Examen de EDP

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1) Resolver el problema de Cauchy, indicando las curvas características planas

$$u u_x + u_t = 0, \quad x \in \mathbb{R}, \quad t > 0$$

$$u(x,0) = h(x), \quad \in \mathbb{R}$$

donde

$$h(x) = \begin{cases} 0, & x < 0 \\ 1, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$$

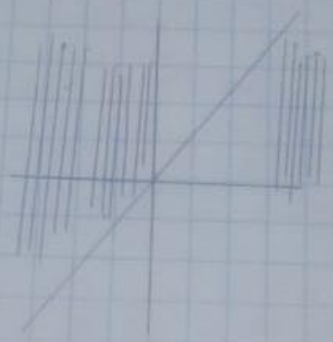
Sol

Tenemos que

$$h(x) = \begin{cases} 0 & x < 0 \Rightarrow x_1 = x < 0 \\ 1 & 0 < x < 1 \Rightarrow x < t < 1 \Rightarrow v_2(x_1) \\ 0 & x > 1 \Rightarrow v_1(x_1) = 0 \end{cases} \quad \begin{cases} \frac{dx}{dt} = 0 \\ \frac{dt}{dt} = 1 \\ \frac{dz}{dt} = 0 \end{cases}$$

$$= \begin{cases} x = \cancel{\frac{1}{c_1}}^{0} + c_1(s) \\ t = t + c_2(s) \\ z = c_3(s) \end{cases} \Rightarrow \begin{cases} s = \frac{x}{c_1} = 0 \\ t = t \\ z = h(x)\frac{x}{c_3} = 0 \end{cases} \Rightarrow u(x,t) = h(x-t) \text{ es sol. cap.}$$

*) donde tendríamos q'



$$\Rightarrow z(t) = u(0, y(t)) \rightarrow z(t) = u(0, y(t))$$

$$\frac{\partial z}{\partial t} = \frac{\partial u}{\partial x}\left(\frac{\partial x}{\partial t}\right) + \frac{\partial u}{\partial y}\frac{dy}{dt} = u_x F_p + u_y F_q$$

$$\Rightarrow = pF_p + \cancel{qF_q}^{0}$$

$$\Rightarrow = pF_p$$

5)

Dado el problema de Cauchy

$$u u_x + u_t = 0 \quad \text{en } \mathbb{R} \times \langle 0,1 \rangle$$

$$u(x,0) = -x, \ \in \mathbb{R}$$

a) encuentre la solución $u = u(x,t)$ en $C^1(\mathbb{R} \times \langle 0,1 \rangle \cap (\mathbb{R} \times [0,1\rangle)$

b) Grafique las curvas características y de la solución $u(x,t)$

c) ¿Que ocurre cuando $t \to 1$? (es decir halle $\lim_{(x,t) \to (x_0,1)} u(x,t)$)

Sol a)

donde $x(s,r)$, $t(s,r)$, $u(s,r) \Rightarrow x(s,0) = s$, $t(s,0) = 0$, $u(s,0) = -s$

$$\Rightarrow \begin{cases} \dfrac{\partial t}{\partial t} = 1 \\ \dfrac{\partial x}{\partial t} = 0 \\ \dfrac{\partial u}{\partial t} = 0 \end{cases} \xRightarrow{r=0} \begin{cases} x(s,t) = s \\ t(s,t) = 0 \\ t(s,t) = -s \end{cases}$$

$$\Rightarrow \frac{\partial t}{\partial r} = 1 \Rightarrow t(s,0) = r + c(s) = t(s,0) = c_1(s) = t(s,t) = t$$

$$\frac{\partial x}{\partial t} = u \Rightarrow \frac{\partial x}{\partial t} = -s \Rightarrow x(s,r) = -rs + c_2(s) \Rightarrow x(s,0) = c_2(s) = -rs + s$$

$$\frac{\partial u}{\partial t} = 0 \Rightarrow u(s,t) = c_3(s) \Rightarrow u(s,0) = c_3(s) = -s \Rightarrow u(s,t) = -s$$

$$\Rightarrow \begin{cases} -rs + s \\ r \\ -s \end{cases} \Rightarrow x = s(1-r) = s(1-t) \Rightarrow s = \frac{x}{1-t} \Rightarrow x = \frac{x}{t-1}$$

Como $u = u(x,t) \equiv u(s,r) \Rightarrow \begin{cases} \text{Para } C_{s_1} = \mathbb{R} \times \langle 0,1 \rangle, \ C_{s_1} = (\mathbb{R} \times [0,1\rangle) \end{cases}$

$$\Rightarrow \begin{matrix} -rs + s = r \\ s(-r+1) = r \end{matrix} \Rightarrow s = \frac{r}{-r+1} \Rightarrow s = \frac{r}{r-1}$$

$$\Rightarrow u(s,r) \in C^1(\mathbb{R} \times \langle 0,1 \rangle \cap (\mathbb{R} \times [0,1\rangle))$$

b) Graficando



4) Dado el problema de cauchy

$$u_x^3 = u_y$$

$$g(x,0) = 2x^{3/2}$$

a) Resolver el problema

b) Indique cual de las dos soluciones

$u(x,t) = 2x^{3/2}(1-2t)^{-3/2}$ o $u(x,t) = 2x^{3/2}(2t-1)^{-3/2}$ es

la respuesta correcta.

Sol(a)

Vemos que $u_x^3 = u_y \iff u_x^3 - u_y = 0$ donde

$F = F(x,y,z,u_x,u_y) = F(x,y,u,p,q)$ entonces

$z(t) = u(x(t), y(t)) = ?$; $p = u_x$, $q = u_y$

$\Rightarrow F(x,y,z,p,q) = p^3 - q$ por lo que

$F_p = 3p^2$, $F_q = -1$, $F_x = 0$, $F_y = 0$, $F_z = 0$

$\frac{dx}{dt} = F_p = 3p^2$ $\Rightarrow \frac{dz}{dt} = pF_p + qF_q = p(3p^2) + q = 3p^3$

$\frac{dy}{dt} = F_q = -1$

$z = 3p^3 + q$ Resolviendo cauchy en el sistema se tiene

$$\begin{cases} \delta^3(s) + \beta(s) = 0 \\ \delta(s)w + \beta(s) = 3s^{1/2} \end{cases} \iff \begin{cases} \delta^3 - \beta = 0 \\ \delta = 3s^{1/2} \end{cases}$$

Asi $\beta(s) = 3s^{1/2}$, $\delta(s) = s$ donde

$\frac{dp}{dt} = p \Rightarrow \frac{dp}{p} = dt \Rightarrow \ln p = t + c_0 \Rightarrow p = ke^t$ como $p(0)$

$\Rightarrow k e^0 = 3s^{1/2} \Rightarrow k_1 = 3s^{1/2}$ Asi $p = 3s^{1/2}e^t = p(s,t)$

$\frac{dq}{dt} = q \Rightarrow \frac{dq}{q} = dt \Rightarrow \ln q = t + c_2 \Rightarrow q = ke^t$ como

$q(0) = \delta(s) \Rightarrow \ln q = k_1 e^0 = s$ Asi $q = se^t$

$\frac{dz}{dt} = 3s^{1/2} \Rightarrow \frac{dz}{z} = 3s^{1/2}dt \rightarrow \ln z = 3s^{1/2}(s^{1/2}) \rightarrow \ln z = 27s^{1/2}e^{t}+c$

$\Rightarrow z = k_1 e^{3s^{1/2}}$

$* \frac{dx}{dt} = 3e^{t} = 3(3s^{1/2}e^{t}) = 9s^{1/2}e^{t} \Rightarrow dx = 9s^{1/2}e^{t}dt$

$\Rightarrow 9s^{1/2}e^{t} = 9s^{1/2}e^{t}+c$

$\Rightarrow \frac{dy}{dt} = 1 = k_2 e^{t} = k_2 e^{t}dy \Rightarrow k_2 e^{t}dt$ como

$y(0) = y(c) \Rightarrow k_2 e^{t} = 0 \Rightarrow k_2 = 0$ por lo que $y = k_2 e^{t}$ desprecio

$9s^{1/2}e^{t} = 1$

a) si $9s^{1/2} = 27s^{1/2}e^{t+c} \Rightarrow 9(s)^{1/2} + 27s^{1/2} \Rightarrow s = x = 36s^{1/2} - 2x^{1/2}$

$2x^{3/2}$ como $u(x,0) = 2x^{3/2} \Rightarrow 2x^{3/2} = (2^{-1/2} - 27y) \Rightarrow (1)$

b) si $1(e^{t}) = y \Rightarrow e^{t} = y$ por lo que tenemos que

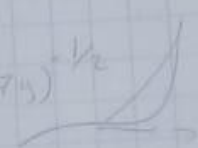
Reemplazando en valor de $u(x,y)$ tenemos

$u(x,y) = 36s^{1/2} - 2s^{1/2} = 34s^{1/2} \pm e^{t}/y \Rightarrow$ reemplazando en

(x,y) tenemos por (1) $\Rightarrow$

$u(x,y) = \frac{2x^{3/2}+c}{(1-27y)^{-1/2}} \Rightarrow 1 \Rightarrow (1-27y)^{-1/2}(2x^{3/2}) = 0$

$\Rightarrow u(x,y) = 2x^{3/2}(1-27y)^{-1/2}$



b) R.T.A es $2x^{3/2}(1-27y)^{-1/2}$

