Chapter One

Introduction to Data Structures and Algorithms

Introduction

A program

- A set of instruction which is written in to solve a problem.
- A solution to a problem actually consists of two things:
  - ✓ A way to organize the data
  - ✓ Sequence of steps to solve the problem
- The way data are organized in a computers memory is said to be **Data Structure.**
- The sequence of computational steps to solve a problem is said to be an **Algorithm.**
- Therefore, a *program* is **Data structures** plus **Algorithm.**

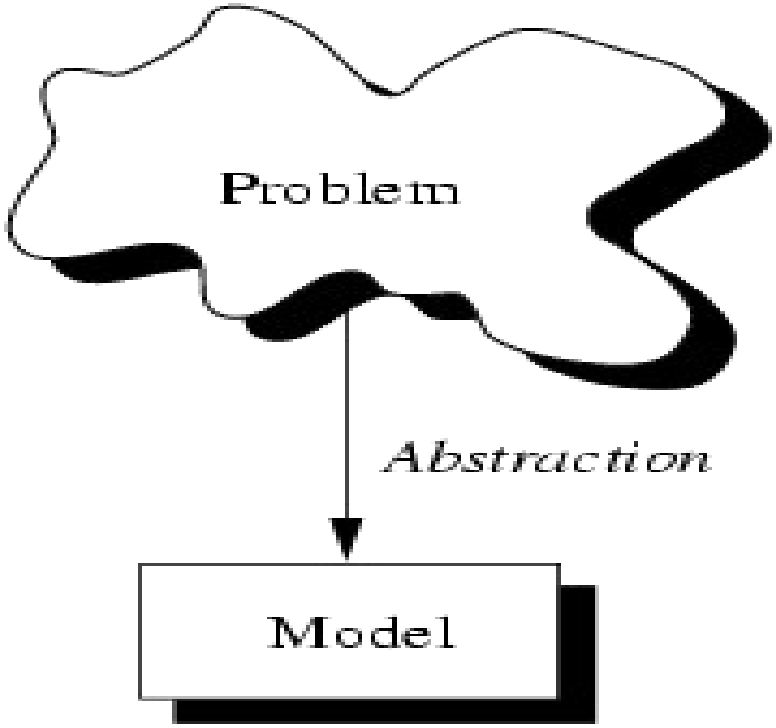
Introduction…

Data structures are used to model the world or part of the world. How?

1. The value held by a data structure represents some specific characteristic of the world.
2. The characteristic being modeled restricts the possible values held by a data structure and the operations to be performed on the data structure.

- The first step to solve the problem is obtaining ones own **abstract view,** or **model,** of the problem.
- This process of modeling is called **abstraction.**

Introduction…



- The model defines an abstract view to the problem.
- The model should only focus on problem related stuff

Abstraction

- Is a process of classifying characteristics as **relevant** and **irrelevant** for the particular purpose at hand and ignoring the **irrelevant** ones.

Example: model students of A University.

- **Relevant:**

  Char Name[15];
  Char ID[11];
  Char Dept[20];
  int Age, year;

- **Non relevant**

  float hieght, weight;

Abstraction…

- Using the model, a programmer tries to define the **properties** of the problem.
- These properties include
  - ✓ The **data** which are affected and
  - ✓ The **operations** that are involved in the problem
- An entity with the properties just described is called an **abstract data type (ADT).**

Abstract Data Types

- Consists of data to be stored and operations supported on them.
- Is a specification that describes a data set and the operation on that data.

- The ADT specifies:
  - ✓ What data is stored.
  - ✓ What operations can be done on the data.
- Does not specify how to store or how to implement the operation.
- Is independent of any programming language

Abstract Data Types…

Example: ADT employees of an organization:

- ✓ This ADT stores employees with their relevant attributes and discarding irrelevant attributes.

Relevant:- Name, ID, Sex, Age, Salary, Dept, Address

Non Relevant :- weight, color, height

- ✓ This ADT supports **hiring, firing, retiring, …**

operations.

Data Structure

- In Contrast a data structure is a language construct that the programmer has defined in order to implement an *abstract data type*.
- What is the purpose of data structures in programs?
  - Data structures are used to model a problem.

Example:

```
struct Student_Record
{
char name[20];
char ID_NO[10];
char Department[10];
int age;
};
```

Data Structure…

- Attributes of each variable:
  - **Name:** Textual label.
  - **Address**: Location in memory.
  - **Scope**: Visibility in statements of a program.
  - **Type**: Set of values that can be stored + set of operations that can be performed.
  - **Size**: The amount of storage required to represent the variable.
  - **Life time**: The time interval during execution of a program while the variable exists.

Algorithm

- Is a concise specification of an operation for solving a problem.
- Is a well-defined computational procedure that takes some value or a set of values as input and produces some value or a set of values as output.

Inputs → **Algorithm** → **Outputs**

- An algorithm is a specification of a behavioral process. It consists of a finite set of instructions that govern behavior step-by-step.
- Is part of what constitutes a data structure

Algorithm…

- Data structures model the static part of the world. They are unchanging while the world is changing.
- In order to model the dynamic part of the world we need to work with algorithms.
- Algorithms are the dynamic part of a program's world model.
- An **algorithm** transforms **data structures** from one state to another state.

Algorithm…

- What is the purpose of algorithms in programs?
  - Take values as input.

  Example: cin>>age;
  - Change the values held by data structures.

  Example: age=age+1;
  - Change the organization of the data structure:

  Example:

  Sort students by name
  - Produce outputs:

  Example: Display student's information

Algorithm…

- The quality of a data structure is related to its ability to successfully model the characteristics of the world (problem).
- Similarly, the quality of an algorithm is related to its ability to successfully simulate the changes in the world.
- However, the quality of data structure and algorithms is determined by their ability to work together well.
- Generally speaking, correct data structures lead to simple and efficient algorithms.
- And correct algorithms lead to accurate and efficient data structures.

Properties of Algorithms

1. Finiteness:

- Algorithm must complete after a finite number of steps.
- Algorithm should have a finite number of steps.

Example of Finite Steps

```
int i=0;
while(i<10)
{
 cout<< i;
 i++;
}
```

Example of Infinite Steps

```
while(true)
{
 cout<<“Hello”;
}
```

2. Definiteness (Absence of ambiguity):

- Each step must be clearly defined, having one and only one interpretation.
- At each point in computation, one should be able to tell exactly what happens next.

3. Sequential:

- Each step must have a uniquely defined preceding and succeeding step.
- The first step (start step) and last step (halt step)

  must be clearly noted.

4. Feasibility:

- It must be possible to perform each instruction.
- Each instruction should have possibility to be executed.

a)
```
for(int i=0; i<0; i++)
        {
        cout<< i;  // there is no possibility that this

        }                  //statement to be executed.
```

b)
```
      if(5>7)
       {
        cout<<“hello”; // not executed.
      }
```

5. Correctness:

- It must compute correct answer for all possible legal inputs.
- The output should be as expected and required and correct.

6. Language Independence:

- It must not depend on any one programming

  language.

7. Completeness:

- It must solve the problem completely.

8. Effectiveness:

- Doing the right thing. It should yield the correct result all the time for all of the possible cases.

9. Efficiency:

- It must solve with the least amount of computational resources such as time and space.
- Producing an output as per the requirement within the given **resources (constraints).**

Example: 1) Write a program that takes a number and displays the square of the number.

```
int x;
cin>>x;
cout<<x*x;
```

2)

```
int x,y;
cin>>x;
y=x*x;
cout<<y;
```

Example:

Write a program that takes two numbers and displays the sum of the two.

Program a

```
cin>>a;
cin>>b;
sum= a+b;

cout<<sum;
```

Program b

```
cin>>a;
cin>>b;
a= a+b;

cout<<a;
```

Program c (the most efficient)

```
cin>>a;
cin>>b;
cout<<a+b;
```

All are **effective** but with **different efficiencies**.

10. Input/output:

- There must be a specified number of input values, and one or more result values.
- Zero or more inputs and one or more outputs.

11. Precision:

- The result should always be the same if the algorithm is given identical input**.**

12. Simplicity:

- A good general rule is that each step should carry out one logical step.
  - What is simple to one processor may not be simple to another.

13. Levels of abstraction:

- Used to organize the ideas expressed in algorithms.
  - Used to hide the details of a given activity and refer to just a name for those details.
  - The simple (detailed) instructions are hidden inside modules.
  - Well-designed algorithms are organized in terms of levels of abstraction.

Algorithm Analysis

- Algorithm analysis refers to the process of determining how much **computing time** and **storage** that algorithms will require.
- In other words, it's a process of predicting the **resource requirement** of algorithms in a given environment.
- In order to solve a problem, there are many possible algorithms.
- One has to be able to choose the best algorithm for the problem at hand using some scientific method.

- To classify some data structures and algorithms as good:
  - we need precise ways of analyzing them in terms of resource requirement.

  The main resources are:
    - **Running Time**
    - **Memory Usage**
    - **Communication Bandwidth**

Note: ***Running time*** is the most important since computational time is the most precious resource in most problem domains.

There are *two* approaches to measure the efficiency of algorithms: Empirical and Theoretical

1. Empirical

- based on the total running time of the program.
- Uses actual system clock time.

Example:

t1= initial time before the program starts

```
for(int i=0; i<=10; i++)
    cout<<i;
```

t2 = final time after the execution of the program is finished

Running time taken by the above algorithm or

Total Time = t2-t1;

- It is difficult to determine efficiency of algorithms using this approach, because clock-time can vary based on many factors.

For example:

a) **Processor speed of the computer**

1.78GHz 2.12GHz

10s <10s

b) **Current processor load**

- Only the work 10s
- With printing 15s
- With printing & browsing the internet >15s

c) **Specific data for a particular run of the program**

- Input size, Input properties

t1

```
for(int i=0; i<=n; i++)
    cout<<i;
```

t2

T=t2-t1;

For n=100, T>=0.5s

n=1000, T>0.5s

d) **Operating System**

- Multitasking Vs Single tasking
- Internal structure

2. Theoretical

- Determining the quantity of resources required using mathematical concept.
- Analyze an algorithm according to the **number of basic operations (time units)** required, rather than according to an absolute amount of time involved.

We use theoretical approach to determine the efficiency of algorithm because:

- The number of operation will not vary under different conditions.
- It helps us to have a meaningful measure that permits comparison of algorithms independent of operating platform.
- It helps to determine the **complexity of algorithm.**

Complexity Analysis

Complexity Analysis is the systematic study of the cost of computation, measured either in:

- Time units
- Operations performed, or
- The amount of storage space required.

Two important ways to characterize the effectiveness of an algorithm are its:

Space Complexity and

Time Complexity.

Time Complexity:

- Determine the approximate amount of time (number of operations) required to solve a problem of size n.
  - The limiting behavior of time complexity as size increases is called the **Asymptotic Time Complexity.**

Space Complexity:

- Determine the approximate memory required to solve a problem of size n.
  - The limiting behavior of space complexity as size increases is called the **Asymptotic Space Complexity.**

- **Asymptotic Complexity** of an algorithm determines the size of problems that can be solved by the algorithm.
- Factors affecting the running time of a program:
  - CPU type (80286, 80386, 80486, Pentium I---IV)
  - Memory used, Computer used, Programming Language - C (fastest), C++ (faster), Java (fast)
    C is relatively faster than Java, because C is relatively
    nearer to Machine language. So, Java takes relatively
    larger amount of time for interpreting or translation to machine code.
  - Algorithm used and Input size

Note: Important factors for this course are

Input size and Algorithm used.

Complexity analysis involves two distinct phases:

- **Algorithm Analysis**: Analysis of the algorithm or data structure to produce a function **T(n)** that describes the algorithm in terms of the operations performed in order to measure the complexity of the algorithm.

Example: Suppose we have hardware capable of executing 10^6 instructions per second. How long would it take to execute an algorithm whose complexity function is $T(n)=2n^2$ on an input size of $n=10^8$?

Solution: $T(n) = 2n^2 = 2(10^8)^2 = 2*10^{16}$

Running time $= T(10^8)/10^6 = 2*10^{16}/10^6 = 2*10^{10}$ seconds.

Order of Magnitude Analysis: Analysis of the function *T (n)* to determine the general complexity category to which it belongs.

- There is no generally accepted set of rules for algorithm analysis.
- However, an exact count of operations is commonly used.
- To count the number of operations we can use the following Analysis Rule.

Analysis Rules:

1. Assume an arbitrary time unit.
2. Execution of one of the following operations takes time 1 unit:
   - Assignment Operation, Example: i=0;
   - Single Input/Output Operation
     Example: cin>>a;
     cout<<“hello”;
   - Single Boolean Operations, Example: i>=10
   - Single Arithmetic Operations, Example: a+b;
   - Function Return, Example: return sum;
3. Running time of a selection statement (if, switch) is the time for the condition evaluation plus the maximum of the running times for the individual clauses in the selection.

Example:

```
int x;
int sum=0;
if(a>b)
 {
    sum= a+b;
    cout<<sum;
 }
else
 {
   cout<<b;
 }
```

T(n) = 1 +1+max(3,1) = 5

4. **Loop statements:**

- The running time for the statements inside the loop * number of iterations + time for setup(1) + time for checking (number of iteration + 1) + time for update (number of iteration)
- The total running time of statements inside a group of nested loops is the running time of the statements * the product of the sizes of all the loops.
- For nested loops, analyze inside out.
- Always assume that the loop executes the maximum number of iterations possible. (Why?)
    - Because we are interested in the worst case complexity.

5. Function call:

1 for setup + the time for any parameter calculations + the time required for the execution of the function body.

Examples:

1)

```
int k=0,n;
cout<<“Enter an integer”;
cin>>n
for(int i=0;i<n; i++)
        k++;
```

T(n)= 3+1+n+1+n+n=3n+5

2)
```
int i=0;
while(i<n)
{
  cout<<i;
  i++;
 }
  int j=1;
  while(j<=10)
{
  cout<<j;
  j++;
}
```

T(n) = 1+n+1+n+n+1+11+2(10)

= 3n+34

3)

```
int k=0;
for(int i=1 ; i<=n; i++)
  for( int j=1; j<=n; j++)
      k++;
```

$T(n)=1+1+(n+1)+n+n(1+(n+1)+n+n)$

$= 2n+3+n(3n+2)$

$= 2n+3+3n^2+2n$

$= 3n^2+4n+3$

4). int sum=0;
for(i=1;i<=n;i++))
sum=sum+i;

T(n)=1+1+(n+1)+n+(1+1)n
=3+4n=O(n)

5). int counter(){
int a=0;
cout<<”Enter a number”;

cin>>n;
for(i=0;i<n;i++)
a=a+1;
return 0; }

T(n)=1+1+1+(1+n+1+n)
+2n+1
=4n+6=O(n)

```
6). void func( ){
        int x=0;  int i=0;  int j=1;
        cout<<"Enter a number";
        cin>>n;
        while(i<n){                    i=i+1;

   }
         while(j<n){
             j=j+1;
         }
      }
```

T(n)=1+1+1+1+1+n+1+2n+n+2(n-1)

= 6+4n+2n-2

=4+6n=O(n)

```
7). int sum(int n){
        int s=0;
        for(int i=1;i<=n;i++)
            s=s+(i*i*i*i);
    return s;
    }
```

T(n)=1+(1+n+1+n+5n)+1

=7n+4=O(n)

```
8). int sum=0;
    for(i=0;i<n;i++)
        for(j=0;j<n;j++)
            sum++;
```

T(n)=1+1+(n+1)+n+n*(1+(n+1)+n+n)

=3+2n+n²+2n+2n²

=3+2n+3n²+2n

=3n²+4n+3=O(n²)

Formal Approach to Analysis

- In the above examples we have seen that analyzing Loop statements is so complex.
- It can be simplified by using some formal approach in which case we can ignore **initializations**, **loop controls**, and **updates**.

Simple Loops: Formally

- **For loop** can be translated to a summation.
- The index and bounds of the summation are the same as the index and bounds of the **for** loop.

- Suppose we count the number of additions that are done. There is 1 addition per iteration of the loop, hence n additions in total.

```
for (i = 1; i < n; i++) {
    sum = sum + a[i];
}
```

$$\sum_{i=1}^{N}$$

Nested Loops: Formally

- Nested **for** loops translate into multiple summations, one for each **For loop**.

```
for(int i = 1; i <= N; i++) {
    for(int j = 1; j <= M; j++) {
        sum = sum + i + j;
    }
}
```

$$\sum_{i=1}^{N}\sum_{j=1}^{M} 2 = \sum_{i=1}^{N} 2M = 2MN$$

Consecutive Statements: Formally

- Add the running times of the separate blocks of your code.

```
for (int i = 1; i <= N; i++) {
    sum = sum+i;
}
for (int i = 1; i <= N; i++) {
    for (int j = 1; j <= N; j++) {
        sum = sum+i+j;
    }
}
```

$$\left[\sum_{i=1}^{N} 1\right] + \left[\sum_{i=1}^{N}\sum_{j=1}^{N} 2\right] = N + 2N^2$$

Conditionals: (Formally take maximum)

Example:

```
if (test == 1) {
   for (int i = 1; i <= N; i++) {
      sum = sum + i;
}}
else for (int i = 1; i <= N; i++) {
      for (int j = 1; j <= N; j++) {
         sum = sum + i + j;
}}
```

$$\max\left(\sum_{i=1}^{N} 1, \sum_{i=1}^{N}\sum_{j=1}^{N} 2\right) =$$

$$\max(N, 2N^2) = 2N^2$$

Recursive: Formally

Usually difficult to analyze.

Example: Factorial

```
long factorial(int n){
    if(n<=1)
        return 1;
    else
        return n*factorial(n-1);
}
```

T(n)=1+T(n-1)=2+T(n-2)=3+T(n-3)=-------

=n-1 (counting the number of multiplication)

Categories of Algorithm Analysis

- Algorithms may be examined under different situations to correctly determine their efficiency for accurate comparison.

Best Case Analysis:

- Assumes the input data are arranged in the most advantageous order for the algorithm.
- Takes the smallest possible set of inputs.
- Causes execution of the fewest number of statements.

Computes the **lower bound of T(n)**, where T(n) is the complexity function.

Examples:

For **sorting algorithm**

- If the list is already sorted (data are arranged in the required order).

For **searching algorithm**

- If the desired item is located at first accessed position.

Worst Case Analysis:

- Assumes the input data are arranged in the most disadvantageous order for the algorithm.
- Takes the worst possible set of inputs.
- Causes execution of the largest number of statements(operations).
- Computes the upper bound of T(n) where T(n) is the complexity function.

Example:

While sorting, if the list is in opposite order.

While searching, if the desired item is located at

the last position or is missing.

Worst Case Analysis:

Worst case analysis is the most common analysis because:

- It provides the upper bound for all input (even for bad ones).
- Average case analysis is often difficult to determine and define.
- If situations are in their best case, no need to develop algorithms because data arrangements are in the best situation.
- Best case analysis can not be used to estimate complexity.
- We are interested in the worst case time since it provides a bound for all input-this is called the “Big-Oh” estimate.

Average Case Analysis:

- Determine the average of the running time overall permutation of input data.
- Takes an average set of inputs.
- It also assumes random input size.
- It causes average number of executions.
- Computes the **optimal bound** of T(n) where T(n) is the complexity function.
- Sometimes average cases are as bad as worst cases and as good as best cases.

Examples:

For sorting algorithms

- While sorting, considering any arrangement (order of input data).

For searching algorithms

- While searching, if the desired item is located at any location or is missing.

The study of algorithms includes:

- How to Design algorithms (Describing algorithms)
- How to Analyze algorithms (In terms of time and memory space)
- How to validate algorithms (for any input)
- How to express algorithms (Using programming language)
- How to test a program (debugging and maintaining)

But, in this course more focus will be given to Design and Analysis of algorithms.

Order of Magnitude

- Refers to the rate at which the storage or time grows as a function of problem size.
- It is expressed in terms of its relationship to some known functions.
- This type of analysis is called **Asymptotic analysis.**

Asymptotic Notations

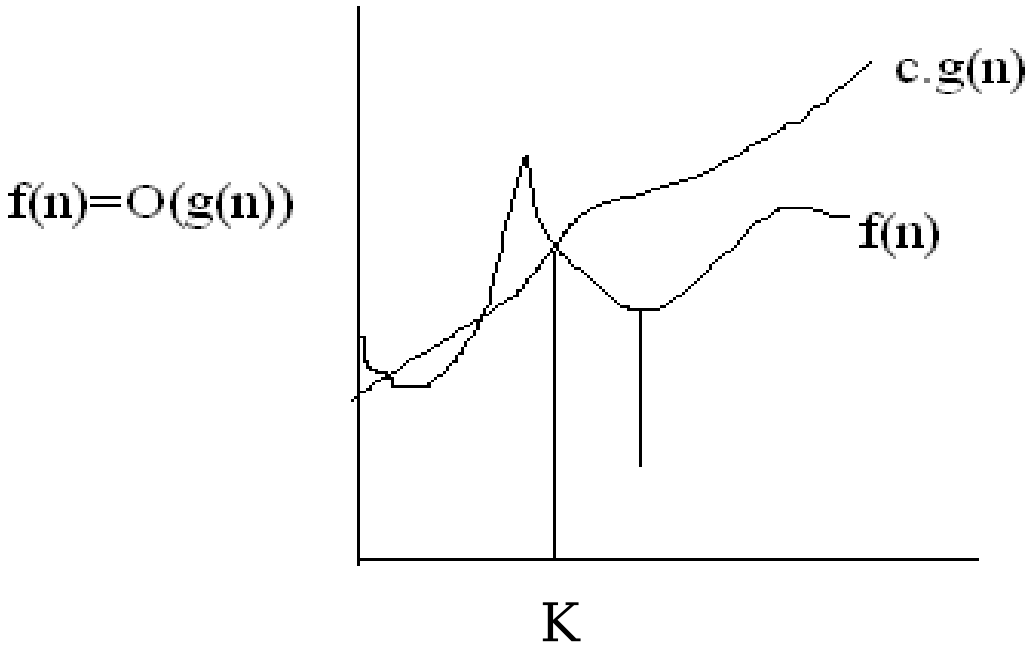
- Asymptotic Analysis is concerned with how the running time of an algorithm increases with the size of the input **in the limit**, as the size of the input increases **without bound!**
- Asymptotic Analysis makes use of **O (Big-Oh) , Ω (Big-Omega), θ (Theta), o (little-o), ω (little-omega)** - notations in performance analysis and characterizing the complexity of an algorithm.
- **Note:** The complexity of an algorithm is a numerical function of the size of the problem (instance or input size).

Types of Asymptotic Notations

1. **Big-Oh Notation (O)**

Definition: We say **f(n)=O(g(n)),** if there are positive constants **k** and **c,** such that to the right of **k**, the value of **f(n)** always lies on or below **c.g(n)**.

- As n increases **f(n)** grows no faster than **g(n).**
- It's only concerned with what happens for very large values of n.
- Describes the worst case analysis.
- Gives an upper bound for a function to within a constant factor.



- O-Notations are used to represent the amount of time an algorithm takes on the worst possible set of inputs, “Worst-Case”

Question-1

f(n)=10n+5 and **g(n)=n.**

Show that **f(n) is O(g(n)).** To show that f(n) is O(g(n)), we must show that there exist constants c and k such that f(n)<=c.g(n) for all n>=k.

10n+5<=c.n ➔ for all n>=k

let c=15, then show that 10n+5<=15n

5<=5n or 1<=n

So, f(n)=10n+5<=15.g(n) for all n>=1

(c=15, k=1), there exist two constants that satisfy the above constraints.

Question-2

$f(n)=3n^2+4n+1$.

Show that **$f(n)=O(n^2)$.**

3$n^2 <= 3n^2$ for all $n>=1$

$4n<=4n^2$ for all $n>=1$ and

$1<=n^2$ for all $n>=1$

$3n^2+4n+1<=3n^2+4n^2+n^2$ for all $n>=1$

$<=8n^2$ for all $n>=1$

So, we have shown that $f(n)<=8n^2$ for all $n>=1$.

Therefore, f(n) is $O(n^2)$, (c=8, k=1), there exist two constants that satisfy the constraints.

2. **Big-Omega (Ω)-Notation** (Lower bound)

- Definition: We write **f(n)= Ω(g(n))** if there are positive constants **k** and **c** such that to the right of **k** the value of **f(n)** always lies on or above **c.g(n).**
- As n increases **f(n)** grows no slower than **g(n).**
- Describes the best case analysis.
- Used to represent the amount of time the algorithm takes on the smallest possible set of inputs-“Best case”.

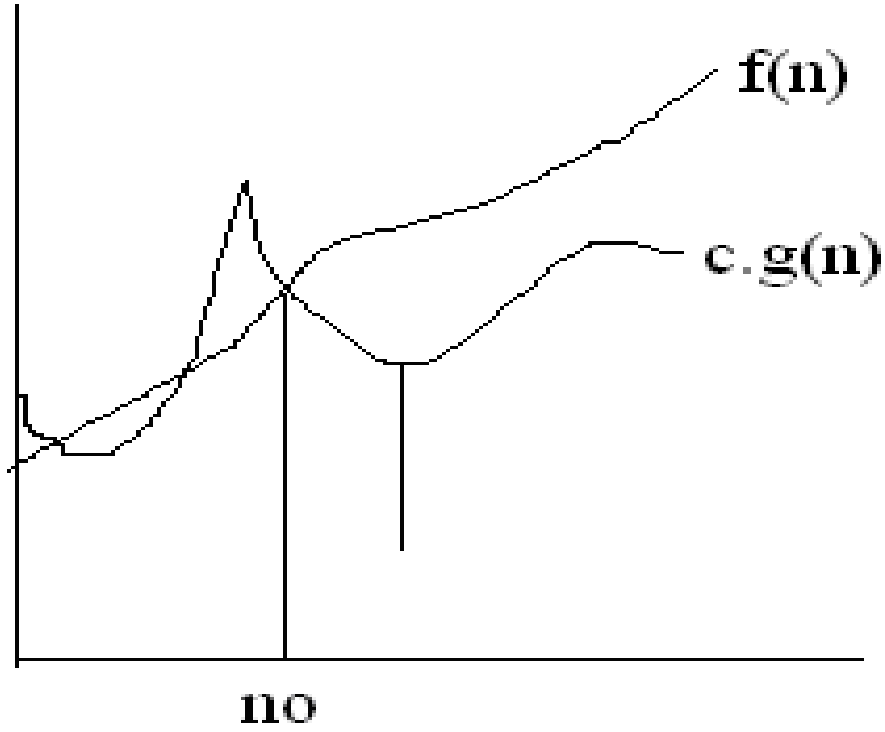
Example:

Find **g(n)** such that **f(n) = Ω(g(n))** for
f(n)=3n+5, g(n) = √n, c=1, k=1.

f(n)=3n+5=Ω(√n)

Big-Omega (Ω)-Notation (Lower bound)

$f(n) = \Omega(g(n))$



3. Theta Notation (θ-Notation) (Optimal bound)

- Definition: We say **f(n)= θ(g(n))** if there exist positive constants **n_o**, **c1** and **c2** such that to the right of **n_o**, the value of **f(n)** always lies between **c1.g(n)** and **c2.g(n)** inclusive, i.e., **c2.g(n)<=f(n)<=c1.g(n),** for all **n>=n_o**.
- As n increases **f(n)** grows as fast as **g(n).**
- Describes the average case analysis.
- To represent the amount of time the algorithm takes on an average set of inputs- "Average case".

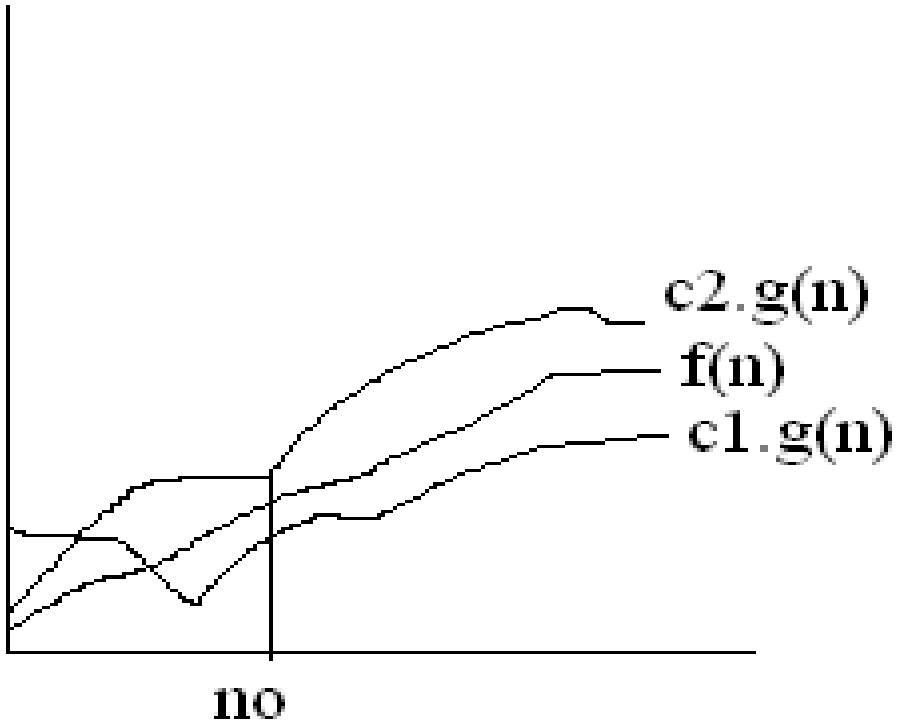
Example: Find **g(n)** such that **f(n) = Θ(g(n)) for f(n)=2n²+3**

→ **$n^2 \leq 2n^2 \leq 3n^2$** → **c1=1, c2=3** and **n_o=1**

→**f(n) = Θ(g(n)).**

Theta Notation (θ-Notation) (Optimal

$f(n) = \theta(g(n))$



4. Little-oh (small-oh) Notation

- Definition: We say **f(n)=o(g(n)),** if there are positive constants $\mathbf{n_o}$ and **c** such that to the right of $\mathbf{n_o}$, the value of **f(n)** lies below **c.g(n).**
- As n increases, **g(n)** grows strictly faster than **f(n).**
- Describes the worst case analysis.
- Denotes an upper bound that is not asymptotically tight.
- Big O-Notation denotes an upper bound that may or may not be asymptotically tight.

Example:

Find **g(n)** such that **f(n) = o(g(n))** for **f(n)** = n^2

$n^2 < 2n^2$, for all $n>1$, ➔ $k=1$, $c=2$, $g(n)=n^2$

$n^2 < n^3$, $g(n) = n^3$, $f(n)=o(n^3)$

$n^2 < n^4$, $g(n) = n^4$, $f(n)=o(n^4)$

5. Little-Omega (ω) notation

- Definition: We write **f(n)=ω(g(n)),** if there are positive constants $\mathbf{n_o}$ and **c** such that to the right of $\mathbf{n_o}$, the value of **f(n)** always lies above **c.g(n).**
- As n increases **f(n)** grows strictly faster than **g(n)**.
- Describes the best case analysis.
- Denotes a lower bound that is not asymptotically tight.
- Big Ω-Notation denotes a lower bound that may or may not be asymptotically tight.

Example: Find **g(n)** such that **f(n)=ω(g(n))** for **$f(n)=n^2+3$**

g(n)=n, Since **$n^2 > n$**, **c**=1, **k**=2.

g(n)=√n, Since $n^2 > \sqrt{n}$, c=1, k=2, can also be solution.

Rules to estimate Big Oh of a given function

- Pick the highest order.
- Ignore the coefficient.

Example:

1. $T(n)=3n + 5$ ➔ $O(n)$
2. $T(n)=3n^2+4n+2$ ➔ $O(n^2)$

Some known functions encountered when analyzing algorithms. (Complexity category for Big-Oh).

See next slide ➔

Rule 1:

If T1(n)=O(f(n)) and T2(n)=O(g(n)), then

a) T1(n)+T2(n)=max(O(f(n)),O(g(n))),

b) T1(n)*T2(n)=O(f(n)*g(n))

Rule 2:

If T(n) is a polynomial of degree k, then $T(n)=\theta(n^k)$.

Rule 3:

$\log^k n = O(n)$ for any constant k. This tells us that logarithms grow very slowly.

- We can always determine the relative growth rates of two functions f(n) and g(n) by computing

  lim n → infinity f(n)/g(n).

- The limit can have four possible values.

- The limit is 0: This means that $f(n)=o(g(n))$.
- The limit is $c \neq 0$: This means that $f(n)=\theta(g(n))$.
- The limit is infinity: This means that $g(n)=O(f(n))$.
- The limit oscillates: This means that there is n_o relation between f(n) and g(n).

Example:

- n^3 grows faster than n^2, so we can say that $n^2=O(n^3)$ or $n^3=\Omega(n^2)$.
- $f(n)=n^2$ and $g(n)=2n^2$ grow at the same rate, so both $f(n)=O(g(n))$ and $f(n)=\Omega(g(n))$ are true.
- If $f(n)=2n^2$, $f(n)=O(n^4)$, $f(n)=O(n^3)$, and **$f(n)=O(n^2)$** are all correct, but the last option is the best answer.

T(n)	Complexity Category functions F(n)	Big-O
c, c is constant	1	C=O(1)
10logn + 5	logn	T(n)=O(logn)
√n +2	√n	T(n)=O(√n)
5n+3	n	T(n)=O(n)
3nlogn+5n+2	nlogn	T(n)=O(nlogn)
$10n^2$ +nlogn+1	n^2	T(n)=O(n^2)
$5n^3 + 2n^2 + 5$	n^3	T(n)=O(n^3)
2^n+n^5+n+1	2^n	T(n)=O(2^n)

Complexity category	Big-Oh	Example
Constant	T(n)=O(1)-constant growth	Determining if a number is even or odd.
Logarithmic	T(n)=O(logn)	Finding an item in a sorted array with a binary search. 10logn+5
Linear	T(n)=O(n)	Finding an item in an unsorted list, adding two n-digit numbers. 5/3n+10
Loglinear	T(n)=O(nlogn)	Merge sort, heap sort 3nlogn+5n+2
Quadratic	$T(n)=O(n^2)$	Adding two nXn matrices, shell sort, insertion sort, multiplying two n-digit numbers by a simple algorithm. $11/7n^2+2n+5$
Cubic	$T(n)=O(n^3)$	Multiplying two nXn matrices by simple algorithm. $3n^3+4n^2+5n+7$
Exponential	$T(n)=O(c^n)$, c>1	2^n+n^2+n+1
Factorial	T(n)=O(n!)	$7n!+n^2+1$
Double exponential	$T(n)=O(c1^{c2n})$, c1 and c2>1	$2^{2n}+2^n+n^2+1$
n^n	$T(n)=O(n^n)$	$8n^n+2^n+n^2+3$

- Arrangement of common functions by growth rate.
- List of typical growth rates.

Function	**Name**
c	Constant
log N	Logarithmic
$\log^2 N$	Log-squared
N	Linear
N log N	Log-Linear
N^2	Quadratic
N^3	Cubic
2^N	Exponential

- The order of the body statements of a given algorithm is very important in determining Big-Oh of the algorithm.

Example: Find Big-Oh of the following algorithm.

1. **for(int i=1;i<=n; i++)**

   sum=sum + i;

   T(n)=2*n=2n=O(n).

2. **for(int i=1; i<=n; i++)**

   for(int j=1; j<=n; j++)

   k++;

T(n)=1*n*n=n^2 = O(n^2).