 red, 3 yellow and 5 green balls. One ball is drawn at random. What is the probability that the drawn ball is NOT yellow?

A. $\frac{3}{10}$

B. $\frac{1}{5}$

C. $\frac{4}{5}$

D. $\frac{7}{10}$

Soln: $P(\text{not yellow}) = \frac{\binom{3}{0}\binom{7}{1}}{\binom{10}{1}} = \frac{3! \cdot 7!}{3!0!} \cdot \frac{1!6!}{10!} = \frac{3!}{3!} \cdot \frac{7 \times 6!}{6!} \cdot \frac{1!9!}{10 \times 9!} = \frac{1 \cdot 7}{10} = \frac{7}{10}$

19. Which of the following is the product AB of the two matrices $A = \begin{bmatrix} -2 & 3 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 5 \\ 3 & 0 \end{bmatrix}$?

A. $\begin{bmatrix} -11 & -10 \\ 11 & 5 \end{bmatrix}$

B. $\begin{bmatrix} -11 & 10 \\ 11 & 5 \end{bmatrix}$

C. $\begin{bmatrix} 11 & -10 \\ 11 & 5 \end{bmatrix}$

D. $\begin{bmatrix} -11 & 10 \\ 11 & -5 \end{bmatrix}$

Soln: $AB = \begin{pmatrix} -2 & 3 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} -1 & 5 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} -2 \cdot -1 + 3 \cdot 3 & -2 \cdot 5 + 3 \cdot 0 \\ 1 \cdot -1 + 4 \cdot 3 & 1 \cdot 5 + 4 \cdot 0 \end{pmatrix} = \begin{pmatrix} 2+9 & -10+0 \\ -1+12 & 5+0 \end{pmatrix}$

$= \begin{pmatrix} 11 & -10 \\ 11 & 5 \end{pmatrix}$

20. Which one of the following is **NOT** true about the determinant of a given square matrix A?

A. If A contains a row of zeros, then its determinant is zero.

B. If A has identical rows, then its determinant is zero.

C. The determinant of A is equal to the determinant of its transpose.

D. Interchanging two rows of A gives the same determinant.

Soln A) True
B) True
C) $|A^T| = |A|$ True
D) false! if B is found by interchanging two rows of A, then $|B| = -|A|$.

21. Which of the following is the inverse of the matrix $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$?

A. $\begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}$

B. $\begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}$

C. $\begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix}$

D. $\begin{bmatrix} -4 & -3 \\ -3 & -2 \end{bmatrix}$

Soln: Let $A = \begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}$, $|A| = 2\cdot4 - 3\cdot3 = 8-9 = -1 \neq 0$ invertible
$C_{11} = 4$ $C_{21} = -3$ $C_{ij} = \begin{pmatrix} 4 & -3 \\ -3 & 2 \end{pmatrix}$
$C_{12} = -3$ $C_{22} = 2$ $adj A = (C_{ij})^T = \begin{pmatrix} 4 & -3 \\ -3 & 2 \end{pmatrix}$
$A^{-1} = \frac{1}{|A|} adj A = \frac{1}{-1}\begin{pmatrix} 4 & -3 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} -4 & 3 \\ 3 & -2 \end{pmatrix}$

22. Consider the system of linear equation $\begin{cases} 3x - ay = 1 \\ 6x + 4y = 5 \end{cases}$. Then which of the following is true?

A. If $a > -2$, then the system has a unique solution.

B. If $a = -2$, then the system has a unique solution.

C. If $a = 2$, then the system has no solution.

D. If $a > 2$, then the system has infinitely many solutions.

$\frac{3}{6} \neq \frac{1}{4} \neq \frac{1}{5}$
let $a = -1$ it has one solution.
Let $a = -2$ $\frac{3}{6} \neq \frac{2}{4} = \frac{1}{5}$ no solution
$\frac{1}{2} = \frac{1}{2} \neq \frac{1}{5}$
$\frac{3}{6} \neq \frac{-2}{4} \neq \frac{1}{5}$ One solution
let $a = 3 \Rightarrow \frac{3}{6} \neq \frac{-3}{4} \neq \frac{1}{5}$ no solution

Note: for any systems of linear equation $\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$
i) if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, one solution
ii) if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, no solution
iii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, Infinite solution.

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23. What is the magnitude of the vector whose initial point is at (1, −3) and terminal point is at (4, 1)?

A. 5 C. 1

B. 6 D. 4

Soln: $|PQ| = \sqrt{(4-1)^2 + (1-(-3))^2} = \sqrt{9+16} = \sqrt{25} = 5$

24. Let T be a translation that takes the origin into (2, 3). Then which of the following is true about the translation T?

A. $(x-4)^2 + (y-3)^2 = 9$ is the translation of $(x-2)^2 + (y+6)^2 = 9$. $(x-2)^2 + (y+6)^2 = 9$ ✗

B. $(x-5)^2 + (y-6)^2 = 9$ is the translation of $(x-3)^2 + (y-3)^2 = 9$. $(x-3)^2 + (y-3)^2 = 9$ ✓

C. $(x-4)^2 + (y+3)^2 = 12$ is the translation of $(x-2)^2 + (y+6)^2 = 12$. $(x-2)^2 + (y+6)^2 = 12$ ✓

D. $(x+4)^2 + (y-3)^2 = 12$ is the translation of $(x-2)^2 + (y+6)^2 = 12$. $(x-2)^2 + (y+6)^2 = 12$ ✗

Soln: $(h, k) = (2-0, 3-0) = (2, 3)$

A) $T(2,-6) \xrightarrow{(2,3)} (4,-3) \Rightarrow (x-4)^2 + (y+3)^2 = 9$

B) $T(3,3) \xrightarrow{(2,3)} (5,6) \Rightarrow (x-5)^2 + (y-6)^2 = 9$

C) $T(2,-6) \xrightarrow{(2,3)} (4,-3) \Rightarrow (x-4)^2 + (y+3)^2 = 12$

D) $T(2,-6) \xrightarrow{(2,3)} (4,-3) \Rightarrow (x-4)^2 + (y+3)^2 = 12$

25. Which of the following functions represents, the graph shown in Figure 2?

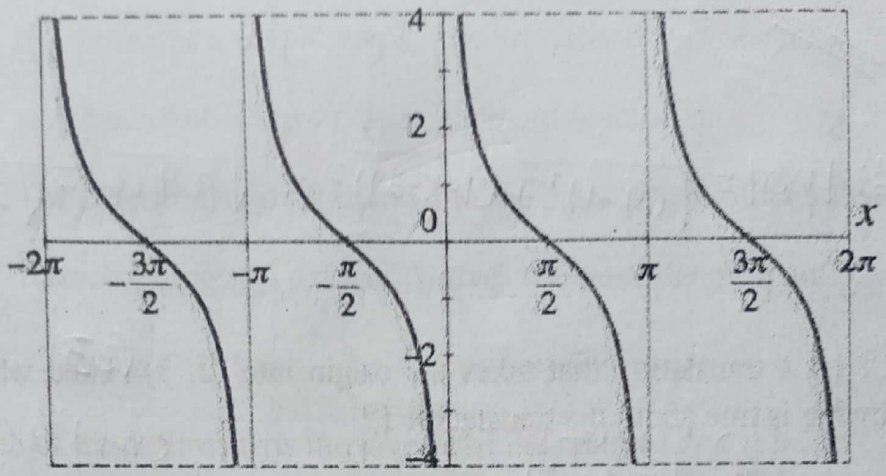


Figure 2

A. $f(x) = \tan x$

C. $f(x) = \csc x$

B. $f(x) = \sec x$

D. $f(x) = \cot x$

Soln: It is $f(x) = \cot x$, because it is decreasing function and its domain is $\mathbb{R}/\{n\pi, n\in\mathbb{Z}\}$.

26. Which of the following is true?

A. The domain of $y = \tan^{-1}x$ is $(-\infty, \infty)$. True ✓

B. The domain of $y = \cos^{-1}x$ is $(-1, 1)$. false! it is $[-1, 1]$

C. $\sin^{-1}(0) = \frac{\pi}{2}$. false! bcs $\sin^{-1}(0) = \pi$, or 2π

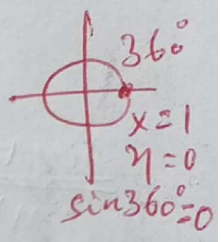
D. $\tan^{-1}(0) = 1$. false! bcs $\tan^{-1}(0) = 0^\circ$

27. Which one of the following is the fourth term of the sequence $\{\cos$

$\{\cos(\frac{n\pi}{2})\}$

A. 0 C. 1

B. -1 D. 0.5



Soln: $\cos\frac{4\pi}{2} = \cos 2\pi = \cos 360^\circ = 1$

28. Which of the following is a geometric sequence?

A. 1, 1.5, 2.5, 3.5,... C. 8, -4, 2, -1 ...

B. 0.1, 0.01, 0.0011, 0.000111, ... D. 0.1, 0.11, 0.12, 0.13,

Soln: $r = \frac{-4}{8} = \frac{2}{-4} = \frac{-1}{2}$, which has a constant common ratio but not for the others.

29. A factory that produces electric cable had sales Birr ~~1,000,00~~ 1,000,000 the first day. During each successive days the sales increased by Birr ~~10~~ 10,000 every day. What is the total sales of the factory in the first 10 days?

A. Birr 10,000,000 C. Birr 20,900,000

B. Birr 1,090,000 D. Birr 10,450,000

Soln: $A_1 = 1,000,000$, $d = 10,000$, $n = 10$

$S_{10} = \frac{10}{2}(2A_1 + (10-1)d) = 5(2,000,000 + 90,000) = 5(2,090,000$

$= 10,450,000$

30. What is the least upper bound of the sequence $\left\{2+\frac{n}{n+1}\right\}_{n=1}^{\infty}$?

A. 1 C. 3

B. 0 D. 2

Soln: Some of the terms of the sequence are:

$\frac{5}{2}, \frac{8}{3}, \frac{11}{4}, ----3$

Upper bound $= (3, \infty)$, lub $= 3$

lower bound $= (-\infty, \frac{5}{2}]$, glb $= \frac{5}{2}$

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31. Which one of the following sequences is a null sequence?

sequence is a null sequence

A. $\left\{n\sin\left(\frac{1}{n}\right)\right\}$

B. $\left\{1-\frac{1}{n}\right\}$

C. $\left\{\frac{n}{n+1}\right\}$

D. $\left\{\frac{\cos n}{n}\right\}$

Soln:- a_n is null sequence iff $\lim_{n\to\infty} a_n = 0$

A) $\lim_{n\to\infty} n\sin\left(\frac{1}{n}\right) = \lim_{n\to\infty} \frac{\sin\left(\frac{1}{n}\right)}{\left(\frac{1}{n}\right)} = 1$, not null

B) $\lim_{n\to\infty}\left(1-\frac{1}{n}\right) = 1$, not null, C.) $\lim_{n\to\infty} \frac{n}{n+1} = 1$, not null

D) $\lim_{n\to\infty} \frac{\cos n}{n} = 0$ null

32. Suppose $\{a_n\}$ and $\{b_n\}$ are sequences with $\lim_{n\to\infty} a_n = L$ and $\lim_{n\to\infty} b_n = M$.

Then which of the following is NOT necessarily true?

$\lim_{n\to\infty} a_n = L$, $\lim_{n\to\infty} b_n = M$

A. $\lim_{n\to\infty}(3a_n + 2b_n) = 3L + 2M$ True

B. $\lim_{n\to\infty}(a_n - b_n) = L - M$ True ($\lim_{n\to\infty}(a_n - b_n) = L - M$)

C. $\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right) = \frac{L}{M}$

D. $\lim_{n\to\infty} a_n b_n = LM$ True ($\lim_{n\to\infty} a_n b_n = LM$)

Soln:- $\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right) = \frac{L}{M}$, this is true only when $M \neq 0$ thus it is not necessarily true.

33. $\lim_{x\to\infty} \frac{3x^2+2x}{5x-x^2}$ is equal to

A. -1

B. -3

C. 1

D. 0

Soln:- $\lim_{x\to\infty} \frac{3x^2+2x}{5x-x^2} = \frac{3}{-1} = -3$ b/s due to a horizontal asymptote rule as $x \to \infty$, it is the quotient of Leading coefficients.

34. Which of the following is the same as the derivative of a function f at a point P, where P is on the graph of f?

A. The continuity of the function at P.

B. The slope of a secant line through P.

C. The slope of the tangent line through P.

D. The graph of f has no jump at P.

SOLN:- The derivative of a function f at a point P means it is a slope of tangent line through P.

35. What is the derivative of the function $f(x) = 3x^2 + 2\sqrt{x} - 4x$ at $x = 1$?

A. -1

B. -3

C. 3

D. 1

SOLN $f'(x) = 6x + 2 \cdot \frac{1}{2\sqrt{x}} - 4 = 6x + \frac{1}{\sqrt{x}} - 4$, $f'(1) = 6(1) + \frac{1}{\sqrt{1}} - 4 = 6 + 1 - 4 = 3$.

36. What are the 2nd and the nth derivative of $f(x) = e^{3x}$ respectively?

A. $4f(x)$ and $2^n f(x)$

B. $2f(x)$ and $nf(x)$

C. $9f(x)$ and $3^n f(x)$

D. $9f(x)$ and $n^2 f(x)$

SOLN: $f'(x) = 3e^{3x}$, $f''(x) = 3^2 e^{3x} = 9e^{3x}$, $f'''(x) = 3^3 e^{3x}$
$f^4(x) = 3^4 e^{3x}$, ---- $f^n(x) = 3^n e^{3x} = 3^n f(x)$.

37. Which of the following is true about the zero of a function?

A. It is the y-intercept of the function.

B. It is the x-intercept of the function.

C. It is the concavity point of the function.

D. It is the critical point of the function.

SOLN :- The Zero of a function means it is x-inter
cept bis Zero of a function is when $f(x) = y =$

38. Which of the following is true about the function defined by $f(x) = x^3 - 3x^2$ on $[-1, 3]$?

Soln: $f'(x) = 3x^2 - 6x = 3x(x-2) = 0$, thus $x = 0$ and $x = 2$ are critical numbers.

A. Its critical numbers are 0 and 2 and its maximum value is 15.

B. Its critical numbers are 0, 2, 3 and its maximum value is 0.

C. It has critical numbers at 0 and 2 and its maximum value is 0.

D. Its critical numbers are 0 and 2 and its maximum value is -4.

$f(0) = 0$, $f(2) = 8 - 12 = -4$, $f(-1) = -1 - 3 = -4$, $f(3) = 27 - 27 = 0$

thus $f(0) = f(3) = 0$ is its absolute maximum whereas -4 is its absolute minimum

39. What is the number that satisfies Rolle's Theorem for $f(x) = x^2 - 3x + 1$ on $[0, 3]$?

Soln: Rolle's theorem states that:

i) f is continuous $[0,3]$

ii) f is differentiable on $(0,3)$.

iii) $f(0) = 1 = f(3)$, then $\exists c \in [0,3]$ such that $f'(c) = 2c - 3 = 0 \Rightarrow 2c = \frac{3}{2} \Rightarrow c = \frac{3}{2}$

A. 1

B. $-\frac{3}{2}$

C. 2

D. $\frac{3}{2}$

40. What are the length L and width W of a rectangle with perimeter 10,000m that maximizes the area?

Soln:

Rectangle

$P = 2(l + w) = 10{,}000$

$\Rightarrow l + w = 5{,}000$

$\Rightarrow l = 5{,}000 - w$

maximize $= A = lw$

$\Rightarrow A(w) = w(5{,}000 - w)$

$= 5000w - w^2$

$A'(w) = 5000 - 2w = 0$

$\Rightarrow w = 2500$ and $l = 5000 - w = 5000 - 2500 = 2{,}500$

A. L = 2,500 m and W = 3,000 m

B. L = 2,500 m and W = 2,500 m

C. L = 5,000 m and W = 5,000 m.

D. L = 3,000 m and W = 2,500 m.

41. If the radius r of a sphere is increasing at the rate of 2 cm/min, then the rate of change of the volume when r = 1 cm is

Soln: $\frac{dr}{dt} = 2\text{cm/min}$, $r = 1\text{cm}$

A. $6\pi\ \text{cm}^3/\text{min}$

B. $4\pi\ \text{cm}^3/\text{min}$

C. $12\pi\ \text{cm}^3/\text{min}$

D. $8\pi\ \text{cm}^3/\text{min}$

$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt} = 4\pi (1\text{cm})^2 \cdot 2\text{cm/min}$

$= 4\pi \cdot 2\ \text{cm}^3/\text{min}$

$= 8\pi\ \text{cm}^3/\text{min}$

42. Which of the following is equal to $\int(\cos x - 2x)dx$ where c is a constant?

A. $\sin x + x^2 + c$.

C. $\sin x - x^2 + c$.

B. $-\sin x - x^2 + c$.

D. $-\sin x + x^2 + c$.

Soln: $\int(\cos x - 2x)dx = \int \cos x\,dx - 2\int x\,dx = \sin x - 2\frac{x^2}{2} + c$

$= \sin x - x^2 + c$

43. What is the area of the region bounded by $y = 0$ and the graph of $f(x) = 4 - x^2$ on the interval $[-1, 1]$?

$f(x) = 4 - x^2$ is larger than $y = 0$ on $[-1, 1]$ thus ↓

A. $\frac{22}{3}$

C. $\frac{26}{3}$

B. $\frac{2}{3}$

D. 8

Soln: Area $= \int_{-1}^{1}((4 - x^2) - (0))dx = \int_{-1}^{1}(4 - x^2)dx = \left(4x - \frac{x^3}{3}\right)\Big|_{-1}^{1}$

$= \left(4 - \frac{1}{3}\right) - \left(-4 + \frac{1}{3}\right) = \left(\frac{11}{3} + \frac{11}{3}\right) = 22/3$

44. Suppose $\int_1^3 f(x)dx = 6$ and $\int_1^2 f(x)dx = 10$, what is the value of $\int_2^3 f(x)dx$?

A. 2

C. 16

B. −4

D. 4

Soln: $\int_1^3 f(x)dx = \int_1^2 f(x)dx + \int_2^3 f(x)dx \Rightarrow \int_2^3 f(x)dx = \int_1^3 f(x)dx - \int_1^2 f(x)dx$

$= 6 - 10 = -4$

45. Which of the following statements has a truth value T?

A. $(p \wedge q) \Rightarrow (p \vee q)$

C. $(p \vee q) \Rightarrow (p \wedge q)$

B. $(p \wedge q) \Rightarrow \neg(p \vee q)$

D. $(p \Rightarrow q) \Rightarrow (p \vee q)$

Soln:

p	q	$p \wedge q$	$p \vee q$	A	$\neg(p \vee q)$	B	C	$p \Rightarrow q$	D
T	T	T	T	T	F	F	T	T	T
T	F	F	T	T	F	T	F	F	T
F	T	F	T	T	F	T	T	T	T
F	F	F	F	T	T	T	T	T	F

always true tautology

not

not

not

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46. The inverse of the function $y = 3x - 5$ is equal to

A. $y = \frac{x+5}{3}$ C. $y = \frac{x-5}{3}$

B. $y = \frac{x+3}{5}$ D. $y = -5x + 3$

Soln:- inverse means interchanging x and y and write y inter ms of x; $f^{-1}(x): x = 3y - 5 \Rightarrow y = \frac{x+5}{3}$

47. Which of the following is the universal set of the expression $\frac{1}{x^2} + \frac{x+1}{x^2+x}$?

A. $\mathbb{R}\backslash\{0, 1\}$ C. $\mathbb{R}\backslash\{0, 1, -1\}$

B. $\mathbb{R}\backslash\{0\}$ D. $\mathbb{R}\backslash\{0, -1\}$

Soln:- Universal Set means domain of a function. $\frac{1}{x^2} + \frac{x+1}{x^2+x} = \frac{1}{x^2} + \frac{x+1}{x(x+1)}$, thus domain = $\mathbb{R}/\{-1, 0\}$

48. The graph of a rational function is given as in the Figure 1 below.

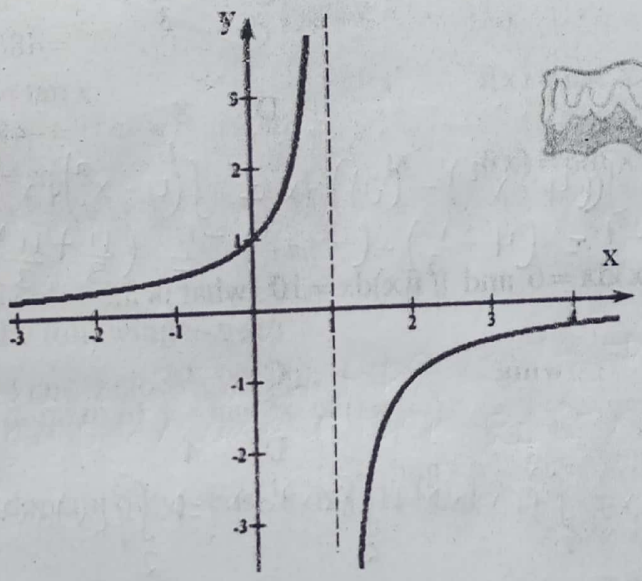


Figure 1

Which of the following is **NOT** true about the graph?

A. $x = 1$ is its vertical asymptote. ✓

B. $y = 0$ is its horizontal asymptote. ✓

C. It is decreasing on $(1, \infty)$. false! bcs it is increasing. from $(1, \infty)$

D. It is increasing on $(-\infty, 1)$. ✓

Note:- we have to observe the graph from left to write to say increasing/decreasing on its domain.

49. The equation of a line is given by $3x - 2y + 6 = 0$. Then which of the following is true about the line?

A. Its y-intercept is –3.

B. Its slope is $\frac{3}{2}$.

C. (2, 5) lies on the line.

D. Its x-intercept is 2.

Soln: A) y-int, when $x=0 \Rightarrow 3(0)-2y+6=0 \Rightarrow -2y=-6 \Rightarrow y=3$

B) $-2y=-3x-6 \Rightarrow y=\frac{3}{2}x+3$, from this slope $=m=3/2$

C) $3(2)-2(5)+6=0$

$\Rightarrow 6-10+6=0 \Rightarrow 2=0$, (2,5) is not on h.

D) x-int when $y=0 \Rightarrow 3x=-6 \Rightarrow x=-2$

50. Which one of the following is the standard form of the equation of the parabola with vertex (4, 6) and focus (–8, 6)?

A. $(x-4)^2=-12(y-6)$

B. $(x-4)^2=12(y-6)$

C. $(y-6)^2=48(x-4)$

D. $(y-6)^2=-48(x-4)$

$(y-6)^2=48(x-4)$.

$(y-6)^2=-48(x-4)$.

Soln: $h=4, k=6$; focus: $f(h-p,k)=(4-p,6)=(-8,6)$

$\Rightarrow 4-p=-8 \Rightarrow p=12$ and sign infront of p is negative and it is x-parabola

$\therefore (y-6)^2=-4(12)(x-4) \Rightarrow (y-6)^2=-48(x-4)$

51. Which of the following is true about the hyperbola given by $\frac{x^2}{4}-y^2=1$?

A. Its foci are at (–5, 0) and (5, 0).

B. Its vertices are at (–2, 0) and (2, 0).

C. Its center is at (2, 1).

D. An asymptote to the hyperbola is $y = 2x$.

Soln: hyperbola: $\frac{x^2}{4}-\frac{y^2}{1}=1 \Rightarrow h=0, k=0, a=2, b=1$

and it is x-hyperbola. $c^2=a^2+b^2=4+1=5$, $c=\sqrt{5}$

A) foci: $f(h\pm c,k)=(-\sqrt{5},0)$ and $(\sqrt{5},0)$.

B) Vertices: $V(h\pm a,k)=(-2,0)$ and $(2,0)$

C) Center: $C(0,0)$.

D) Asymptote: $y-k=\pm\frac{b}{a}(x-h)$

$\therefore y-0=\pm\frac{1}{2}(x-0) \therefore y=\pm\frac{1}{2}x$.

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52. Two statements p and q are given by:

p: $\frac{22}{7}$ is an irrational number. T

q: The sum of two odd integers is odd. F

Then which of the following has a truth value T?

A. $p \wedge \neg q = T \wedge T = T$ C. $\neg p \wedge q = F \wedge F = F$

B. $p \vee \neg q = T \vee T = T$ D. $p \wedge q = T \wedge F = F$

Soln: There is two true statements A and B and Two false statements i.e C and D.

53. Which one of the following is the contrapositive of the statement: If 21 is a prime number, then 4 is an odd number?

A. If 4 is an odd number, then 21 is not a prime number.

B. If 4 is an odd number, then 21 is a prime number.

C. If 4 is not an odd number, then 21 is not a prime number.

D. If 21 is not a prime number, then 4 is an even number.

Soln: The contrapositive of $P \Rightarrow Q$ is $\neg Q \Rightarrow \neg P$, then "If 4 is not an odd number, then 21 is not a prime number"

54. Which of the following is true where c is a constant?

A. $\int \frac{\sin\sqrt{x}}{\sqrt{x}} dx = -2\cos\sqrt{x} + c$ C. $\int \ln x\,dx = \frac{1}{x} + c$ ✗, $\int \ln x\,dx = x\ln x - x + C$

B. $\int (3x+5)^6 dx = \frac{(3x+5)^7}{7} + c$ ✗ D. $\int xe^x dx = \frac{x^2}{2}e^x + c$ ✗ = Let $f(x) = x$

Soln

A) Let $u = \sqrt{x}$, then $du = \frac{1}{2\sqrt{x}} dx$

$\Rightarrow 2du = \frac{1}{\sqrt{x}} dx$

thus $\int \frac{\sin\sqrt{x}}{\sqrt{x}} dx = \int \sin u \cdot 2du$

$= -2\cos u + C$

$= -2\cos\sqrt{x} + C$

$\int xe^x dx = xe^x - \int e^x dx$

$= xe^x - e^x + C$

$g'(x) = e^x$

$f'(x) = 1$

$g(x) = e^x$

B) $\int (3x+5)^6 dx = \frac{1}{3} \cdot \frac{(3x+5)^7}{7} + C$

55. The acceleration of a body at any time t is given by $a(t)=4-2t+3t^2$ with initial velocity $v(0)=5$. Then what is the velocity v of the body as a function of time?

* $a=\frac{dv}{dt}$ $\Rightarrow \int a\,dt = V$

A. $v(t)=1+4t-t^2+t^3$ C. $v(t)=-2+6t$

B. $v(t)=5+4t-t^2+t^3$ D. $v(t)=9-t^2+t^3$

Soln: $a(t)=4-2t+3t^2$, $V(0)=5$

$\Rightarrow V(t)=\int(4-2t+3t^2)dt = 4t-t^2+t^3+c$, if $V(t=0)=5$, then $4(0)-0^2+0^3+c=5 \Rightarrow c=5$ $\therefore V(t)=4t-t^2+t^3+5$

56. Which of the following argument is valid?

A. $p,\neg q \vdash p \Rightarrow q$ $p,\neg q \vdash p \Rightarrow q$

B. $\neg p,q \vdash p \Rightarrow q$ $\neg p,q \vdash p \Rightarrow q$

C. $p,q \vdash p \Rightarrow \neg q$ $p,q \vdash p \Rightarrow \neg q$

D. $p,\neg q \vdash p \wedge q$

A) $P=T$, $\neg q=T \Rightarrow q=F$ $\therefore P \Rightarrow q=F$ invalid

B) $\neg p=T \Rightarrow p=F$ and $q=T$ $\therefore P \Rightarrow q=T$ Valid

C) $P=T$ and $q=T$ $\therefore P \Rightarrow \neg q=F$, invalid

D) $P=T$, $\neg q=T$, $q=F$ $\therefore P \wedge q=F$ thus it is invalid.

57. Which of the following proof is more suitable to prove the assertion:

$1+2^3+3^3+\ldots+n^3=(1+2+3+\ldots+n)^2$

A. Proof by counter example.

B. Proof by the principle of mathematical induction.

C. Proof by contradiction.

D. Proof by indirect proof.

Solution: i) it is true for $n=1$, $1=1^2=1$

ii) Suppose it is true for $n=k$, then we need to show that it is true for $n=k+1$ or $1+2^3=(1+2)^2=9$

$1+2^3+3^3=(1+2+3)^2=36$

It is true for every n.

58. Math test scores distribution of 20 students is given as in Table 2 below ?

C.M.	3	8	13	18
Scores	1 - 5	6 - 10	11 – 15	16 - 20
Frequency	4	6	8	2

Table 2

What is the standard deviation of the scores ?

A. 5 C. $\sqrt{21}$

B. 4 D. $\sqrt{20}$

Soln: $\bar{x} = \frac{f_1x_{c_1} + \cdots + f_nx_{c_n}}{f_1 + \cdots + f_n} = \frac{12+48+104+36}{4+6+8+2} = \frac{200}{20} = 10$

Variance $= \frac{f_1(x_{c_1}-\bar{x})^2 + \cdots + f_n(x_{c_n}-\bar{x})^2}{20} = \frac{4(49)+6(4)+8(9)+2(64)}{20} = \frac{420}{20} = 21$ ∴ SD $= \sqrt{\text{variance}} = \sqrt{21}$

59. What is the coefficient of x^2 in the binomial expansion of $\left(2x+\frac{3}{x^3}\right)^8$?

Because there is no x^2 term.

A. 6 C. 576

B. 0 D. 500

Soln: $\left(2x+\frac{3}{x^3}\right)^8 = (2x)^8 + \binom{8}{1}(2x)^7\left(\frac{3}{x^3}\right)^1 + \binom{8}{2}(2x)^6\left(\frac{3}{x^3}\right)^2 + \binom{8}{3}(2x)^5\left(\frac{3}{x^3}\right)^3 + \binom{8}{4}(2x)^4\left(\frac{3}{x^3}\right)^4 + \binom{8}{5}(2x)^3\left(\frac{3}{x^3}\right)^5 + \binom{8}{6}(2x)^2\left(\frac{3}{x^3}\right)^6 + \binom{8}{7}(2x)^1\left(\frac{3}{x^3}\right)^7 + \left(\frac{3}{x^3}\right)^8$

60. Which of the following is true about the graph of $y = -4\cos\left(\frac{2x}{3}\right)$?

A. Its period is 3π. ✓

B. Its amplitude is -4. ✗

C. The graph completes one cycle on the interval $\left[0, \frac{3\pi}{2}\right]$. ✗

D. The graph completes one cycle on the interval $\left[0, \frac{2\pi}{3}\right]$. ✗

Soln: $y = -4\cos\left(\frac{2}{3}x - 0\right) + 0$, from this.

A) Period $= \frac{2\pi}{2/3} = 2\pi \cdot \frac{3}{2} = 3\pi$

B) amp $= |-4| = 4$

C) it completes one cycle on $[0, 3\pi]$ from origin to its period.

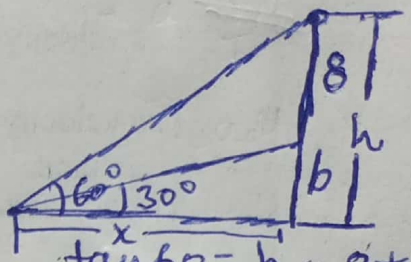
61. The angle of elevation of the top of a tree is found to be 60° as measured from a point on a level ground. What is the height of the tree if the angle of elevation of a point on the tree that is 8m below the top is 30° as measured from the same point on the ground?

A. 8m

B. 4m

C. 14m

D. 12m



$\tan 30° = \frac{b}{x} = \frac{\sqrt{3}x - 8}{x} = \frac{1}{\sqrt{3}} \Rightarrow 3x - 8\sqrt{3} = x$

$\therefore h = \sqrt{3}x = \sqrt{3}(4\sqrt{3}) = 12m \Rightarrow 2x = 8\sqrt{3}$

$\Rightarrow x = 4\sqrt{3}$

$\tan 60 = \frac{h}{x} = \frac{8+b}{x}$

$\Rightarrow h = \sqrt{3}x$

$\sqrt{3}x = 8 + b$

$\Rightarrow b = \sqrt{3}x - 8$

62. Which of the following series is convergent?

A. $\sum_{n=1}^{\infty} 5^{11-2n} = \sum_{n=1}^{\infty} 5^{11} \cdot 5^{-2n} = \sum_{n=1}^{\infty} \frac{1}{5^{11}}\left(\frac{1}{25^n}\right)$ It is convergent because $|r| = \frac{1}{25} < 1$

B. $\sum_{n=1}^{\infty}\left(\frac{2}{5}\right)^{-n}$

B) $\sum_{n=1}^{\infty}\left(\frac{2}{5}\right)^{-n} = \sum_{n=1}^{\infty}\left(\frac{5}{2}\right)^{n}$ and $|r| = 5/2 > 1$ divergent.

C. $\sum_{n=1}^{\infty}\left(\frac{3}{2}\right)^{n}$, $r = \frac{3}{2} > 1$ divergent

D. $\sum_{n=1}^{\infty} 2^{2n-5} = \sum_{n=1}^{\infty} 2^{2n} \cdot 2^{-5} = \sum_{n=1}^{\infty}\left(\frac{1}{32}\right)(4^n)$

from this $r = 4 > 1$ thus divergent.

63. Let f be a function defined on a closed bounded interval $[a, b]$. Which of the following describes that f is continuous on $[a, b]$?

A. f is continuous on (a, b), f is continuous from the right at a and from the left at b.

B. f is continuous at each point in (a, b) and at both a and b.

C. f is continuous on (a, b), f is continuous from the left at a and from the right at b.

D. f is continuous on (a, b), f is continuous from the left at a and from the left at b.

Soln:- f is continuous on $[a,b]$ iff.

i) f is continuous on (a,b).

ii) f is continuous at a to the right.

iii) f is continuous at b to the left.

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64. A position of a particle is given by the equation $S(t)=4t^4-4t+1$ where t is measured in seconds and S in meters. Which of the following is **NOT** true about the motion of the particle?

$S(t)=4t^4-4t+1$

Soln

A. The velocity at 2 seconds is given by 124 m/sec. ✓ $v=\frac{ds}{dt}$, $a=\frac{dv}{dt}$

B. The velocity v at any time t is given by $v(t)=16t^3-4$. ✓ $V(t)=16t^3-4$

C. The acceleration of the particle at 1 sec is 48 m/sec^2. ✓ $a(t)=48t^2$

D. The particle is at rest at $t=0.5$ seconds.

A) $V(2)=16(2^3)-4=16(8)-4=128-4=124 m/s$

B) $V(t)=16t^3-4$

C) $a(t=1)=48(1)^2=48 m/s^2$

D) $V(t)=0 \Rightarrow 16t^3-4=0 \Rightarrow 16t^3=4 \Rightarrow t^3=\frac{4}{16}=\frac{1}{4} \Rightarrow t=\sqrt[3]{\frac{1}{4}} \neq 0.5$

65. If $f'(1)=4$, $f(1)=3$ and $g'(3)=5$, then $(gof)'(1)$ is equal to

A. 5 C. 20

B. 3 D. 15

Soln: $(gof)'(1)=g'(f(1))\cdot f'(1)=g'(3)\cdot f'(1)$

$=5.4$

$=20$

THE END

CamScanner