



Answer Key



ADDIS ABABA SCIENCE AND TECHNOLOGY UNIVERSITY (AASTU)

COLLEGE OF NATURAL AND SOCIAL SCIENCES (MATHEMATICS DEPARTMENT)

APPLIED MATHEMATICS II FOR PRE ENGINEERING STUDENTS

May 12/2021

Maximum marks: 15%

Duration: 60 Minutes

For instructor's use only:

Part-I (6%)	Part-II (9%)	Total-15(%)
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Name: _____ IDNo _____

Part-I Short Answer (6%)

For each of the following problems, write the most simplified answers. (1 Point for each blank)

- Let $f(x, y) = \frac{x}{y}$. Then a) $\frac{\partial f}{\partial x}(x, y) = \underline{\frac{1}{y}}$
b) $\frac{\partial f}{\partial y}(x, y) = \underline{-\frac{x}{y^2}}$
- The directional derivative of $f(x, y) = x^2 + y^2$ at the point $(2, 1)$ in the direction of the unit vector $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ is $\underline{\frac{3\sqrt{2}}{2} \text{ or } 6/\sqrt{2}}$.
- $\int_1^2 \int_0^1 xy \, dy \, dx = \underline{3/4}$
- The area of the region bounded by the curves $y = x^2$ and $x = y^2$ is $\underline{1/3}$
- The cylindrical coordinate of the point $(1, 1, 1)$ is $\underline{(\sqrt{2}, \frac{\pi}{4}, 1)}$

Part-II Work Out (9%)

Show all the necessary steps in the space provided for each of the following questions. (3 pts each)

- Evaluate the following limit

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^2}{x^3 + y^3}$$

In polar forms, $x = r \cos \theta$

28. If $y = x \sin(x + y)$, then the value of $y' =$ _____
29. $\frac{d}{dx} [x^2 + 3xe^{2x}] =$ _____
 A. $2x + 6xe^{2x} + 3e^2$ B. $2x + 6xe^{2x}$
 C. $2x + 6xe^{2x} + 3e^{2x}$ D. $2x + 3e^{2x}$ E. None of the above
30. If $y = x^2 \sin\left(\frac{1}{x}\right)$, then $\frac{dy}{dx} =$ _____
 A. $2x \sin\left(\frac{1}{x}\right) - x^2 \cos\left(\frac{1}{x}\right)$ B. $\frac{-2}{x} \cos\left(\frac{1}{x}\right)$
 C. $2x \cos\left(\frac{1}{x}\right)$ D. $2x \sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right)$ E. $-\cos\left(\frac{1}{x}\right)$
31. Let $y = \sqrt{3 - 2x}$, then $\frac{dy}{dx} =$ _____
 A. $\frac{1}{2\sqrt{3 - 2x}}$ B. $-\frac{1}{\sqrt{3 - 2x}}$ C. $-\frac{(3 - 2x)^{5/2}}{3}$ D. $-\frac{1}{3 - 2x}$ E. $\frac{2}{3}(3 - 2x)^{5/2}$
32. At what value of x is $f(x) = \frac{1}{3}x^3 - 2x^2 - 3x + 5$ have a relative minimum?
 A. -1 only B. 0 only C. 1 only D. 3 only E. -1 and 3
33. If $f'(x) = \frac{(x+1)x^2}{(x-1)^{1/3}}$, then which interval(s) is the continuous function $f(x)$ is increasing?
 A. $(-1, 1)$ B. $(-\infty, -1) \cup (1, \infty)$ C. $(-\infty, 0) \cup (1, \infty)$ D. $(-\infty, -1) \cup (0, \infty)$ E. $(1, \infty)$
34. Find the derivative $\frac{dy}{dx}$. Simplify your answer, if possible
 a) $y = \left(\frac{3x-2}{2x+1}\right)^5$ b) $y = \frac{(x^2+2)^2}{(x^2+x^{-1})^3}$
35. Differentiate the following trigonometric functions.
 a. $f(x) = \cot(4x + 5)$ b. $f(x) = \cosh^{-1}(\sec x)$ c. $f(x) = \frac{1 - \tan 2x}{\sec 2x}$ d. $f(x) = \sqrt{\cos 2x + 1}$
36. Find the derivative for the following functions
 (a) $f(x) = \ln \sqrt{1 + \sin^2 x}$ (b) $f(x) = e^{\ln \sqrt{x}}$ (c) $f(x) = \sin(\ln(\tan^{-1} x))$

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Part II: Choose the best answer from the given alternatives. (Each question worth 1.5 Pts)

- 1) If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, then A^{-1} is equal to
 (A) $\frac{-1}{19}A$ (B) A (C) $\frac{1}{19}A$ (D) $-A$
- 2) For real numbers x and y , which one of the following statements is true?
 (A) $(\forall x)(\exists y)[x^2 + y + 1 = 0]$ (C) $(\exists y)(\forall x)[x^2 + y + 1 = 0]$
 (B) $(\exists x)(\forall y)[x^2 + y + 1 = 0]$ (D) $(\forall y)(\exists x)[x^2 + y + 1 = 0]$
- 3) Suppose that A and B are 3×3 matrices, I_3 is the identity matrix of order 3 such that $AB = 2I_3$. If $\det(B) = 2$, what is the $\det(A^t)$?
 (A) 8 (B) 4 (C) 12 (D) 1
- 4) Which one of the following functions is continuous at the given point?
 (A) $f(x) = \frac{1}{x^2}$ at $c = 1$ (C) $f(x) = \frac{1}{x}$ at $c = 0$
 (B) $f(x) = \frac{1}{x-2}$ at $c = 2$ (D) $f(x) = \sqrt{x}$ at $c = 0$
- 5) $\lim_{x \rightarrow 2} \frac{4-x^2}{x-2}$ is equal to
 (A) 4 (B) -2 (C) 2 (D) -4
- 6) $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{x+4}$ is equal to
 (A) e^4 (B) e (C) $e+1$ (D) 1
- 7) Which one of the following matrices is in row-reduced echelon form?
 (A) $\begin{bmatrix} 0 & 1 & -1 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \end{bmatrix}$
 (B) $\begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
- 8) For the function f if $\lim_{x \rightarrow \infty} f(x) = 2$ and $\lim_{x \rightarrow 3} f(x) = -\infty$. Then which one of the following is false?
 (A) The line $y = 3$ is a horizontal asymptote (C) The line $y = 2$ is a horizontal asymptote
 (B) The line $x = 3$ is a vertical asymptote (D) None of the above
- 9) The cofactor of the element 5 in the matrix $A = \begin{bmatrix} -2 & 2 & 1 \\ 4 & 3 & 2 \\ 5 & 1 & -3 \end{bmatrix}$ is equal to
 (A) 1 (B) 2 (C) 3 (D) 4

11. If $\cos(xy) = y^2$, then $\frac{dy}{dx} =$ _____

12. If $f(x) = e^{4x}$, then $f'''(0) =$

- A) $\frac{6}{4^5}$ B) 6×4^5 C) 4^5 D) $(4^5)^{4^5}$ E) None of these

13. Find all the local extrema of $f(x) = x - \cos x$ in $[-2\pi, 2\pi]$

14. Let $f(x) = (x^2 - 1)e^x$, then find the critical points and inflection points

15. $\frac{d}{dx} \tan \sqrt{4x} =$ _____

16. $\frac{d}{dx} x^{\sqrt{x}} =$ _____

17. $\frac{d}{dx} \left(\int_1^{2x-1} \ln(\ln t) dt \right) =$ _____

18. Let $f(x) = \tan^{-1}(\sin^{-1} x^2)$ find $f'(x)$

19. $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{e^{x-1}} \right)^{e^x} =$ _____

- A) e^{-e} B) e^{-1} C) e D) e^e E) none of these

20. Find The equation of the tangent line to the ellipse $x^2 + 2y^2 = 6$ at $(2, 1)$

21. The slope of the line tangent to the curve $y^3 + x^2y^2 - 3x^3 = 9$ at $(1, 2)$ is _____

22. The critical point(s) of $f(x) = x^2e^{-x}$ is(are): _____

23. Find the interval of monotonic and the inflection point(s) of $f(x) = 2x^3 - 6x^2 - 144x + 1$

24. The equation of the line tangent to the graph of a differentiable function f at $x = 0$ is $y = 3x + 4$

Determine the value of $\lim_{x \rightarrow 0} \frac{x f(x)}{\sin(2x)}$

25. Let f be a strictly increasing continuous function with $f(3) = 5$, find $\lim_{x \rightarrow 3} \frac{(f(x))^2 - 25}{f(x) - 5}$

26. The derivative of the function $f(x)$ is $f'(x) = (x-1)^2(x-2)$, then which of the following is true

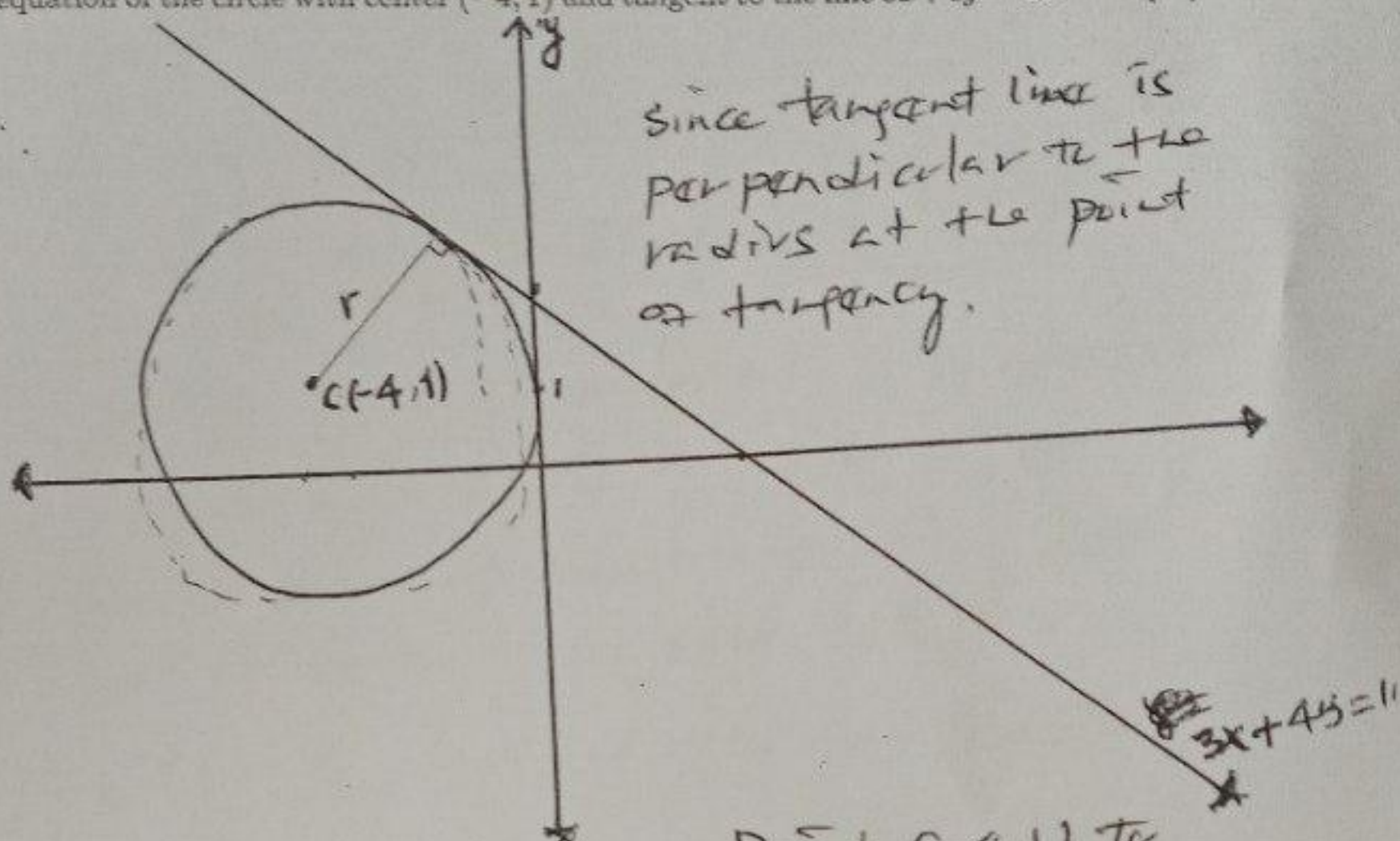
- A) f is increasing for all x B) f has local maximum at $x = 1$
C) f has local maximum at $x = 2$ D) f has local minimum at $x = 2$ E) None

27. If $f(x) = x^2 \ln(x^3)$, then $f'(x) =$ _____

- A. $3x + \ln(x^3)$ B. $3x(1 + \ln(x^2))$ C. $\frac{1}{x}$ D. $2x$ E. $2x \ln(3x^3)$

15. Find the equation of the circle with center $(-4, 1)$ and tangent to the line $3x + 4y = 10$. [05]

Solⁿ :



Since tangent line is perpendicular to the radius at the point of tangency.

By distance formula from a point $(-4, 1)$ to the line $3x + 4y - 10 = 0$ is the radius of the circle.

$$3 \text{ pt } \left\{ \begin{aligned} r &= \frac{|3(-4) + 4(1) - 10|}{\sqrt{9 + 16}} = \frac{|-18|}{5} = 18/5 \end{aligned} \right.$$

Hence, the equation of a circle will be

$$(x-h)^2 + (y-k)^2 = r^2 \quad \left\{ \begin{aligned} & (1 \text{ pt}) \end{aligned} \right.$$

$$\underline{1 \text{ pt}} \left\{ \begin{aligned} & (x+4)^2 + (y-1)^2 = (18/5)^2 \end{aligned} \right.$$

Part I: - Write true if the statement is correct or false if it is incorrect (1 pt each)

- ___ 1. The real part of any complex number z and its conjugate are the same.
- ___ 2. Every integer power of a complex number i is a member of the set $\{-i, -1, i, 1\}$.
- ___ 3. The subset of a Cartesian product of any two nonempty set is a function.
- ___ 4. The inverse of any one to one correspondence function is also function.
- ___ 5. The Exponential function has horizontal asymptote but not vertical.
- ___ 6. The function $f: \mathbb{N} \rightarrow \mathbb{N}$ given by $f(x) = 3x$ is onto function.
- ___ 7. $10^{2n-1} + 1$ is divisible by 11 for all $n \in \mathbb{N}$.
- ___ 8. Logarithmic function has y -intercept but not x -intercept.
- ___ 9. Trigonometric functions are periodic functions with period $p = 2\pi$.
- ___ 10. The value of x which satisfies $\log x + \log(x + 3) = 1$ is 2.

Parts II: - Fill the simplified answer in the given space for the following questions (1 pts each)

1. The range of the function $f(x) = \frac{x}{|x|}$ is _____
2. The radian measure of 25° is _____
3. If $f(x) = x^3 - \frac{1}{x^3}$, then $f(x) + f\left(\frac{1}{x}\right)$ is equal to _____
4. Let $f(x) = x + 1$ and $g(x) = \sqrt{x - 1}$, then $(f \circ g)(5)$ is equal to _____
5. The modulus of $z = \frac{1-i}{3+i} + \frac{4i}{5}$ is _____
6. the principal argument of the complex number $z = -2 + 2\sqrt{3}i$ is _____
7. The amplitude of a function $f(x) = \cos\left(\frac{x}{4}\right)$ is _____
8. The polar form of the complex number $z = (2 + 2i)^{1000}$ is _____
9. If $f(x) = x - 3$ and $g(x) = x^2 + 3$, then $(gf)(x) =$ _____
10. If $f(x) = \frac{x^3 - 1}{(x^2 - 1)(x^4 - 16)}$ then the vertical asymptote(s) of f is(are) _____

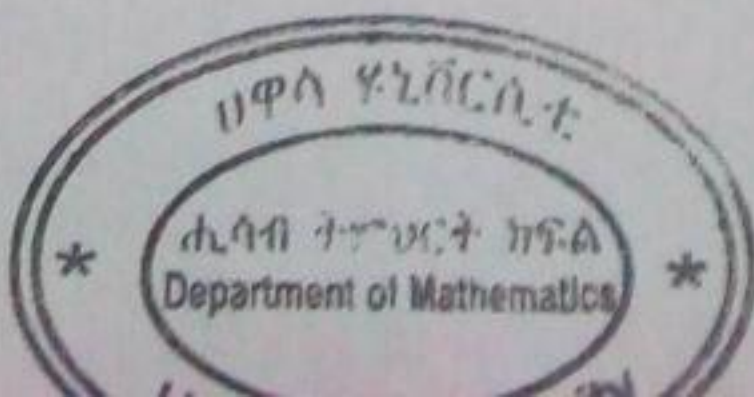
Part II. Give the simplified answers to each of the following questions on the space provided in the answer sheet [1.5 points each]

1. If $z = 1 - \sqrt{3}i$, then the principal argument of $z^4 =$ _____
2. Give the equation of the line that passes through the point $(1, 0)$ and perpendicular to the line $-2x + 4y = 2$ _____
3. If $GCF(a, b) = 5$ and $ab = 250$, then $LCM(a, b) =$ _____
4. The inverse of $f(x) = \sqrt{2-x} + 5$ is _____
5. Let $l_1 : 2y + x = 3$ and $l_2 : 2y + x = 1$. The distance between l_1 and l_2 is _____
6. If $x = 1$ and $x + 1$ are a root and a factor of $p(x) = 2ax^5 - x^3 + bx^2 + 1$ respectively, then the value of $a =$ _____ and $b =$ _____
7. Suppose $f(x) = \frac{x}{1-x}$ and $g(x) = \frac{x-1}{x+1}$ then $f \circ g(x) =$ _____
8. The value of $\coth(\ln 5)$ is _____
9. The domain of the function $f(x) = \frac{\sqrt{x-2}}{\sqrt{3-x}}$ is _____
10. The amplitude and period of $y = -2\sin(\frac{3}{2}x)$ respectively are $A =$ _____, $P =$ _____
11. Let $f = \{(2, 4), (3, 6), (4, 2), (6, 0)\}$ and $g = \{(0, 3), (3, 5), (2, -4), (4, 8), (6, 0)\}$.
Then $\frac{f}{g} =$ _____
12. If $\sqrt{3}^{[4 + \log_{\sqrt{5}} k]} = \frac{1}{3}$, then $k =$ _____

Part III. Work on the following problems by showing all the pertinent information.

1. If $2 + 3i$ is a root of $p(x) = x^4 - 4x^3 + 12x^2 + 4x - 13$, then find all the other roots [4 points]
2. Discuss whether the lines $l_1 : y = -x + 3$, $l_2 : 3y = 5x + 15 = 0$ and $l_3 : y - 2x + 1 = 0$ have two points or one point or no point in common with the circle $x^2 + y^2 + 2x - 4y + 3 = 0$ [4 points]
3. Find equation of the circle passing through the points $(1, 1)$, $(-1, 1)$ and $(-2, -1)$ [4 points]
4. Let $f(x) = \frac{x^2 - x - 6}{x - 2}$. Find [1+2+2]
 a. Its intercepts b. Asymptotes c. Sketch the graph

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29. $\frac{d}{dx} [x^2 + 3xe^{2x}] =$ _____
 A. $2x + 6xe^{2x} + 3e^2$ B. $2x + 6xe^{2x}$
 C. $2x + 6xe^{2x} + 3e^{2x}$ D. $2x + 3e^{2x}$ E. None of the above
30. If $y = x^2 \sin\left(\frac{1}{x}\right)$, then $\frac{dy}{dx} =$ _____
 A. $2x \sin\left(\frac{1}{x}\right) - x^2 \cos\left(\frac{1}{x}\right)$ B. $\frac{-2}{x} \cos\left(\frac{1}{x}\right)$
 C. $2x \cos\left(\frac{1}{x}\right)$ D. $2x \sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right)$ E. $-\cos\left(\frac{1}{x}\right)$
31. Let $y = \sqrt{3 - 2x}$, then $\frac{dy}{dx} =$ _____
 A. $\frac{1}{2\sqrt{3 - 2x}}$ B. $-\frac{1}{\sqrt{3 - 2x}}$ C. $-\frac{(3 - 2x)^{5/2}}{3}$ D. $-\frac{1}{3 - 2x}$ E. $\frac{2}{3}(3 - 2x)^{5/2}$
32. At what value of x is $f(x) = \frac{1}{3}x^3 - 2x^2 - 3x + 5$ have a relative minimum?
 A. -1 only B. 0 only C. 1 only D. 3 only E. -1 and 3
33. If $f'(x) = \frac{(x+1)x^2}{(x-1)^{1/3}}$, then which interval(s) is the continuous function $f(x)$ is increasing?
 A. $(-1, 1)$ B. $(-\infty, -1) \cup (1, \infty)$ C. $(-\infty, 0) \cup (1, \infty)$ D. $(-\infty, -1) \cup (0, \infty)$ E. $(1, \infty)$
34. Find the derivative $\frac{dy}{dx}$. Simplify your answer, if possible
 a) $y = \left(\frac{3x-2}{2x+1}\right)^5$ b) $y = \frac{(x^2+2)^2}{(x^2+x^{-1})^3}$

35. Differentiate the following trigonometric functions.

a. $f(x) = \cot(4x + 5)$ b. $f(x) = \cosh^{-1}(\sec x)$ c. $f(x) = \frac{1 - \tan 2x}{\sec 2x}$ d. $f(x) = \sqrt{\cos 2x + 1}$

36. Find the derivative for the following functions

(a) $f(x) = \ln \sqrt{1 + \sin^2 x}$ (b) $f(x) = e^{\ln \sqrt{x}}$ (c) $f(x) = \sin(\ln(\tan^{-1} x))$

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Addis Ababa Science and Technology University

Department Of Mathematics

Applied Mathematics I

Worksheet (Matrices, Determinants and System of Linear Equations)

1. Construct a 3×4 matrix $A = (a_{ij})$ such that $a_{ij} = \begin{cases} -3i + 4j, & i < j \\ i^2 + 2j, & i \geq j \end{cases}$

2. Given the matrices $A = \begin{pmatrix} 1 & -2 \\ 3 & 4 \\ 0 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ -1 & 4 \end{pmatrix}$, $C = \begin{pmatrix} 2 & 2 & 3 \\ 1 & 4 & 5 \end{pmatrix}$ & $D = \begin{pmatrix} 2 & 1 & 2 \\ 0 & 1 & 2 \\ 8 & 1 & 2 \end{pmatrix}$.

Compute the following (if possible).

(a) AB

(b) $3A - 2C^T$

(c) $2B - 4C$

(d) $(DA)^T$

3. If $\begin{pmatrix} a+b & c+d \\ c-d & a-b \end{pmatrix} = \begin{pmatrix} 2 & 10 \\ 6 & 4 \end{pmatrix}$, then find a, b, c & d .

4. Let $A = \begin{pmatrix} 4 & -2 & 2 \\ 2 & 4 & -4 \\ 1 & 1 & 0 \end{pmatrix}$ and $U = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$.

(a) Show that $AU = 2U$

(b) Find $A^5 U$

(c) Give the formula for $A^n U$, $n \in \mathbb{N}$

5. Determine all 2×2 matrices A over $\mathbb{R}$ such that $A^2 = 0_{2 \times 2}$.

6. Let $A = \begin{pmatrix} 7 & 4 \\ -9 & -5 \end{pmatrix}$. Prove that $A^n = \begin{pmatrix} 1+6n & 4n \\ -9n & 1-6n \end{pmatrix}$ if $n \geq 1$.

7. Express $A = \begin{pmatrix} 1 & 0 & -6 \\ -2 & 4 & 7 \\ 3 & 5 & 2 \end{pmatrix}$ as a sum of symmetric and skew symmetric matrices.

8. Reduce each of the following to echelon form and determine their ranks.

(a) $\begin{pmatrix} 2 & 3 & -4 \\ -1 & -5 & 1 \\ 4 & 6 & -2 \end{pmatrix}$ (b) $\begin{pmatrix} -1 & -2 & 3 \\ -2 & -4 & 6 \\ 3 & 6 & -9 \\ -4 & -8 & 12 \end{pmatrix}$ (c) $\begin{pmatrix} -2 & 1 \\ 3 & 0 \\ 1 & -4 \\ 8 & 2 \end{pmatrix}$ (d) $\begin{pmatrix} -2 \\ -1 \\ 1 \\ 4 \end{pmatrix}$

9. Let $A = \begin{pmatrix} 4 & 5 & 3 & 7 \\ -1 & 3 & 2 & -1 \\ 2 & 3 & 2 & 1 \end{pmatrix}$. Find the row reduced echelon form of A .

Part II Show all the necessary steps of all questions in this section using the answer space provided.

11. Show that $\sqrt{2}$ is an irrational number.

[03]

proof: suppose that $\sqrt{2}$ is rational number.

$\Rightarrow \sqrt{2} = \frac{a}{b}$ for some $a, b \in \mathbb{Z}$ and $\text{G.C.F.}(a, b) = 1$

$$\Rightarrow 2 = \frac{a^2}{b^2} \Rightarrow a^2 = 2b^2$$

$$\Rightarrow a^2 \text{ is even}$$

$$\Rightarrow a \text{ is even}$$

$$\Rightarrow a = 2k, k \in \mathbb{Z}$$

$$\Rightarrow 4k^2 = 2b^2$$

$$\Rightarrow b^2 = 2k^2$$

$$\Rightarrow b^2 \text{ is even}$$

$$\Rightarrow b \text{ is even}$$

$$\Rightarrow \text{GCF}(a, b) \geq 2$$

i.e. a & b are not relatively prime or $\frac{a}{b}$ is not in simplest form.

$$\Rightarrow \text{Which is a contradiction}$$

$$\Rightarrow \sqrt{2} \text{ is not rational no.}$$

Hence, $\sqrt{2}$ is an irrational no.

12. Find the fourth root of $-1 + \sqrt{3}i$.

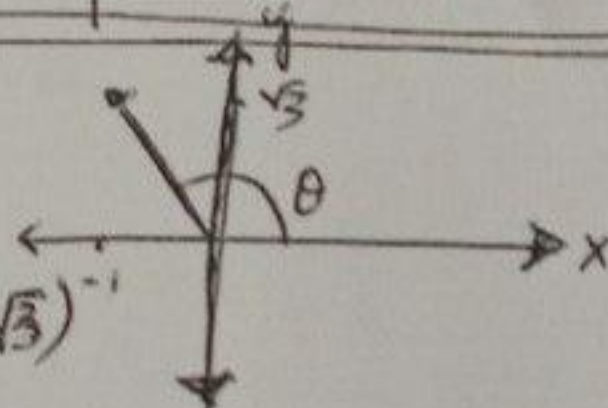
[04]

$$\text{Let } z = -1 + \sqrt{3}i$$

$$\Rightarrow r = |z| = 2$$

$$\Rightarrow \theta = \text{Arg}(z) = \tan^{-1}(\sqrt{3})$$

$$\therefore \theta = \frac{5\pi}{6}$$



$$\Rightarrow \sqrt[n]{z} = r^{1/n} e^{i(\frac{\theta + 2k\pi}{n})}$$

$$= \sqrt[n]{2} e^{i(\frac{5\pi/6 + 2k\pi}{4})}$$

$$, n = 4, k = 0, 1, 2, 3$$

$$, k = 0, 1, 2, 3$$

$$= \sqrt[n]{2} \left[\cos\left(\frac{5\pi/6 + 2k\pi}{4}\right) + i \sin\left(\frac{5\pi/6 + 2k\pi}{4}\right) \right]$$

$$Z_0 = \sqrt[n]{2} \left(\cos\left(\frac{5\pi}{24}\right) + i \sin\left(\frac{5\pi}{24}\right) \right)$$

$$Z_1 = \sqrt[n]{2} \left[\cos\left(\frac{17\pi}{24}\right) + i \sin\left(\frac{17\pi}{24}\right) \right]$$

$$Z_2 = \sqrt[n]{2} \left[\cos\left(\frac{29\pi}{24}\right) + i \sin\left(\frac{29\pi}{24}\right) \right]$$

$$Z_3 = \sqrt[n]{2} \left[\cos\left(\frac{41\pi}{24}\right) + i \sin\left(\frac{41\pi}{24}\right) \right]$$

Best Wishes!!!

Please go on to the next page

28. If $y = x \sin(x + y)$, then the value of $y' =$ _____
29. $\frac{d}{dx} [x^2 + 3xe^{2x}] =$ _____
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30. If $y = x^2 \sin\left(\frac{1}{x}\right)$, then $\frac{dy}{dx} =$ _____
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33. If $f'(x) = \frac{(x+1)x^2}{(x-1)^{1/3}}$, then which interval(s) is the continuous function $f(x)$ is increasing?
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Part I: Write "True" if the statement is correct or "False" if the statement is incorrect on the space provided (1 pt. each)

False

1. For any function f and g , $(f \circ g)(x) = (g \circ f)(x)$.

True

2. The domain of a relation R is equal to the range of the inverse relation of R .

True

3. Any relation is a function.

True

4. The set of real numbers is a subset of the set of complex numbers.

True

5. Any set is a subset of itself.

True

6. The principal argument of a complex number $Z = i$ is $\frac{\pi}{2}$.

False

7. The set of natural numbers is closed under division.

False

8. The sum of two irrational numbers is always irrational number.

False

9. The range and codomain of any function are the same.

True

10. The simplified form of $(1 - i)^{10}$ and $(1 - i)^{18}$ are equal.

Part II: Choose the correct answer from the given alternatives and write your answer on the space provided (1.5 pts. each)

D

11. Let $A = \{1, 2\}$ and $B = \{3, 4\}$, then which one of the following relations is a relation from A to B ?

A. $\{(3, 1), (4, 2)\}$

B. $\{(1, 3), (2, 5)\}$

C. $\{(3, 1)\}$

D. $\{(1, 3), (2, 4)\}$

B

12. Which one of the following statements is **not true** about integers a , b and c ?

A. $a + b = b + a$

B. $a - b = b - a$

C. $(a \times b) \times c = a \times (b \times c)$

D. $a \times (b + c) = a \times b + a \times c$

A

13. The GCF of 100 and 150 is

A. 50

B. 300

C. 25

D. 150

C

14. Which one of the following numbers is **not** rational number?

A. $\frac{1}{2}$

B. $3.\overline{14}$

C. $\sqrt{2}$

D. $\sqrt{25}$



- A 15. The greatest lower bound of a sequence $\left\{\frac{(-1)^n}{n}\right\}$ is
- A. -1 B. 1 C. $\frac{1}{2}$ D. $-\frac{1}{2}$

- D 16. Which one of the following statements is not true about any complex numbers Z_1 and Z_2 ?
- A. $\overline{\overline{Z_1}} = Z_1$ B. $Z_1 + \overline{Z_1} = 2\text{Re}(Z_1)$
C. $\left|\frac{Z_1}{Z_2}\right| = \frac{|Z_1|}{|Z_2|}; Z_2 \neq 0$ D. $|Z_1 + Z_2| \leq |Z_1| - |Z_2|$

- B 17. Let order pairs $(x, 2)$ and $(2x - 9, 2)$ are equal, then the value of x is
- A. -9 B. 9 C. 3 D. -3

18. The codomain of a relation $R: \mathbb{N} \rightarrow \mathbb{R}$ is
- A. $\mathbb{N}$ B. $\emptyset$ C. $\mathbb{R}$ D. $\mathbb{N} \cap \mathbb{R}$

- D 19. Which one of the following is a function?

- A. $\{(x, y): x \text{ is a father of } y\}$ B. $\{(x, y): x = y^2\}$
C. $\{(x, y, x + y): \forall x, y \in \mathbb{Z}\}$ D. $\{(x, y): x + y = 1\}$

20. Which one of the following statements is true about sets A , B and C ?
- A. $A \times B = B \times A$ B. $A \setminus B = B \setminus A$
C. $(A \Delta B) \Delta C = A \Delta (B \Delta C)$ D. $(A \times B) \times C = A \times (B \times C)$

21. Let $A = [-1, \infty)$, $B = (-\infty, 1]$ and $C = (1, \infty)$, then $C \cup (A \cap B)$ is
- A. $(-\infty, 1]$ B. $[-1, \infty)$ C. $\mathbb{R}$ D. $(1, \infty)$

- D 22. Which one of the following is not true about functions $f(x) = x + 1$ and $g(x) = \frac{1}{x}$?

A. $(f + g)(x) = \frac{x^2 + x + 1}{x}$

B. $(f \cdot g)(x) = \frac{x + 1}{x}$

C. $(f \circ g)(x) = \frac{x + 1}{x}$

D. Domain of $(f \circ g)(x)$ is $\mathbb{R}$

46. Use the basis information to Sketch the graph of $f(x) = \frac{x}{(x+1)^2}$

47. The value of $\int_0^{\pi/4} \sin x \, dx =$ _____

A. $-\frac{\sqrt{2}}{2}$

B. $\frac{\sqrt{2}}{2}$

C. $-\frac{\sqrt{2}}{2} - 1$

D. $1 - \frac{\sqrt{2}}{2}$

E. $\frac{\sqrt{2}}{2} - 1$

48. $\int x^2 \cos(x^3) dx =$ _____

A. $-\frac{1}{3} \sin(x^3) + C$

B. $\frac{1}{3} \sin(x^3) + C$

C. $-\frac{x^3}{3} \sin(x^3) + C$

D. $\frac{x^3}{3} \sin(x^3) + C$

E. $\frac{x^3}{3} \sin\left(\frac{x^4}{4}\right) + C$

49. $\int \frac{1}{x^2 - 6x + 8} dx =$ _____

A. $\frac{1}{2} \ln \left| \frac{x-4}{x-2} \right| + C$

B. $\frac{1}{2} \ln \left| \frac{x-2}{x-4} \right| + C$

C. $\frac{1}{2} \ln |(x-2)(x-4)| + C$

D. $\frac{1}{2} \ln |(x-4)(x+2)| + C$

E. $\ln |(x-4)(x-2)| + C$

50. $\int \tan x \ln(\cos x) dx =$ _____ $\frac{d}{dx} \left(\frac{\int e^{x^2} dx}{\int t' dt} \right) =$ _____

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51. Find $\int_a^a \frac{dx}{x^2}$ when $a \neq 0$.

A) $\frac{a-1}{a}$

B) $\frac{a+1}{a^2}$

C) $\frac{a-1}{a^2}$

D) $\frac{a^2}{a-1}$

E) None of these

52. Solve the integration of the following

a. $\int x \sin x dx$

b. $\int x^2 \cos x dx$

c. $\int x \sin(2x) dx$

e. $\int \sin^2 x dx$

53. Find

a. $\int \cos 4x \cos 3x dx$

b. $\int \cos^3 x \sin^8 x dx$

c. $\int x^2 2^x dx$

d. $\int \frac{x}{\sqrt{1-x^2}} dx$

54. Evaluate the following.

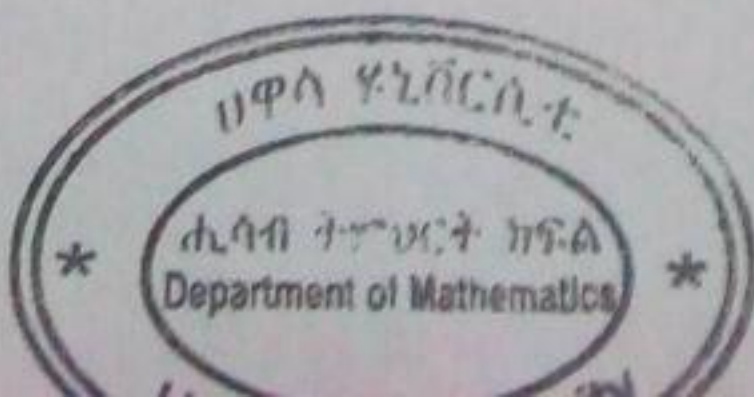
Part II. Give the simplified answers to each of the following questions on the space provided in the answer sheet [1.5 points each]

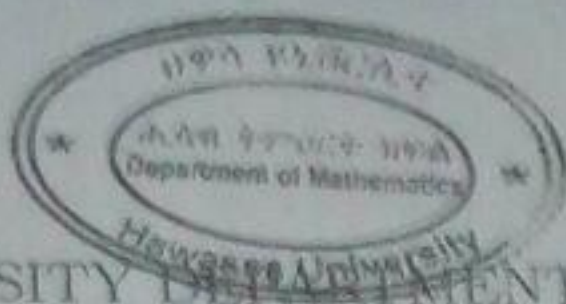
1. If $z = 1 - \sqrt{3}i$, then the principal argument of $z^4 =$ _____
2. Give the equation of the line that passes through the point $(1, 0)$ and perpendicular to the line $-2x + 4y = 2$ _____
3. If $GCF(a, b) = 5$ and $ab = 250$, then $LCM(a, b) =$ _____
4. The inverse of $f(x) = \sqrt{2-x} + 5$ is _____
5. Let $l_1 : 2y + x = 3$ and $l_2 : 2y + x = 1$. The distance between l_1 and l_2 is _____
6. If $x = 1$ and $x + 1$ are a root and a factor of $p(x) = 2ax^5 - x^3 + bx^2 + 1$ respectively, then the value of $a =$ _____ and $b =$ _____
7. Suppose $f(x) = \frac{x}{1-x}$ and $g(x) = \frac{x-1}{x+1}$ then $f \circ g(x) =$ _____
8. The value of $\coth(\ln 5)$ is _____
9. The domain of the function $f(x) = \frac{\sqrt{x-2}}{\sqrt{3-x}}$ is _____
10. The amplitude and period of $y = -2\sin(\frac{3}{2}x)$ respectively are $A =$ _____, $P =$ _____
11. Let $f = \{(2, 4), (3, 6), (4, 2), (6, 0)\}$ and $g = \{(0, 3), (3, 5), (2, -4), (4, 8), (6, 0)\}$.
Then $\frac{f}{g} =$ _____
12. If $\sqrt{3}^{[4 + \log_{\sqrt{5}} k]} = \frac{1}{3}$, then $k =$ _____

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Part III. Work on the following problems by showing all the pertinent information.

1. If $2 + 3i$ is a root of $p(x) = x^4 - 4x^3 + 12x^2 + 4x - 13$, then find all the other roots [4 points]
2. Discuss whether the lines $l_1 : y = -x + 3$, $l_2 : 3y = 5x + 15 = 0$ and $l_3 : y - 2x + 1 = 0$ have two points or one point or no point in common with the circle $x^2 + y^2 + 2x - 4y + 3 = 0$ [4 points]
3. Find equation of the circle passing through the points $(1, 1)$, $(-1, 1)$ and $(-2, -1)$ [4 points]
4. Let $f(x) = \frac{x^2 - x - 6}{x - 2}$. Find
a. Its intercepts b. Asymptotes c. Sketch the graph [1+2+2]





HAWASSA UNIVERSITY DEPARTMENT OF MATHEMATICS

BASIC MATH FOR NATURAL SCIENCE FINAL EXAM -Jan. 18-2012 E.C.

Name: _____ ID.No. _____ Sec. _____ Time 2 : 30 hrs.

Part I. Choose the best answer to each of the following questions and write your answer on the space provided in the answer sheet [1.5 points each]

- Which of the following compound statement is contradiction?
 A) $(p \Leftrightarrow q) \Leftrightarrow (\neg p \Leftrightarrow \neg q)$ B) $(p \Rightarrow q) \Leftrightarrow (p \vee q)$
 C) $(p \Rightarrow q) \Leftrightarrow (p \wedge \neg q)$ D) $(\neg p \wedge \neg q) \Rightarrow \neg(p \vee q)$
- Which of the following is equal to $\sin 280^\circ$?
 A) $\sin 80^\circ$ B) $\cos -10^\circ$ C) $-\cos 80^\circ$ D) $-\cos 10^\circ$
- If $\cos \theta = \frac{5}{6}$ and $0 < \theta < \frac{\pi}{2}$, then $\csc \theta$ is A) $\frac{36}{11}$ B) $\frac{11}{36}$ C) $\frac{6}{\sqrt{11}}$ D) $\frac{\sqrt{11}}{6}$
- Suppose $f(x) = 2x^3 + 4x^2 + 5x - 9$. A root of f lies between
 A) -1 and 0 B) 0 and 1 C) 1 and 2 D) -2 and -1
- Which of the following is a one-to-one correspondence function?
 A) $k : (-\infty, 2] \mapsto [0, \infty) : k(x) = \sqrt{2-x}$ B) $g : \mathbb{R} \mapsto [0, \infty) : g(x) = 2^x + 1$
 C) $h : \mathbb{R} \setminus \{0\} \mapsto \mathbb{R} : h(x) = \ln x^2$ D) $f : (-\infty, 1] \mapsto \mathbb{R} : f(x) = x^2 - 2x + 3$
- The simplest form of $\log_3 4 \times \log_4 5 \times \log_5 6 \times \log_6 7 \times \log_7 8 \times \log_8 9$ is
 A) 1 B) 2 C) $\log_8 9$ D) 3
- Which of the following pair of functions are equal?
 A) $f(x) = \frac{1}{x}$ and $g(x) = \log_2 x$ B) $h(x) = \frac{1}{x}$ and $k(x) = \frac{x+1}{x^2+x}$
 C) $m(x) = \frac{x}{x^2}$ and $n(x) = \frac{1}{x}$ D) B and C
- Which of the following is not a correct identity?
 A) $\frac{1}{1-\cos x} + \frac{1}{1+\cos x} = 2 \csc^2 x$ B) $\frac{\csc^2 x - 1}{\csc^2 x} = \cos^2 x$
 C) $\cos^2 x - \sin^2 x = 1 - 2 \sin^2 x$ D) $(\cot^2 x + 1)(\sin^2 x - 1) = \cot^2 x$
- Which of the following is not true about the graph of $f(x) = -\left(\frac{1}{3}\right)^{x-1} - 2$?
 A) Its horizontal asymptote is $y = -2$ B) The range of $f(x)$ is $(-2, \infty)$
 C) Its y -intercept is $(0, -5)$ D) The graph of $f(x)$ is increasing in its domain
- The equation of the circle with center at $(2, 1)$ and tangent to the line $l : 3y - 4x = 5$ is
 A) $x^2 + y^2 - 4x - 2y + 1 = 0$ B) $x^2 + y^2 + 4x - 2y - 16 = 0$
 C) $x^2 + y^2 + 4x + 2y + 4 = 0$ D) $x^2 + y^2 - 4x + 2y - 1 = 0$

16. Let the length of the transverse of a hyperbola is 16 and the end points of the conjugate axis are $(5, -5)$ and $(5, 3)$, then find:

- The equation of the hyperbola
- The vertex and the foci of the hyperbola.
- Sketch the graph

Solⁿ: (i) eqⁿ of the hyperbola is

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

1PT

Where the center $C(h, k)$ = mid point of conjugate axis

$$\Rightarrow C(h, k) = (5, -1)$$

$a = 8$ (half of length of transverse axis)

$b = 4$ (half of length of conjugate axis)

$$\Rightarrow \frac{(x-5)^2}{64} - \frac{(y+1)^2}{16} = 1 \quad \text{is eqⁿ of the hyperbola.}$$

ii) the vertices are

$$V(h-a, k) \text{ and } V'(h+a, k)$$

$$= V(-3, -1) \text{ and } V'(13, -1)$$

1PT

$$c = \sqrt{a^2 + b^2} = \sqrt{80}$$

$\Rightarrow$ the foci are $F(h-c, k) = F(5-\sqrt{80}, -1)$ and

$$F'(h+c, k) = F'(5+\sqrt{80}, -1)$$

1PT

