

Ampère's law

Ampère's law is useful relation used to find magnetic field it is closely analogous to Gauss' law. Consider first a long straight wire carrying a current I . we know that the lines $\mathbf{B}$ form circles around the wire and by symmetry the magnitude of B is the same everywhere on a circular path (Amperian loop) and is given by

$$B = \frac{\mu_0 I}{2\pi r}$$

Its direction at each point is tangent to the circle. Δl is an element of the circle in this special case the tangent component $B_{||}$ of $\mathbf{B}$ is equal to B itself. Now we multiply $B_{||}$ by the length of the arc Δl and sum these products for all the line elements $\Delta l_1, \Delta l_2, \dots$, which together make up the circle

$$\sum B_{||} \Delta l = B_{||} \Delta l_1 + B_{||} \Delta l_2 + \dots,$$

Since $B_{||}$ is the same it can be factored out and we have

$$\sum B_{||} \Delta l = B_{||} (\Delta l_1 + \Delta l_2 + \dots)$$

The term in the bracket is the circumference of the circle.

Thus we get

$$\sum B_{||} \Delta l = \frac{\mu_0 I}{2\pi r} (2\pi r) = \mu_0 I$$

If the angle between the parallel component and the path is θ then the expression becomes

$$\sum B_{||} \Delta l \cos \theta = \mu_0 I$$

Ampère's law states that in any loop path (*amperian loop*), the length element times the magnetic field in the direction of the length element is equal to the permeability of free space times the electric current enclosed in the loop. If the loop contains any number of current carrying conductor, Ampère's law takes the form

$$\sum B_{||} \Delta l = \mu_0 \sum I$$

Application of Ampere's law

Magnetic field produced by a long-straight current carrying wire

Take a circular amperian loop of radius r

$$\sum B_{||} \Delta l = \mu_0 I$$

Since the magnetic field is uniform along the circle, one can write

$$B_{||} \sum \Delta l = \mu_0 I$$

Using the circumference of a circle, we have

$$B_{||} (2\pi r) = \mu_0 I$$

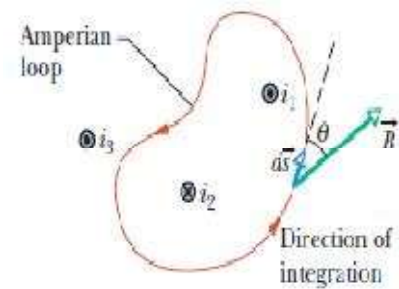


Figure 6.12 Ampère's law applied to an arbitrary Amperian loop.

Finally, taking into account that the magnetic field is always tangent to the circle, we obtain

$$B = \frac{\mu_0 I}{2\pi r}$$

Magnetic field of a solenoid

A solenoid is a long straight coil of wire. If the length of the solenoid L is large compared with its cross-sectional diameter, the internal field near its center is very nearly uniform and parallel to the axis and the external field near the center is very small.

The field along the center can be found using Ampere's law

The field along side ab is parallel to this side and is constant.

($B_{||} = B$) and $\sum B_{||} \Delta h = Bh$

Along side ad and bc B perpendicular to h

$$\sum B_{||} \Delta h = 0$$

And along side cd

$$\sum B_{||} \Delta h = 0$$

If n is the number of turns per unit length, the number of turns in the length h is nh . Each of these turns passes once through the rectangle $abcd$ and carries current I . the total current through the rectangle is then nhI and from Ampere's law

$$Bh = \mu_0 nhI$$

Hence, the magnetic field turns out to be

$$B = \mu_0 nI = \frac{\mu_0 IN}{L}$$

Field of a toroid

The field B is tangent to the path and has constant magnitude B . Thus

$$\sum B_{||} \Delta l = B(2\pi r)$$

The total current through the loop is NI ,

where N is the total number of loops. Then, from Ampere's law

$$B(2\pi r) = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{2\pi r}$$

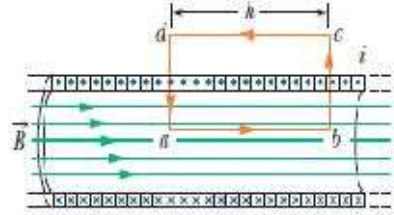


Figure 6.13 A long solenoid carrying current i . The amperian loop is the rectangle $abcd$

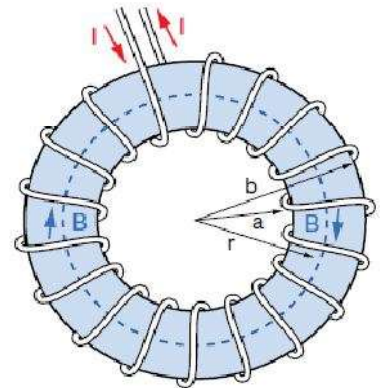


Figure 6. 14 A toroid

Magnetic properties of Matter

The magnetic state of a substance is described by a quantity called the magnetization vector $\mathbf{M}$ (the magnetic moment per unit volume of the substance $\mathbf{M} = \frac{\mu}{V} = \frac{NI\mathbf{A}}{V}$). When a substance is placed in a magnetic field, the total magnetic field in the region is

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_m = \mathbf{B}_0 + \mu_0 \mathbf{M} = \mu_0 (\mathbf{H} + \mathbf{M}),$$

$\mathbf{H}$ is the magnetic field strength within the substance. It is related to the magnetic field due to the current in the wire

Classification of Magnetic substances

Substances can be classified as belonging to one of the three categories, depending on their magnetic properties; paramagnetic and ferromagnetic materials are those made of atoms that have permanent magnetic moments (due to unpaired electrons). Diamagnetic materials are those made of atoms that do not have permanent magnetic moments.

For paramagnetic and diamagnetic substances ($\mathbf{M} \sim \mathbf{H}$)

$$\mathbf{M} = \chi \mathbf{H},$$

where χ is magnetic susceptibility. It is a measure of how susceptible a material is to being magnetized. For paramagnetic substances, χ is positive and $\mathbf{M}$ is the same direction as $\mathbf{H}$. (Al, Li, Mg, Ca, O_2 ...). For diamagnetic substances, χ is negative and $\mathbf{M}$ is opposite to $\mathbf{H}$. (Bismuth, Cu, diamond, Au, Pb, Hg, N_2 ...).

$$\mathbf{B} = \mu_0 (\mathbf{H} + \chi \mathbf{H}) = \mu_0 (1 + \chi) \mathbf{H} = \mu_m \mathbf{H},$$

where $\mu_m = \mu_0 (1 + \chi)$. χ is very small for paramagnetic and diamagnetic materials $\Rightarrow \mu_m \approx \mu_0$. For Ferromagnetic materials μ_m is typically several thousands times greater than μ_0 (χ is very large). Nickel, iron, cobalt and their alloys are ferromagnetic.

Earth's Magnetism

Circulating electric currents in the molten core of the earth produce the magnetic field. The rotation of the earth may play a part in generating the currents that are presumed to be the source of the magnetic field.

The magnetic north pole is near the geographic south pole of the earth. They are about 1000 Km apart.

Horizontal and vertical components of the Earth's magnetic field

Except near the equator, the field lines of the Earth's magnetic field are at an angle to the Earth's surface. At the magnetic poles, the field lines pass through the Earth's surface vertically. However, at any other point on the Earth's surface the Earth's magnetic field has a vertical and a horizontal component.

The Earth's magnetic field is a vector quantity; at each point in space it has a strength and a direction.

The strength of the field at the Earth's surface ranges from less than 30,000 nT in South America and South Africa to over 60,000 nT around the magnetic poles in northern Canada and south of