

**ETHIOPIAN UNIVERSITY ENTRANCE EXAMINATION (EUEE)
MATHEMATICS FOR SOCIAL SCIENCE GINBOT 2011/JUNE 2019**

DIRECTIONS: For each of the following problems, choose the best answer from the given alternatives and carefully blacken the letter of your best choice on the separate answer sheet provided

1. At what vales of x does the function $f(x) = \frac{4x}{3} - x^4$ attains its maximum value?

- (A) -1
- (B) 1
- (C) 0
- (D) 4/3

Solution:-

Since $f(x) = \frac{4}{3}x^3 - x^4$ is differentiable function it attains its maximum at x where $f'(x) = 0$ so that

$$f'(x) = 0 \Rightarrow 4x^2 - 4x^3 = 0 \Rightarrow 4x^2(1 - x) = 0 \Rightarrow x = 0 \text{ or } x = 1$$

Using second derivative test:

$$f''(x) = 8x - 12x^2 \text{ and } f''(1) = 8(1) - 12(1)^2 = -4$$

so that

$f''(1) < 0 \Rightarrow f$ has local maximum at $x = 1$ and since f is a polynomial with even

degree & negative leading coefficient, $f(1)$ is also maximum value of f .

Answer:-B

2. What is the limit of the sequence $1, \frac{2}{2^2-1^2}, \frac{3}{3^2-2^2}, \frac{4}{4^2-3^2} \dots$?

- (A) $\frac{1}{2}$
- (B) 0
- (C) $\frac{3}{2}$

Solution:-

Searching for the general form of the sequence:

$$a_1 = 1 = \frac{1}{1^2-(1-1)^2}, a_2 = \frac{2}{3^2-(3-1)^2}, a_3 = \frac{3}{3^2-(3-1)^2}, a_4 = \frac{4}{4^2-(4-1)^2}, a_5 = \frac{5}{5^2-(5-1)^2}$$

and so

$$\text{on gives as } a_n = \frac{n}{n^2-(n-1)^2} \text{ i. e. } a_n = \frac{n}{2n-1}$$

$$\text{Then, } \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n}{2n-1} = \lim_{n \rightarrow \infty} \frac{1}{2-\frac{1}{n}} = \frac{1}{2}$$

Answer:- A

3. Suppose 2,500 items are produced by a machine and 2% of the product are randomly selected and tested. If 5 of the tested items have defect, then what is the probability that an item produced by the machine has NO defect?

- (A) 0.90
- (B) 0.85
- (C) 0.80
- (D) 0.95

Solution: -

The 2% of 25,000 is 50 items are selected randomly and 5 of them is defective. Thus, the probability that an item produced by the machine has 5

defect is: $P(\text{defect}) = \frac{5}{50} = 0.1$

Then, the probability that an item produced by the machine has no defect is:

$$P(\text{No defect}) = 1 - P(\text{defect}) = 1 - 0.1 = 0.90$$

Answer:- A

4. For real numbers x and y, which one of the following is **NOT** true?

- (A) $(\forall x)(\exists y)(x^2 + y^2 \geq -1)$
- (B) $(\forall x)(\exists y)(y \geq x^2 + 1)$
- (C) $(\exists x)(\exists y)(y \geq x^2 + 1)$
- (D) $(\forall x)(\exists y)(y \geq x^2 + 1)$

Solution: -

(A) Since $x^2 \geq 0$ and $y^2 \geq 0$ for all $x, y \in \mathbb{R}$,
 $x^2 + y^2 \geq 0 \Rightarrow x^2 + y^2 \geq -1$ for all $x, y \in \mathbb{R}$, Thus, $(\forall x)(\exists y)(y^2 + x^2 \geq -1)$ is True.

(B) $\forall(x)(\exists y)(y \geq x^2 + 1)$ is True, because for every x there exist some y such that

$$y \geq x^2 + 1 \text{ to hold (i.e. For all } x \in \mathbb{R}, x^2 \geq 0 \Rightarrow x^2 + 1 \geq 1 \text{ and } y \geq x^2 + 1 \Rightarrow y \geq 1)$$

(C) $(\exists x)(\exists y)(y \geq x^2 + 1)$ is True. For instance take $x = 0$ and $y = 1$, then $1 \geq 0^2 + 1$ is true.

(D) $(\exists x)(\exists y)(y \geq x^2 + 1)$ is False. For instance take $y = 0$, then $0 \geq x^2 + 1 \Rightarrow x^2 \leq -1$ is false

Answer: -B

5. Which one of the following is a valid argument?

- (A) If I am literate, then I can read and write. I can read but I can't write. Thus, I am not literate.
- (B) If I don't change my oil regulatory, my engine will die. My engine died. Thus, I didn't change my oil regulatory.
- (C) If you do every problem in the book, then you will learn the subject.
You learned the subject. Thus, you do every problem in the book.
- (D) If it rains or snows, then my roof leaks. My roof is leaking. Thus, it is raining and snowing.

Solution: -

(A) If I am literate, then I can read and write. I can read but I can't write. Thus, I am not literate.

Let p : I am literate, q : I can read and r : I can write.

Then, the argument can be written as $p \Rightarrow (q \wedge r), q \wedge \neg r \vdash \neg p$

p Method I Formal Proofing

- a) $q \wedge \neg p \equiv \text{True} \dots$ Premise,
- b) $q \equiv \text{True}$ and $\neg r \equiv \text{True} \dots$ By principle of detachment and step (a),
- c) $r \equiv \text{False} \dots$ Rule of negation and step (b),
- d) $q \wedge r \equiv \text{False} \dots$ By laws of conjunction and steps (b) & (c)
- e) $p \Rightarrow (q \wedge r) \equiv \text{True} \dots$ Premise
- f) $\neg p \equiv \text{True} \dots$ By Modens Tollens and steps (d) & (e) Therefore the argument is Valid.

Method II Truth Table

p	q	r	$\neg r$	$q \wedge r$	$p \Rightarrow (q \wedge r)$	$q \wedge \neg r$	$\neg p$
T	T	T	F	T	T	F	F
T	T	F	T	F	F	T	F
T	F	T	F	F	F	F	F
T	F	F	T	F	F	F	F
F	T	T	F	T	T	F	T
F	T	F	T	F	T	T	T

F	F	T	F	F	T	F	T
F	F	F	T	F	T	F	T

Only in the sixth row the premises are true, and the argument also in this row is true.

Therefore the argument is valid.

(B) If I don't change my oil regulatory, my engine will die. My engine died.
Thus, I didn't change my oil regulatory.

Let p : I change my oil regulatory and q : My engine will die.

Then, the argument can be written as: $\Box p \Rightarrow q, q \vdash \Box p$

Method I Formal Proofing

- a) $q \equiv \text{True} \dots$ Premise,
 - b) $\Box p \Rightarrow q \equiv \text{True} \dots$ Premise, and
 - c) $\Box p \equiv \text{True or False}.$
- Therefore the argument is invalid.

Method II Truth Table

p	q	$\Box p$	$\Box p \Rightarrow q$
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

In the first and the third rows the premise are true but the argument is true or false.

Therefore the argument is invalid.

(C) If you do every problem in the book, then you will learn the subject. You learned the subject. Thus, you do every problem in the book.

Let p : Do every problem in the book and q : Learn the subject.

Then, the argument can be written as $\Rightarrow q, q \vdash p.$

Method I Formal Proofing

- a) $q \equiv \text{True} \dots$ Premise,
- b) $p \Rightarrow q \equiv \text{True} \dots$ Premise, and

- c) $p \equiv \text{True or False}$ from step (a) & (b) above. Therefore the argument is invalid.

Method II Truth Table

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

In the first and the third rows the premise are true but the argument is true or false. Therefore the argument is invalid

(D) If it rains or snows, then my roof leaks. My roof is leaking. Thus, it is raining and snowing.

Let p : It is rain, q : It is snows, r : The roof is leaks.

Then the argument can be written as $(p \vee q) \Rightarrow r, r \vdash p \wedge q$

Method I Formal Proofing

- $r \equiv \text{True} \dots$ Premise,
- $(p \vee q) \Rightarrow r \equiv \text{True} \dots$ Premise,
- $p \vee q \equiv \text{True or False} \dots$ by steps (a) and (b) above, and
- $p \wedge q \equiv \text{True or False} \dots$ by step (c) above Therefore the argument is invalid.

Method II Truth Table

p	q	r	$p \vee q$	$(p \vee q) \Rightarrow r$	$p \wedge q$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	T	T	F
T	F	F	T	F	F
F	T	T	T	T	F
F	T	F	T	F	F
F	F	T	F	T	F
F	F	F	F	T	F

The premises are true in the 1, 3rd, 5th and 7th rows but the argument is not.

Therefore the argument is invalid.

Answer:- A.

6. If the second and fifth terms of a geometric progression are $-\frac{1}{2}$ and $-\frac{1}{16}$ respectively, what is the sum of the first eight terms of the sequence?

- (A) 256/255
(B) 255/256
(C) 128/85
(D) 85/128

Solution:-

Given a geometric sequence the n^{th} -term

$\{G_n = G_1 r^{n-1}\}$, $G_2 = -\frac{1}{2}$ and $G_5 = \frac{1}{16}$, then

$$\frac{G_5}{G_2} = \frac{G_1 r^4}{G_1 r} = \frac{\frac{1}{16}}{-\frac{1}{2}} \Rightarrow r^3 = -\frac{1}{8} \Rightarrow r = -\frac{1}{2}$$

, and $G_2 = G_1 r \Rightarrow G_1 = \frac{G_2}{r} = \frac{-\frac{1}{2}}{-\frac{1}{2}} = 1$

Thus, the sum of the first n^{th} -terms (S_n) is found by $S_n = \frac{G_1(1-r^n)}{1-r}$ and for $n = 8$

$$S_8 = \frac{1 \left(1 - \left(-\frac{1}{2} \right)^8 \right)}{1 - \left(-\frac{1}{2} \right)} = \frac{85}{128}$$

Answer: -D

7. There are three children in a room with, ages four, five, and six. If five-year-old child enters the room, then which of the following statements is correct?

- (A) Mean age and standard deviation will increase.
(B) Mean age will stay the same but the standard deviation will increase.
(C) Mean age will stay the same but the standard deviation will decrease.
(D) Mean age and standard deviation will stay the same. Solution:-

Given old data: 4,5,6; $\bar{x} = \frac{4+5+6}{3} = 5$ and

$$\sigma = \sqrt{\frac{(4-5)^2 + (5-5)^2 + (6-5)^2}{3}} = \sqrt{\frac{2}{3}}$$

The new data: 4,5,6,5; $\bar{x}_n = \frac{4+5+6+5}{4} = 5$

$$\text{and } \sigma_n = \sqrt{\frac{(4-5)^2 + (5-5)^2 + (6-5)^2 + (5-5)^2}{4}} = \sqrt{\frac{1}{2}}$$

Therefore $\bar{x} = \bar{x}_n$ and $\sigma > \sigma_n$.

Answer: -C

8. The marks of 50 students are given below.

Marks	0 – 10	20 – 30	30 – 40	40 – 50	60 – 70
No. of Students	5	8	f_1	10	f_2

The median of the data is 26, what are the values of f_1 and f_2 ?

- (A), $f_1 = 7$ $f_2 = 20$
 (B), $f_1 = 20$ $f_2 = 7$
 (C), $f_1 = 15$ $f_2 = 12$
 (D) $f_1 = 7$ $f_2 = 20$

,

Solution:-

The median m_d is 26 which is in the class 20 – 30 , with

$$B_L = 20, \frac{n}{2} = 25, f_c = f_1,$$

$$cf_b = 13$$

and $i = 10$.

$$m_d = B_L + \left(\frac{\frac{n}{2} - cf_b}{f_c} \right) i \Rightarrow 26 = 20 + \left(\frac{25 - 13}{f_1} \right) 10 \Rightarrow f_1 = 20$$

Since the sum of all frequencies is 50;

$$5 + 8 + f_1 + 10 + f_2 = 50 \Rightarrow 5 + 8 + 20 + 10 + f_2 = 50 \Rightarrow 43 + f_2 = 50 \Rightarrow f_2 = 7$$

Answer. B

9. What is the sum of the infinite series $\sum_{k=0}^{\infty} 5 \left(\frac{2^k + 5^k}{10^k} \right)$?

- (A) 65/4
 (B) 25/4
 (C) 5
 (D) 15

Solution:-

The expression $\frac{2^k + 5^k}{10^k}$ simplified as $\frac{2^k + 5^k}{10^k} = \frac{2^k}{10^k} + \frac{5^k}{10^k} = \left(\frac{2}{10} \right)^k = \left(\frac{1}{5} \right)^k + \left(\frac{1}{2} \right)^k$

. Then

$$\sum_{k=0}^{\infty} \frac{5(2^k + 5^k)}{10^k} = 5 \sum_{k=0}^{\infty} \frac{(2^k + 5^k)}{10^k} = 5 \sum_{k=0}^{\infty} \left(\left(\frac{1}{5} \right)^k + \left(\frac{1}{2} \right)^k \right) = 5 \left(\sum_{k=1}^{\infty} \left(\frac{1}{5} \right)^k + \sum_{k=1}^{\infty} \left(\frac{1}{2} \right)^k \right)$$

and

both $\sum_{k=0}^{\infty} \left(\frac{1}{5} \right)^k$ and $\sum_{k=0}^{\infty} \left(\frac{1}{2} \right)^k$ are convergent series (i.e. S_{∞} is convergent)
 ,since

the absolute value of their common ratios $\left|\frac{1}{5}\right|$ and $\left|\frac{1}{2}\right|$ respectively are less than

one. They converge to $\frac{G_1}{1-r}$ and $S_\infty = \frac{G_1}{1-r}$.

For the $\sum_{k=0}^{\infty} \left(\frac{1}{5}\right)^k$ cases $G_1 = r = \frac{1}{5}$ and $S_\infty = \frac{\frac{1}{5}}{1-\frac{1}{5}} = \frac{1}{4}$. Similarly for the $\sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k$

cases $G_1 = r = \frac{1}{2}$ and $S_\infty = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$.

Thus $\sum_{k=0}^{\infty} \frac{5(2^k+5^k)}{10^k} = 5 \left(\sum_{k=1}^{\infty} \left(\frac{1}{5}\right)^k + \sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^k \right) = 5 \left(\frac{1}{4} + 1 \right) = \frac{25}{4}$

Answer: B

10. A water tank is a rectangular parallelepiped with base length 3 m, width 2 m and height 2.5 m.

If water is flowing in to the tank at a rate of $0.12 \text{ m}^3/\text{sec}$, then how fast does the level of the water rise up in the tank?

- (A) 0.03 m/sec
- (B) 0.02 m/sec
- (C) 0.04 m/sec
- (D) 0.06 m/sec

Solution:-

Since the level of water is rise up in the water tank, the base length ($b = 3m$) & the base width ($w = 2m$) of the water tank are constant. Then the volume (v) of a rectangular parallelepiped is $v = bwh$; where $h = 2.5m$ is its height. Let the rate of

change of the volume of the water tank is $\frac{dv}{dt} = 0.12 \frac{m^3}{sec}$ and the rate of change of the

$\frac{dh}{dt}$ water level in the tank is $\frac{dh}{dt}$. Thus;

$$\frac{d}{dt} v = \frac{d}{dt} (bwh) \Rightarrow \frac{dv}{dt} = bw \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{bw} \frac{dv}{dt} = \frac{1}{(2m)(3m)} \left(0.12 \frac{m^3}{sec} \right) = 0.02 \frac{m}{sec}$$

Answer. B

11. The earth's orbit has a semi-major axis $a \approx 149.6$ Gm (gigameters) and an eccentricity of $e \approx 0.017$. What is the approximate value of the semi-minor axis?

- (A) 152.14 Gm
- (B) 145.32 Gm
- (C) 149.58 Gm
- (D) 149.06 Gm

Solution:-

Let semi-minor axis is b , semi-major axis is a and focal length c . Then, since earth's orbit is elliptic, $a^2 = b^2 + c^2$ and given $a \approx 149.6$ Gm (gigameters) and an eccentricity of

$$e = \frac{c}{a} \approx 0.017 \quad ; \quad c = ea = 0.017(149.6 \text{ Gm}) = 2.5432 \text{ Gm}. \text{ From } a^2 = b^2 + c^2;$$

$$b^2 = a^2 - c^2 \Rightarrow b = \sqrt{a^2 - c^2} = \sqrt{(149.6)^2 - (2.5432)^2} \text{ Gm} = 149.58 \text{ Gm}$$

Answer: -C

12. If f is the greatest integer function and g is the absolute value function, then what is the value of $(f \circ g)\left(\frac{3}{2}\right) + (g \circ f)\left(-\frac{4}{3}\right)$?

- (A) 1
- (B) -1
- (C) 3
- (D) 2

Solution:-

Given greatest integer function $(x) = [x]$ and absolute value function $(x) = |x|$.

Then,

$$(f \circ g)(x) = ((x)) = [g(x)] = [|x|] \text{ and } (f \circ g)\left(\frac{3}{2}\right) = \left[\left|\frac{3}{2}\right|\right] = \left[\frac{3}{2}\right] = 1$$

.Similarly

$$(g \circ f)(x) = (|f(x)|) = [|x|] \text{ and } (g \circ f)\left(-\frac{4}{3}\right) = \left|\left[-\frac{4}{3}\right]\right| = |-2| = 2.$$

Thus

$$(f \circ g)\left(\frac{3}{2}\right) + (g \circ f)\left(-\frac{4}{3}\right) = 1 + 2 = 3$$

Answer: -C

13. Which one of the following is true about the graph of $f(x) = \frac{x^2+5x+6}{x^2-4} + 3$?

- (A) The graph has a hole at $x = 2$.
- (B) The vertical asymptote of the graph are $x = -2$ and $x = 2$.
- (C) The graph has y -intercept at $(0, -3/2)$.
- (D) The horizontal asymptote of the graph is $y = 4$. Solution:-

For the given function

$$f(x) = \frac{x^2+5x+6}{x^2-4} + 3 = \frac{4x^2+5x-6}{x^2-4} = \frac{(4x-3)(x+2)}{(x+2)(x-2)} = \frac{4x-3}{x-2}, x \neq \pm 2.$$

Then,

- i. Since $x + 2$ is the common factor for the numerator and the denominator of (x) , it has a hole at $x = -2$ and $(-2, f(-2)) = (-2, \frac{11}{4})$ is the hole. Therefore A is false.
- ii. The only vertical asymptote of (x) is $x = 2$, since the denominator of (x) is zero when $x = 2$. Therefore B is false.
- iii. The y -intercept of $f(x)$ is $(0, f(0)) = (0, \frac{3}{2})$. Therefore C is false.
- iv. The horizontal asymptote of (x) is the ratio of the leading coefficient that is $y = 4$.

Therefore D is true.

Answer D

14. 19. If $2 \begin{pmatrix} 2x & x \\ -5 & -3 \end{pmatrix}^{-1} = \begin{pmatrix} 3 & 2 \\ -5 & -4 \end{pmatrix}$, then what is the value of x ?

- (A) -2
- (B) $\frac{1}{2}$
- (C) 2
- (D) -1/2

Solution:-

$$\begin{pmatrix} 2x & x \\ -5 & -3 \end{pmatrix}^{-1} \begin{pmatrix} 2x & x \\ -5 & -3 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ -5 & -4 \end{pmatrix} \begin{pmatrix} 2x & x \\ -5 & -3 \end{pmatrix} \quad \text{Multiplying both sides by } \begin{pmatrix} 2x & x \\ -5 & -3 \end{pmatrix}$$

$$2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 6x-10 & 3x-6 \\ -10x+20 & -5x+12 \end{pmatrix}$$

Thus, by equality of matrices

$$6x - 10 = 2 \Rightarrow x = 2 \text{ or } 3x - 6 = 0 \Rightarrow x = 2$$

$$-10x + 20 = 0 \Rightarrow x = 2 \text{ or } -5x + 12 = 2 \Rightarrow x = 2$$

Answer: -C

15. Among 2000 students who took a regional exam, the percentile of certain student's score is 90. Which one of the following is correct about the student's score?
- (A) The students have answered 90% of the question correctly.
 (B) The student's score is the same as the top 10% of the scores.
 (C) The student's score is greater than or equal to that of 1800 students.
 (D) The score of the student is as good as that of 90% of the students.

Solution:-

Answer:- C

16. If $f(x) = ax - b$ and $f^{-1}(x) = \frac{1}{2}x + 2$, for each $x \in \mathbb{R}$, then what must be the values of a and b ?
- A. $a = 2, b = 3$
 B. $a = 2, b = 2$
 C. $a = 1, b = 1$
 D. $a = \frac{1}{2}, b = -2$

Solution:-

Given $f(x) = ax - b$ and $f^{-1}(x + 1) = \frac{1}{2}x + 2$, for $x \in \mathbb{R}$.

Let $y = f(x)$. Then, $y = ax - b \Rightarrow y + b = ax \Rightarrow x = \frac{y+b}{a}$ and interchange x and y to get $f^{-1}(x)$ so that $y = \frac{x+b}{a}$. Thus $f^{-1}(x) = \frac{x+b}{a}$ and $f^{-1}(x + 1) = \frac{(x+1)+b}{a} = \frac{1}{a}x + \frac{b+1}{a}$.

From the given above we have $\frac{1}{a}x + \frac{(1+b)}{a} = \frac{1}{2}x + 2$, and by fundamental theorem of polynomials $\frac{1}{a}x = \frac{1}{2}x$ and $\frac{b+1}{a} = 2$. Therefore $a = 2$ and $b = 3$.

Answer. -A

17. Fatuma can solve 90% of the problems given in a book and Mesfin can solve 70%. What is the probability that at least one of them will solve the problem selected at random from the book?

- a. 0.97
 b. 0.87
 c. 0.77
 d. 0.67

Solution:-

Let F is Fatuma, M is Mesfin and p is the probability. Given

$$p(F) = 90\% = 0.9 \text{ and}$$

$$p(M) = 70\% = 0.7. \text{ Then}$$

$$\begin{aligned} p(F \text{ or } M) &= p(F) + p(M) - p(F \text{ and } M) \\ &= p(F) + p(M) - p(F)p(M); \text{ where } p(F \text{ and } M) = p(F)p(M) \\ &= 0.9 + 0.7 - 0.9 \times 0.7 = 0.97 \end{aligned}$$

Answer:- A

18. If $f(x) = x^3 - 2x + 1$, then which one of the following is **NOT** true?

e. $f(c) = 0$ for some $c \in [-2, 0]$

f. $f(c) = \frac{1}{2}$ for some $c \in [0, 1]$

g. $f(c) = 3$ for some $c \in [1, 2]$

h. $f(c) = \frac{1}{4}$ for some $c \in [-1, 0]$

Solution:-

Given $f(x) = x^3 - 2x + 1$. Then,

(A) Since f is continuous on $[-2, 0]$; $f(-2) = -3$ and $f(0) = 1$ implies $f(-2) \leq 0 \leq f(0)$.

So by intermediate value theorem there exist $c \in [-2, 0]$ such that $f(c) = 0$.

(A) is True

(B) Since f is continuous on $[0, 1]$; $f(0) = 1$ and $f(1) = 0$ implies $f(1) \leq 0 \leq f(0)$.

So by intermediate value theorem there exist $c \in [0, 1]$ such that $f(c) = 0$. (B) is True

(C) Since f is continuous on $[1, 2]$; $f(1) = 0$ and $f(2) = 5$ implies $f(1) \leq 0 \leq f(2)$.

So by intermediate value theorem there exist $c \in [1, 2]$ such that $f(c) = 0$. (C) is True

(D) Since f is continuous on $[-1, 0]$; $f(-1) = 2$ and $f(0) = 1$ have the same sign. So by intermediate value theorem there may not exist $c \in [-1, 0]$ such that

$f(c) = 0$. (D) is False

Answer:- D

19. A man wants to fence two identical rectangular enclosures in a field alongside a straight river as shown in the following figure.



What is the maximum area of each enclosure that he can make with 192 meter fencing material if the side along the river does not need a fence?

- (A) 1530 m²
- (B) 1536 m²
- (C) 1664 m²
- (D) 1564 m²

Solution:-

Given 192 meter fencing material $\Rightarrow 3y + 2x = 192 \Rightarrow y = 64 - \frac{2x}{3}$

If A is the combined area, then $A(x, y) = 2xy$ for $y = 64 - \frac{2x}{3}$

$$A(x) = 2x(64 - \frac{2}{3}x) = 128x - \frac{4}{3}x^2$$

Then,

$$A'(x) = 0 \Rightarrow (128x - \frac{4}{3}x^2)' = 0 \Rightarrow 128 - \frac{8}{3}x = 0 \Rightarrow x = 48$$

and

$$A''(x) = -\frac{8}{3} < 0 \text{ for } x = 48 \in [0, 192] \text{ so } A(x)$$

$$A(48) = 128(48) - \frac{4}{3}(48)^2 = 3072$$

is maximum which is

The maximum area of each is $\frac{1}{2}A(48) = 1,536 \text{ m}^2$ since the rectangles are identical.

Answer. -B

20. Which one of the following is equal to $f(x) = \sqrt{(x+4)^2}$ for every $x \in \mathbb{R}$?

- A. $g(x) = x + 4$
- B. $g(x) = x + 2$
- C. $g(x) = |x| + 4$
- D. $g(x) = |x + 4|$

Solution:-

Given $f(x) = \sqrt{(x+4)^2}$. Then $g(x) = |x+4| = f(x)$, Since $\sqrt{x^2} = |x|$

Answer: -D

21. What is the polar form of; $\frac{7-i}{3-4i}$?

- A. $\sqrt{2}(\cos \frac{\pi}{4} + i \sin \frac{\pi}{2})$
 B. $\sqrt{2}(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4})$
 C. $2(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2})$
 D. $2(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4})$

Solution:-

Let $z = \frac{7-i}{3-4i}$. Then the rectangular form of z (i.e. $z = x + yi$ where $x, y \in \mathbb{R}$) is

$$z = \frac{7-i}{3-4i} = \frac{7-i}{3-4i} \times \frac{3+4i}{3+4i} = \frac{25+25i}{25} = 1 + i$$

The modulus of z , $|z| = r = \sqrt{x^2 + y^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$ and the principal argument of

$$z \text{ is } \text{Arg } z = \theta = \tan^{-1}\left(\frac{y}{x}\right) + k\pi, \text{ where } k = \begin{cases} -1; & \text{if } x, y < 0 \\ 0; & \text{if } x > 0 \\ 1; & \text{if } x < 0, y > 0 \end{cases} \quad 0 = \tan^{-1}\left(\frac{1}{1}\right) = \tan^{-1}(1) = \frac{\pi}{4}.$$

Thus, the polar form of z is

$$z = r(\cos \theta + i \sin \theta) = \sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right) = \sqrt{2}\left(\frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2}\right) = 1 + i$$

Answer: -B

22. The center of a circle is on the line $y = 2x$ and the line $x = 1$ is tangent to the circle at $(1, 6)$. How long is the radius of the circle?

- a. 5
 b. 2
 c. 3
 d. 4

Solution

The radius of a circle is perpendicular to the tangent line $x = 1$ at the point of tangency $P(1, 6)$ which is on the line $y = 6$. The point of intersection of $y = 2x$ and $y = 6$ is $(3, 6)$ the center of the circle and the distance between the point of tangency and the center is the radius of the circle (r). Thus, $r = |3 - 1| \text{ units} = 2 \text{ units}$.

Answer: B

23. If $a_1 = 2, a_2 = 6, a_3 = 10, a_4 = 14, \dots$, then $\sum_{n=1}^{100} a_n$?

- A. 20,020
 B. 20,200

C. 22,000

D. 20,000

Solution:-

If $a_1 = 2, a_2 = 6, a_3 = 10, a_4 = 14, \dots$ is an arithmetic sequence with $a_1 = 2$ and $d = 4$. Then, $\sum_{n=1}^{100} a_n$ implies the sum of the first 100 terms $S_{100} = 100$ is found by

$$S_n = \frac{n}{2}(2a_1 + (n-1)d)$$

$$S_{100} = \sum_{n=1}^{100} a_n = \frac{100}{2}(2(2) + (100-1)4) = 20,000.$$

Answer: D

24. If $\neg p \Rightarrow r$ is False and $p \Leftrightarrow q$ is True, which of the following is True?

A. $\neg p \wedge (q \Rightarrow r)$

B. $p \vee (\neg q \wedge r)$

C. $\neg p \Rightarrow (q \vee r)$

D. $p \Rightarrow (\neg q \vee r)$

Solution:-

Given $\neg p \Rightarrow r \equiv F$ implies that $\neg p \equiv T$ and $r \equiv F$ by the rule of implication " $\Rightarrow$ ", and $p \equiv F$ by the rule of negation " $\neg$ ". Again $p \Leftrightarrow q \equiv T$ implies that $q \equiv F$ since $p \equiv F$ and rule of Bi-implication " $\Leftrightarrow$ ".

$$F \vee F \equiv F$$

$$F) \equiv F \Leftrightarrow T \equiv F$$

$$(A) \neg p \wedge (q \Rightarrow r) \equiv \neg F \wedge (F \Rightarrow F) \equiv T \wedge T \equiv T$$

$$(B) p \vee (\neg q \wedge r) \equiv F \vee (\neg F \wedge F) \equiv F \vee (T \wedge F) \equiv F$$

$$(C) \neg p \Rightarrow (q \vee r) \equiv \neg F \Rightarrow (F \vee F) \equiv T \Rightarrow F \equiv F$$

$$(D) p \Leftrightarrow (\neg q \vee r) \equiv F \Leftrightarrow (\neg F \vee F) \equiv F \Leftrightarrow (T \vee F) \equiv F$$

Answer: A

25. Which one of the following is the set of critical numbers of $f(x) = \frac{3}{8}x^{8/3} - 6x^{2/3}$?

A. $\{-2, 2\}$

B. $\{-2, 0, 2\}$

C. $\{-1, 0, 1\}$

D. $\{-1, 1\}$

8 Solution:-2

Given $f(x) = \frac{3}{8}x^{\frac{5}{3}} - 6x^{\frac{1}{3}}$. Then, $f'(x) = x^{\frac{5}{3}} - \frac{4}{x^{\frac{1}{3}}} = \frac{x^2-4}{x^{\frac{1}{3}}}$. Thus:

i) $f'(x) = \frac{x^2-4}{x^{\frac{1}{3}}}$ is undefined at $x = 0$ and $0 \in \text{Domain of } f$. So 0 is the critical number.

ii) $f'(x) = 0 \Rightarrow \frac{x^2-4}{x^{\frac{1}{3}}} = 0 \Rightarrow (x+2)(x-2) = 0 \Rightarrow x = \pm 2$. So -2 and 2 are critical numbers.

Therefore, the set of critical numbers of f is $\{-2, 0, 2\}$

Answer: -B

26. 35. If $h(x) = \sqrt{1 + \sqrt{x}}$, then which of the following is equal to $h'(x)$?

- A. $\frac{2\sqrt{1+\sqrt{x}}}{x}$
- B. $\frac{\sqrt{x+x\sqrt{x}}}{x}$
- C. $\frac{\sqrt{x+x\sqrt{x}}}{1}$
- D. $\frac{1}{4\sqrt{x+x\sqrt{x}}}$

Solution:- Given $h(x) = \sqrt{1 + \sqrt{x}}$. Then,
 $h'(x) = \frac{1}{2\sqrt{1+\sqrt{x}}} (1 + \sqrt{x})'$, by the rule $\sqrt{x}' = \frac{1}{2\sqrt{x}}$ and chain rule.

$$h'(x) = \frac{1}{2\sqrt{1+\sqrt{x}}} \left(0 + \frac{1}{2\sqrt{x}} \right)$$

$$h'(x) = \frac{1}{4\sqrt{x+x\sqrt{x}}}, \text{ again by the rule } \sqrt{x} = \frac{1}{2\sqrt{x}}$$

Answer -D

27. 37. If $f(x) = ax^3 + \frac{b}{x} + 5$ has local minimum at $(2, -3)$ what are the values of a and b ?

- A. $a = -\frac{1}{4}, b = 12$
- B. $a = \frac{1}{4}, b = -12$
- C. $a = \frac{1}{4}, b = 12$

D. $a = -\frac{1}{4}, b = -12$

Solution:-

Given $f(x) = ax^3 + \frac{b}{x} + 5$. Then

i) f has local minimum at $(2, -3)$ so point $(2, -3)$ is on f so that

$$f(2) = -3 \Rightarrow a(2)^3 + \frac{b}{2} + 5 = -3 \Rightarrow 16a + b = -16 \dots (1)$$

ii) $f'(x) = 3ax^2 - \frac{b}{x^2}$ and f is differentiable at $x = 2$ and has local minimum at

$$(2, -3)$$

$$\text{so that } f'(2) = 0 \Rightarrow 3a(2)^2 - \frac{b}{2^2} = 0 \Rightarrow b = 48a \dots (2)$$

Substituting (2) into (1),

$$16a + 48a = -16 \Rightarrow 64a = -16 \Rightarrow a = -\frac{1}{4} \text{ and } b = 48\left(-\frac{1}{4}\right) = -12$$

Answer. -D

28. If A is a 3×3 matrix and $\det(A) = 5$, then how much is $\det(2A^T A)$?

- A. 100
- B. 200
- C. 50
- D. 20

Solution: -

Given a 3×3 matrix A of $\det(A) = |A| = 5$. Then $d(2A^T A) = |2A^T A|$ is By the property $|kA| = k^n |A|$, where n the order of A and $k \in \mathbb{R}$ and $k \neq 0$.

$$\begin{aligned} |2A^T A| &= 2^3 |A^T| |A| = 8 |A|^2, \text{ since } |A^T| = |A| \\ &= 8 \times 5^2 = 8 \times 25 = 200 \end{aligned}$$

Answer:- B

$$\frac{a^{-1} b^{-1}}{a^{-3} - b^{-3}}? \quad - \quad -$$

29. What is the simplified form of

- A. $\frac{a^2 b^2}{b^3 - a^3}$
- B. $\frac{b^2 - a^2}{a^3 - b^3}$
- C. $\frac{ab}{a^3 - b^3}$
- D. $\frac{a - b}{a - b}$

Solution: -

The given expression $\frac{a^{-1}b^{-1}}{a^{-3}-b^{-3}}$ simplified as

$$\frac{a^{-1}b^{-1}}{a^{-3}-b^{-3}} = \frac{(ab)^{-1}}{a^{-3}-b^{-3}} = \frac{\frac{1}{ab}}{\frac{1}{a^3}-\frac{1}{b^3}} = \frac{\frac{1}{ab}}{\frac{b^3-a^3}{(ab)^3}} = \frac{1}{ab} \times \frac{(ab)^3}{b^3-a^3} = \frac{(ab)^2}{b^3-a^3} = \frac{a^2b^2}{b^3-a^3}$$

Answer: -A

30. If there are two children in a family, what is the probability that there is at least one girl in the family?

- A. $\frac{3}{4}$
- B. $\frac{1}{4}$
- C. $\frac{1}{2}$
- D. 3

Solution:-

Let G be a girl and B be a boy. Then, the sample space is $S = \{GG, GB, BG, BB\}$ and the event of getting at least one girl is $E = \{GG, GB, BG\}$ so that the probability that there is at least one girl is

$$P = \frac{n(E)}{n(S)} = 3/4$$

Answer: A

31. Consider the following system of equations: $\begin{cases} x - 2y + z = 1 \\ x + y + z = 3 \end{cases}$ How much must be the value of k so that the system has a solution?

- A. 7
- B. -1
- C. 0
- D. 1

Solution:-

$x - 2y + z = 1$
 Given $\begin{cases} -x + y + z = 3 \\ 3x - 5y + z = k \end{cases}$. Then, the determinant of coefficient matrix is zero $3x - 5y + z = k$

$$\begin{vmatrix} 1 & -2 & 1 \\ -1 & 1 & 1 \\ 3 & -5 & 1 \end{vmatrix} = 0$$

i. e. $\begin{vmatrix} -1 & 1 & 1 \\ 3 & -5 & 1 \end{vmatrix} = 0$ then system can't have a unique solution. But the

determinant is not zero. By using elementary (Gaussian) row operations we can write the augmented matrix in Row Echelon form as follow.

The augmented matrix is : $\begin{pmatrix} 1 & -2 & 1 & 1 \\ -1 & 1 & 1 & 3 \\ 3 & -5 & 1 & k \end{pmatrix}$

By pivoting $R_2 \rightarrow R_2 + R_1$ and $R_3 \rightarrow R_3 - 3R_1$: $\begin{pmatrix} 1 & -2 & 1 & 1 \\ 0 & -1 & 2 & 4 \\ 0 & 1 & -2 & k-3 \end{pmatrix}$

By pivoting $R_3 \rightarrow R_3 + R_2$: $\begin{pmatrix} 1 & -2 & 1 & 1 \\ 0 & -1 & 2 & 4 \\ 0 & 0 & 0 & k+1 \end{pmatrix}$

Thus, from the last row, the system has solution (particularly; infinitely many solution) if $k+1 = 0 \Rightarrow k = -1$.

Answer: -B

32. Which one of the following is the multiplicative inverse of $z = \frac{3+4i}{4-5i}$?

- (A) $\frac{8}{25} - \frac{31}{25}i$
- (B) $\frac{-8}{25} + \frac{31}{25}i$
- (C) $\frac{8}{25} + \frac{31}{25}i$
- (D) $\frac{-8}{25} - \frac{31}{25}i$

Solution:-

$$\text{The multiplicative inverse of } z \text{ is } \frac{1}{z} \text{ for } z = \frac{3+4i}{4-5i};$$

$$\frac{1}{z} = \frac{4-5i}{3+4i} = \frac{4-5i}{3+4i} \times \frac{3-4i}{3-4i} = \frac{(3-4i)(4-5i)}{3^2+4^2} = \frac{-8-31i}{25} = -\frac{8}{25} - \frac{31}{25}i$$

Answer: -B

44. If $\begin{vmatrix} -1 & 1 & 2 \\ 3 & 2 & x \\ 2 & 4 & 1 \end{vmatrix} = \begin{vmatrix} -x & 3 & 2 \\ 2 & 2 & 3 \\ 1 & -1 & -2 \end{vmatrix}$

33. $\frac{2}{x}$, then what is the value of x ?

- A. $3\frac{2}{5}$
- B. $2\frac{2}{5}$
- C. 5
- D. $-\frac{2}{3}$

Solution:-

Expanding along the first row

$$\begin{array}{ccccccc}
 & & & 2 & x & & 3 & x & & 3 & 2 \\
 | & 3 & & 2 & & x & | & = & -1 & | & & & & -1 & | & & & +2 & | & & & | \\
 & & & 4 & 1 & & 2 & 1 & & 2 & 4 \\
 & 2 & 4 & 1 \\
 \hline
 & = & -1(2 - 4x) - (3 - 2x) + 2(12 - 4) = 6x + 11 \dots
 \end{array}$$

(1) Expanding along the first row

$$\begin{array}{ccccccc}
 & & & 2 & 3 & & 2 & 3 & & 2 & 2 \\
 | & 2 & & 2 & & 3 & | & = & -x & | & & & & -3 & | & & & +2 & | & & & | \\
 & & & -1 & -2 & & 1 & -2 & & 1 & -1 \\
 & 1 & -1 & -2 \\
 \hline
 & = & -(-4 - (-3)) - 3(-4 - 3) + 2(-2 - 2) = x + 13 \dots (2)
 \end{array}$$

Then, setting (1) = (2) $\Rightarrow 6x + 11 = x + 13 \Rightarrow x = \frac{2}{5}$

Answer: -C

34. Everyday a person saves 5 cents more than the amount he saved on the previous day. His target is to save the total amount of 3,225 cents by the end of 30 days. How much must be the starting saving to meet the target?

- A. 25 cents
- B. 35 cents
- C. 50 cent
- D. 60 cent

Solution:-

Everyday a person saves 5 cents more than the amount he saved on the previous day so that it is an arithmetic progression of

$$\begin{array}{l}
 a_1 = x, a_2 = x + 5, a_3 = \\
 x + 10 \dots \text{in cents} \\
 \text{and } d = 5. \text{ For the given } S_{30} = 3,225 \text{ cents, then } a_1 \text{ is found by}
 \end{array}$$

$$S_n = \frac{n}{2}(2a_1 + (n-1)d)$$

$$3,225 = \frac{30}{2}(2x + (30-1)5) \Rightarrow 215 = 2x + 145 \Rightarrow 2x = 70 \Rightarrow x = 35$$

Therefore $a_1 = 35 \text{ cents}$

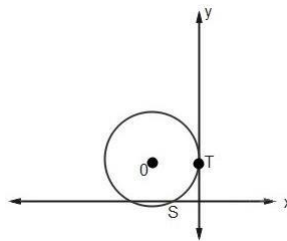
Answer: -B

35. If the circle passing through the point $(-1, 0)$ touches the y -axis at $(0, 2)$, then what is the equation of the circle?

- A. $x^2 + y^2 + 5x - 4y + 4 = 0$
 B. $x^2 + y^2 - 5x - 4y + 4 = 0$
 C. $x^2 + y^2 - 5x + 4y + 4 = 0$
 D. $x^2 + y^2 + 5x + 4y + 4 = 0$

Solution:-

Let $O(h, k)$ be the center of the circle. Given y -axis touches the circle at point $T(0, 2)$ means $\overline{OT}$ is perpendicular to y -axis so that $k = 2$ & $O(h, k) = O(h, 2)$.



Since $S(-1, 0)$ and $T(0, 2)$ are points on the circle

$$OS = OT \Rightarrow \sqrt{(-1-h)^2 + (2-0)^2} = |h-0|$$

$$\Rightarrow h^2 + 2h + 5 = h^2 \Rightarrow h = -5/2$$

Thus, the center is at $O(-\frac{5}{2}, 2)$ and the radius is $r = \left| -\frac{5}{2} - 0 \right| = \frac{5}{2}$

By $(x-h)^2 + (y-k)^2 = r^2$; standard form of equation of a circle

$$\left(x - \left(-\frac{5}{2} \right) \right)^2 + (y - 2)^2 = \left(\frac{5}{2} \right)^2 \Rightarrow x^2 + y^2 + 5x - 4y + 4 = 0$$

Answer. -A

36. Which one of the following is true about the pair of lines: $3x + 9y - 24 = 0$ and $4x + 12y + 32 = 0$?

- A. perpendicular lines
 B. intersecting lines
 C. parallel and distinct line
 D. representing the same lines

37. Let z be a complex number and $w = 3 + 4i$. If $\frac{z^2+1}{z+i} = z|w| - \frac{1}{i}\bar{w}$, then what is the value of z ?

- A. $-4 - 2i$
 B. $4 - 2i$
 C. $-1 - i$
 D. $-1 + i$

Solution:-

Given $w = 3 + 4i$. Then, $|w| = \sqrt{3^2 + 4^2} = 5$ and

$$\frac{1}{i}\bar{w} = \frac{1}{i}(3 - 4i) = \frac{3-4i}{i} \times \frac{-i}{-i} = -4 -$$

3i. Thus

$$\frac{z^2 + 1}{z + i} = |w|z - \frac{1}{i}\bar{w}$$

$$\frac{z^2 + 1}{z + i} = 5z - (-4 - 3i)$$

$$\frac{(z+i)(z-i)}{z+i} = 5z + 4 + 3i, \text{ since}$$

$$z^2 + 1 = (z + i)(z - i)$$

$$z - i = 5z + 4 + 3i$$

$$\Rightarrow z - 5z = 4 + 4i \Rightarrow -4z = 4 + 4i \Rightarrow z = -1 - i$$

Answer: -C

38. In a company, a regular work time is 40 hours per week. For a service above regular work time, the normal overtime pay rate (per hour) is 1.5 times regular rate and the holyday overtime rate is twice of the regular rate. If an employee whose regular pay is Birr 50 per hour, has worked 55 hours in a week among which 5 hours are holyday overtime, how much (in Birr) is his payment for the week?

- A. 2500
- B. 3125
- C. 3500
- D. 3250

Solution:-

Let be the regular overtime work time per week $t_{ro} = 10 \text{ Hrs}$, the holyday overtime work time per week $t_{ho} = 5 \text{ Hrs}$ and the total work time per week $t = 55 \text{ Hrs}$. Then

Total payment

$$= 40 \text{ Hrs} \times \text{Birr } 50 + 10 \text{ Hrs} \times \text{Birr } 75 + 5 \text{ Hrs} \times \text{Birr } 100 = \text{Birr } 3,250.$$

Answer :- D

39. A young student borrows a certain amount of money from money lender. The money lender charges interest of 20% compounded yearly. If the money lender sends a letter of demand of 400,000 birr after 2 years, how much did the student borrow?

- A. 277,777.80 birr

- B. 384,467.50 birr
C. 240,000 birr
D. 320,000 birr

Solution:-

Givens:- Amount: $A = \text{Birr } 400,000$, rate: $r = 20\%$, time: $t = 2 \text{ years}$ and the interest is compounded yearly ($m = 1$).

Required:- the principal P ?

$$\text{Solution: } A = P \left(1 + \frac{r}{m}\right)^{mt} \Rightarrow P = \frac{A}{(1+r)^t}, \text{ since } m = 1$$

$$P = \frac{\text{Birr } 400,000}{(1+0.2)^2} = \frac{\text{Birr } 400,000}{1.44} = \text{Birr } 277,777.80$$

Answer: -A

40. The following table represents the mean weight (in kg) of four group students

Group	I	II	III	IV
Mean Weight	72	80	75	80
Number of students in the group	10	12	8	10

What is the mean weight (in kg) of all students?

- A. 77
B. 77.4
C. 76.6
D. 76

Solution:-

Given mean weight (in Kg) of four groups of students as follows

$$\text{Then, combined mean} = \frac{10(72) + 12(80) + 8(75) + 10(80)}{10 + 12 + 8 + 10} = \frac{3,080}{30} = 77$$

Answer. -A

41. The following table shows a simple frequency distribution of the mark that students scored in a test.

Score (Marks)	4	5	6	7	8	9	10
Frequency	5	8	9	14	7	5	3

Which of the following is NOT true about the frequency distribution?

- (A) The inter-quartile of the mark is 3.
 (B) The 75th percentile of the mark is 7.
 (C) The 25th percentile of the mark is 5...
 (D) The top 25% of the scores are 8 and above. Solution:-

Then, $n = \sum_{i=1}^7 f_i = 5 + 8 + 9 + 14 + 7 + 5 + 3 = 51$ (n is odd).

Then,

$$Q_k = \left(\frac{k(n+1)}{4} \right)^{th} \text{ item} \Rightarrow Q_1 = \left(\frac{1(51+1)}{4} \right)^{th} \text{ item} = 13^{th} \text{ item} = 5$$

(A) First quartile:

Third quartile: $Q_k = \left(\frac{k(n+1)}{4} \right)^{th} \text{ item} \Rightarrow Q_3 = \left(\frac{3(51+1)}{4} \right)^{th} \text{ item} = 39^{th} \text{ item} = 8$

Inter-Quartile-Range = $Q_3 - Q_1 = 8 - 5 = 3$, So A is true

(B) The 75th - percentile:

$$P_t = \left(\frac{t(n+1)}{100} \right)^{th} \text{ item} \Rightarrow P_{75} = \left(\frac{75(51+1)}{100} \right)^{th} \text{ item} = 39^{th} \text{ item} = 8. \text{ So B is false}$$

(C) The 25th - percentile:

$$P_t = \left(\frac{t(n+1)}{100} \right)^{th} \text{ item} \Rightarrow P_{25} = \left(\frac{25(51+1)}{100} \right)^{th} \text{ item} = 13^{th} \text{ item} = 5. \text{ So C is true.}$$

(D) The $Q_3 = P_{75} = 8$, means the top 25% score is 8 and above. So D is true.

Answer:- C

42. You are in the process of buying a house, and you need a mortgage loan of Birr 200,000. You got approved for 15-year fixed rate loan at a rate of 4.5%. How much, in total, will you pay for this house? Given: $(1.00375)^{180} =$

$$1.96, (1.00375)^{-180} = 0.51, \frac{0.00375}{0.49} = 7.65 \times 10^{-3}$$

- A. Birr 275,000
 B. Birr 250,000
 C. Birr 275,40
 D. Birr 250,400

Solution:-

Given:- Mortgage loan (principal) $P = \text{Birr } 200,000$, Time $t = 15 \text{ years}$
 and $(n = mt = 12 \times 15 = 180 \text{ where } m \text{ is the fixed payment by month})$ Fixed annual

rate $r = 4.5\%$ and the interest rate per period $i = \frac{r}{m} = \frac{4.5\%}{12} = \frac{0.045}{12} = 0.00375$. Then
 the periodic payment ($p.p$):

$$p.p = P \frac{i}{1 - (1 + i)^{-n}} = \text{Birr } 200,000 \times \frac{0.00375}{1 - (1 + 0.00375)^{-180}}$$

$$= \text{Birr } 200,000 \times \frac{0.00375}{1 - (1.00375)^{-180}}$$

$$= \text{Birr } 200,000 \times \frac{0.00375}{1 - 0.51}, \text{ where } (1.00375)^{-180} = 0.51$$

$$= \text{Birr } 200,000 \times \frac{0.00375}{0.49} = \text{Birr } 200,000 \times 0.00765$$

$$= \text{Birr } 1,530.61$$

$$\text{Total payment} = n \times p.p = 180 \times \text{Birr } 1530 = \text{Birr } 275,400$$

Answer: -C

43. Hagos bought a bike from a manufacturer by 10,000 Birr and sold it to Abraha by making a profit of 10%. Abraham also sold the bike by making a profit of 15%. What is the selling price of Abraha?

- A. 12,500 Birr
- B. 11,500 Birr
- C. 11,550 Birr
- D. 12,650 Birr

Solution:-

Given:- The original price is $P = \text{Birr } 10,000$.

Let P_1 is additional 10% price of P and P_2 is additional 15% price of P_1 .

Then,

$$P_1 = P + 10\%P = P + 0.1P = P(1 + 0.1) = 1.1P$$

and

$$P_2 = P_1 + 15\%P_1 = P_1 + 0.15P_1 = P_1(1 + 0.15) =$$

$$P_1 = 1.15(1.1P) = 1.265P;$$

Where $P_2 = 1.265 \times \text{Birr } 10,000 = \text{Birr } 12,650$.

Answer:- D

44. A company wants to buy two cars from Marathon Motors. The price of each car including VAT(15%) is Birr 402,500. If the company wants to subtract a 2% withholding tax before VAT, what is the amount to be subtracted?

- a. 14,000 Birr
- b. 14,250 Birr
- c. 16,000 Birr
- d. 16,250 Birr

Solution:-

Given: Price including VAT (15%): $P = \text{Birr } 402,500$ and 2% Withhold tax before VAT (T).

Required:- T

Solution:- Let C is the original price (the cost of the car before VAT).

Then,

$$P = C + 15\%(C) = C + 0.15C = 1.15C \Rightarrow C = \frac{P}{1.15} = \frac{\text{Birr } 402,500}{1.15} = \text{Birr } 350,000$$

and

$$T = 2\% \text{ of } C = 0.02 \times \text{Birr } 350,000 = \text{Birr } 7,000 \text{ and for the two cars}$$

$$2T = 2 \times \text{Birr } 7,000 = \text{Birr } 14,000$$

Answer:- A

45. A sweatshirt with regular price of 340 Birr is first discounted by 20% and then by 10%. What is the final selling price of the sweatshirt after the discount?

- A. 238 Birr
- B. 244.80 Birr
- C. 102 Birr
- D. 95.20 Birr

Solution:-

Given:- The original price is $P = \text{Birr } 340$.

Let P_1 is 20% discount of P and P_2 is 10% discount of P_1 . Then

$$P_1 = P - 20\%(P) = P - 0.2P = P(1 - 0.2) = 0.8P$$

$$P_2 = P_1 - 10\%(P_1) = P_1 - 0.1P_1 = P_1(1 - 0.1) = 0.9P_1 = 0.9(0.8P) = 0.72P; \text{ where } P_1 = 0.8P. \text{ Thus } P_2 = 0.72 \times \text{Birr } 340 = \text{Birr } 244.80$$

Answer:- A

46. If Birr 2000 is deposited in a Bank with interest rate 10% per annum compounded semi-annually, then how much Birr will it be by the end of one year?

- A. 2100
- B. 2205
- C. 2200
- D. 2210

Solution:-

Given:- The principal $P = \text{Birr}2,000$, rate $r = 0.1 = 10\%$, compounded semi-annually $m = 2$ and time $t = 1 \text{ year}$.

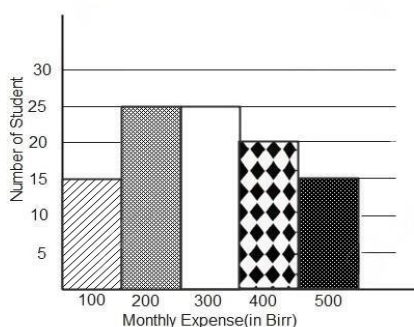
Required:- Amount

$$\text{Solution:- } A = P \left(1 + \frac{r}{m}\right)^{mt} = \text{Birr}2,000 \left(1 + \frac{0.1}{2}\right)^{2 \times 1} = \text{Birr}2,000 \times 1.1025 =$$

$\text{Birr}2,205$

Answer:- B

47. The following histogram gives the monthly expense of 100 students.



Monthly Expense (in Birr)

Which one of the following is true? (G 12 Unit 9)

- (A) Only 40 students have monthly expense less than Birr 400.
- (B) 60% of the students have monthly expense less than 400.
- (C) 35% of the students have monthly expense more than 300.
- (D) 60 students have monthly expense more than Birr 200.

Solution:-
From the given a histogram for monthly expense of 100 students the frequency distribution table will be as follows

Monthly Expense	100	200	300	400	500
Frequency	15	25	25	20	15

(A) Number of students with monthly expense of less than Birr 400:

$$n(\text{Birr } 100) + n(\text{Birr } 200) + n(\text{Birr } 300) = 15 + 25 + 25 = 65. \text{ So}$$

A is False (B) The percent of students with monthly expense of less than Birr 400:

$$\frac{65}{100} \times 100\% = 65\% \text{ So B is false.}$$

C. The percent of students with monthly expense of greater than Birr 300

$$\frac{20+15}{100} \times 100\% = 35\%. \text{ So C is true.}$$

D. The number of students with monthly expense of greater than

Birr 200 is

$$= n(\text{Birr } 300) + n(\text{Birr } 400) + n(\text{Birr } 500) = 25 + 20 + 15 = 60. \text{ So}$$

D is false.

Answer: -C

48. What is the maximum value of $z = -3x + 2y$, subject to $-x + y \leq 1$, $x - y \leq 1$ and $x + y \geq 3$?

(A) It has no maximum

(B) 4

(C) 1

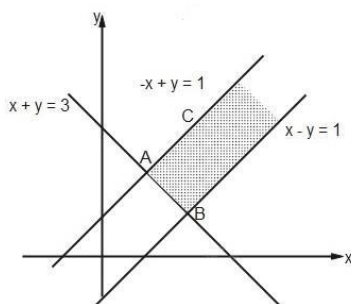
(D) 6

Solution:-

We need the maximum value of $z = -3x + 2y$

Subject to $-x + y \leq 1$, $x - y \leq 1$ and $x + y \geq 3$

The graph of the given constraints is as a region given below



If it exists optimal solution exists at vertices like points A or B as in the figure.

Solving the simultaneous equation $\begin{cases} -x + y = 1 \\ x + y = 3 \end{cases}$ we get A(1, 2)

And, solving the simultaneous equation we get $\begin{cases} x - y = 1 \\ x + y = 3 \end{cases}$ we get B(2, 1)

($x + y = 3$ Then, at A(1, 2): $Z = -3x + 2y = -3$

at B(2, 1): $Z = -3x + 2y = -3(2) + 2(1) = -4$

Since point A gives larger value of z than at point B, if there exists the maximum value of z is on the line $-x + y = 1$. Since the region is unbounded take $x = 1 + m$ for $m > 0$ like at point C. Then,

$-x + y = 1 = -(1 + m) + y = 1 \Rightarrow y = 2 + m$ so that $z = -3(1 + m) + 2(2 + m) = 1 + m > 1$ which shows 1 is not maximum value of z . As $m \rightarrow \infty$, also $\rightarrow \infty$. So that it has no maximum.

Answer:- B

49. Which one of the following points is in the solution region of the linear inequalities

$$-x + 3y \leq 10, x + y \leq 6, x - y \leq 2, x \geq 0, y \geq 0$$

- A. (1,1)
- B. (3,4)
- C. (3,0)
- D. (0,4)

Solution:-

Given:- The inequalities are

$$-x + 3y \leq 10, x + y \leq 6, x - y \leq 2, x \geq 0, \text{ and } y \geq 0.$$

Then

Points (x, y)	$-x + 3y \leq 10$	$x + y \leq 6$	$x - y \leq 2$	$x \geq 0$	$y \geq 0$
A.(1, 1)	$-1 + 3(1) \leq 10 \equiv T$	$1 + 1 \leq 6 \equiv T$	$1 - 1 \leq 2 \equiv T$	$1 \geq 0 \equiv T$	$1 \geq 0 \equiv T$
B.(3, 4)	$-3 + 3(4) \leq 10 \equiv T$	$3 + 4 \leq 6 \equiv F$	$3 - 4 \leq 2 \equiv T$	$3 \geq 0 \equiv T$	$4 \geq 0 \equiv T$
C.(3, 0)	$-3 + 3(0) \leq 10 \equiv T$	$3 + 0 \leq 6 \equiv T$	$3 - 0 \leq 2 \equiv F$	$3 \geq 0 \equiv T$	$0 \geq 0 \equiv T$
D.(0, 4)	$0 + 3(4) \leq 10 \equiv F$	$0 + 4 \leq 6 \equiv T$	$0 - 4 \leq 2 \equiv T$	$0 \geq 0 \equiv T$	$4 \geq 0 \equiv T$

Since (1,1) is only satisfying all the inequalities the answer is A.

Answer:- A

50. Which one of the following points is in the solution region of the linear inequalities?

$$-x + 3y \leq 10, x + y \leq 6, x - y \leq 2, x \geq 0, y \geq 0$$

?

- A. (1,1)
- B. (3,4)
- C. (3,0)
- D. (0,4)

Solution: -

Given:- The inequalities are
 $-x + 3y \leq 10$, $x + y \leq 6$, $x - y \leq 2$, $x \geq 0$, and $y \geq 0$.

Then

Points (x, y)	$-x + 3y \leq 10$	$x + y \leq 6$	$x - y \leq 2$	$x \geq 0$	$y \geq 0$
A.(1, 1)	$-1 + 3(1) \leq 10 \equiv T$	$1 + 1 \leq 6 \equiv T$	$1 - 1 \leq 2 \equiv T$	$1 \geq 0 \equiv T$	$1 \geq 0 \equiv T$
B.(3, 4)	$-3 + 3(4) \leq 10 \equiv T$	$3 + 4 \leq 6 \equiv F$	$3 - 4 \leq 2 \equiv T$	$3 \geq 0 \equiv T$	$4 \geq 0 \equiv T$
C.(3, 0)	$-3 + 3(0) \leq 10 \equiv T$	$3 + 0 \leq 6 \equiv T$	$3 - 0 \leq 2 \equiv F$	$3 \geq 0 \equiv T$	$0 \geq 0 \equiv T$
D.(0, 4)	$0 + 3(4) \leq 10 \equiv F$	$0 + 4 \leq 6 \equiv T$	$0 - 4 \leq 2 \equiv T$	$0 \geq 0 \equiv T$	$4 \geq 0 \equiv T$

Since (1,1) is only satisfying all the inequalities the answer is A.

Answer:- A.

51. A man owns a field of area 1000 sq.m. He wants to plant fruit trees in it. He has sum of Birr 1400 to purchase young trees. He has the choice of two types of trees. Type A requires 10 sq.m of ground per tree and cost Birr 20 per tree and Type B requires 20 sq.m of ground per tree and cost Birr 25 per tree. When fully grown, type A produces an average of 20 kg fruit which can be sold at a profit of Birr 2 per kg and type B produces an average of 40 kg fruit which can be sold at a profit of 1.5 Birr per kg. How many of each type should he plant to achieve maximum profit when the trees are fully grown?

- A. 40 trees of type A and 20 trees of type B.
- B. 20 trees of type A and 40 trees of type B.
- C. 30 trees of type A and 30 trees of type B.
- D. 35 trees of type A and 25 trees of type B.

Solution:-

Given the information as in the table below

Tree	No.of Trees to produced	Required Area (sq.m per tree)	Cost (Birr per tree)	Average fruit produced (kg)	Profit (Birr per kg)
A	x	10	20	20	2
B	y	20	25	40	1.5

$$\text{Available area} = 1000 \text{ sq.m} \Rightarrow 10x + 20y \leq 1000 \text{ (Column 3)}$$

$$x + 2y \leq 100 \dots (1)$$

and

Available money = Birr 1400 sq.m $\Rightarrow 20x + 25y \leq 1400$ (Column 4)

and

$$4x + 5y \leq 280 \dots (2)$$

$$\text{And } x \geq 0, y \geq 0$$

Type A produces average 20kg with profit 2 Birr/kg

$$\Rightarrow (20\text{kg}) (2 \text{ Birr/kg}) = \text{Birr } 40 \dots (3)$$

Type B produces average 40kg with profit 1.5 Birr/kg

$$\Rightarrow (40\text{kg}) (1.5 \text{ Birr/kg}) = \text{Birr } 60 \dots (4)$$

Number of each type to maximize the profit P

Combining (1), (2), (3) & (4), the man wants to maximize the profit P, where

$$P = P(x, y) = 40x + 60y, \text{ subjected to: } x + 2y \leq 100$$

$$4x + 5y \leq 280, x \geq 0, y \geq 0$$

The region R shown in the figure below describes the above system. And, if it exists, the optimal solution of P occurs at any of the vertices of the region R; at point O, A, B or C. Then,

i) x - intercept of $4x + 5y = 280 \Rightarrow A(70, 0)$

ii) y - intercept of $x + 2y = 100$

$$\Rightarrow C(0, 50) \text{ for } 4x + 5y = 280$$

iii) Solving the system

$$\begin{cases} x + 2y = 100 \\ 4x + 5y = 280 \end{cases} \Rightarrow B(20, 40)$$

$$\text{At } O(0, 0) P = 40x + 60y = 40(0) + 60(0) = 0$$

$$\text{At } A(70, 0) P = 40x + 60y = 40(70) + 60(0) = 2800$$

$$\text{At } B(20, 40) P = 40x + 60y = 40(20) + 60(40) = 3200$$

$$\text{At } C(0, 50) P = 40x + 60y = 40(0) + 60(50) = 3000$$

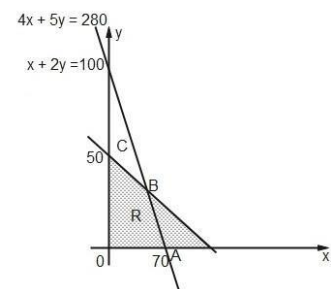
maximizes the profit P to Birr 3200.

$$\begin{aligned} 2800 \\ = 3200 \\ 3000 \end{aligned}$$

Then,

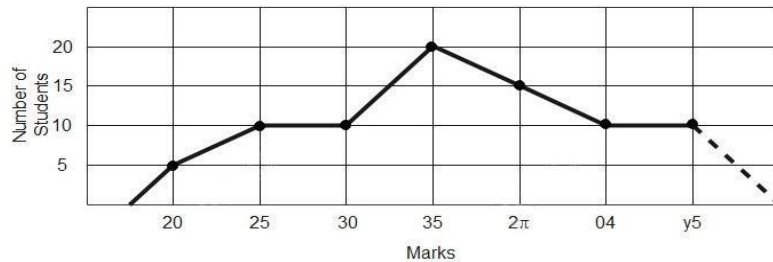
Therefore planting 20 type A & 40 type B

Answer: -B



$\Rightarrow \text{Max } P$

52. Marks scored by students in exam are grouped in to seven class intervals of equal width and represented by the following frequency polygon.



Which of the following is true about the marks?

- (A) The distribution of the mark is positively skewed.
- (B) The mean and the mode of the marks are equal.
- (C) The mode of the marks is 35.
- (D) 75% of the students scored greater than 30.

Solution:-

First, summarize the frequency polygon in to class intervals and the class marks as follows

Class Boundaries	15.5-22.5	22.5-27.5	27.5-32.5	32.5-37.5	37.5-42.5	42.5-47.5	47.5-52.5
Class Mark (x_c)	20	25	30	35	40	45	50
Frequency (f)	5	10	10	20	15	10	10
Cumulative Frequency (cf)	5	15	25	45	60	70	80

Then,

Mean

$$\bar{x} = \frac{\sum_{i=1}^7 f_i x_i}{\sum_{i=1}^7 f_i} = \frac{5(20) + 10(25) + 10(30) + 20(35) + 15(40) + 10(45) + 10(50)}{10 + 20 + 15 + 5} = \frac{2,900}{50} = 58$$

$$d_1 = f_{Mo} - f_b = 20 - 10 = 10 \text{ \& } d_2 = f_{mo} - f_s = 20 - 15 = 5$$

$$Mo = B_L \left(\frac{d_1}{d_1 + d_2} \right) i = 32.5 + \left(\frac{10}{10 + 5} \right) 5 \approx 35.83$$

The M_d class is

$$\left(\frac{n}{2} \right)^{th} \text{ item class} = \left(\frac{\sum f}{2} \right)^{th} \text{ item class} = 40^{th} \text{ item class} = 4^{th} \text{ class}$$

So $B_L = 32.5$, $cf_b = 25$, $f_c = 20$ and $i = 5$. Thus

$$M_d = B_L + \left(\frac{\frac{n}{2} - cf_b}{f_c} \right) i = 32.5 + \left(\frac{40 - 25}{20} \right) 5 = 36.25$$

The P_{25} class is

$$\left(\frac{tn}{100} \right)^{th} \text{ item class} = \left(\frac{25 \sum f}{100} \right)^{th} \text{ item class} = 20^{th} \text{ item class} = 3^{rd} \text{ class}$$

So $B_L = 27.5$, $cf_b = 15$, $f_c = 10$ and $i = 5$. Thus

$$P_{25} = B_L + \left(\frac{\frac{tn}{100} - cf_b}{f_c} \right) i = 27.5 + \left(\frac{20 - 15}{10} \right) 5 = 30 = Q_1$$

- (A) Mean $\bar{x} = 58$, Median $M_d = 36.25$, and Mode $M_o = 35.83$ so that Mean > Median > Mode So the distribution of the mark is positively skewed. Therefore (A) is True.
- (B) Mean $\neq$ Mode Therefore (B) is False
- (C) Mode $M_o = 35.85 \Rightarrow$ (C) is False
- (D) The 25th percentile $P_{25} = 30 > 30$. Therefore (D) is False.

Answer:- A