12. Theorems on Area

Let I do 12.1

1. Question

If any triangle and rectangle are on the same base and between the same parallel then let us prove logically that the area of triangular region is half the area in the shape of rectangular.

Answer

Given.

If any triangle and rectangle are on the same base and between the same parallel.

Formula used.

Area of rectangle = Length × Breadth

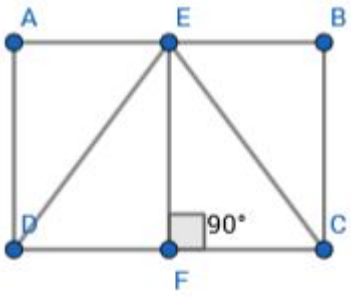
Area of triangle = $\frac{1}{2}$× Base × Height

⇒ Property of parallel lines

Perpendicular distance between 2 parallel is always same.

Draw a rectangle ABCD between 2 parallel lines PQ and RS

Draw triangle with base CD and point E on common line AB



As we know that rectangle possess each angle to be 90°

So Breadth AD and BC act as perpendicular between parallel lines

Hence, Height of triangle(EF) = Breadth of rectangle(AD)

⇒ By Property of parallel lines

As both rectangle and triangle possess same base CD

Which is also length of rectangle

∴ Length of rectangle = Base of triangle = CD

Area of triangle = $\frac{1}{2}$× Base × Height

$\frac{1}{2}$× CD × EF

$\frac{1}{2}$× CD × AD ∵ EF = AD

$\frac{1}{2}$ × [Length of rectangle × Breadth of rectangle]

$\frac{1}{2}$ × [Area of rectangle]

Hence proved;

2. Question

If any triangle and any parallelogram are on the same base and between same parallels let us prove logically that the area of triangular region is half the area in the shape of parallelogram region.

Answer

Given.

If any triangle and parallelogram are on the same base and between the same parallel.

Formula used.

Area of parallelogram = Base × Perpendicular

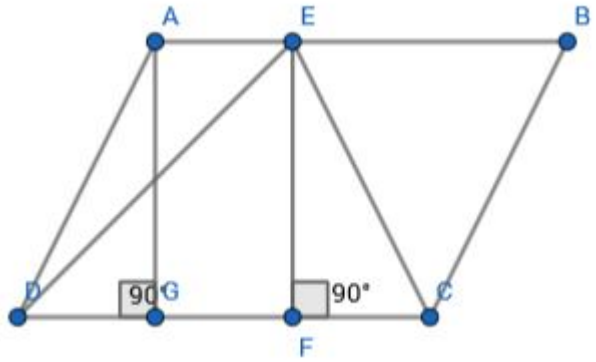
Area of triangle = $\frac{1}{2}$× Base × Height

⇒ Property of parallel lines

Perpendicular distance between 2 parallel is always same.

Draw a parallelogram ABCD between 2 parallel lines PQ and RS

Draw triangle with base CD and point E on common line AB



As we know that if both parallelogram and triangle lies on same 2 parallel lines

Perpendicular height of both will be same

Height of triangle(EF) = perpendicular of parallelogram (AG)

⇒ By Property of parallel lines

As both parallelogram and triangle possess same base CD

Which is also Base of parallelogram

∴ Base of parallelogram = Base of triangle = CD

Area of triangle = $\frac{1}{2}$× Base × Height

$\frac{1}{2}$× CD × EF

$\frac{1}{2}$× CD × AG ∵ EF = AG

$\frac{1}{2}$ × [Perpendicular × Base of parallelogram]

$\frac{1}{2}$ × [Area of Parallelogram]

Hence proved;

Lets Prove 12

1. Question

P and Q are the midpoints of sides AB and DC of parallelogram ABCD, let's prove that the area of quadrilateral shaped $\mathrm{APCQ} = \frac{1}{2} \times$ area of parallelogram shaped region ABCD.

Answer

Given.

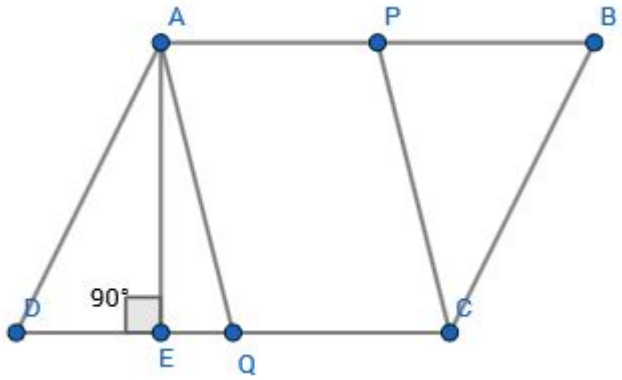
P and Q are the midpoints of sides AB and DC of parallelogram ABCD

Formula used

Area of parallelogram = Base × Perpendicular

⇒ Property of parallel lines

Perpendicular distance between 2 parallel is always same.



As AB = CD and AB || CD ∵ property of parallelogram

Their mid points will also be equal and parallel

∴ AP=CQ and AP || CQ

∴ APCQ is a parallelogram

As we know that if both parallelogram lies on same 2 parallel lines

Because opposite lines are parallel in parallelogram

AB || CD

Perpendicular height of both will be same

⇒ By Property of parallel lines

As both parallelogram are on same base line

But CD = 2 × CQ ∵ Q is mid-point of CD

CQ = $\frac{CD}{2}$

Area of parallelogram ABCD = CD × AE

Area of parallelogram APCQ = Base × Height

= CQ × AE

=$\frac{CD}{2}$ × AE

$=\frac{1}{2}\times CD\times AE$

$\frac{1}{2}\times$ [Area of Parallelogram ABCD]

Hence proved;

2. Question

The distance between two sides AB and DC of rhombus ABCD is PQ and distance between sides AD and BC is RS; let's prove that PQ = RS.

Answer

Given.

The distance between two sides AB and DC of rhombus ABCD is PQ and distance between sides AD and BC is RS

Formula used.

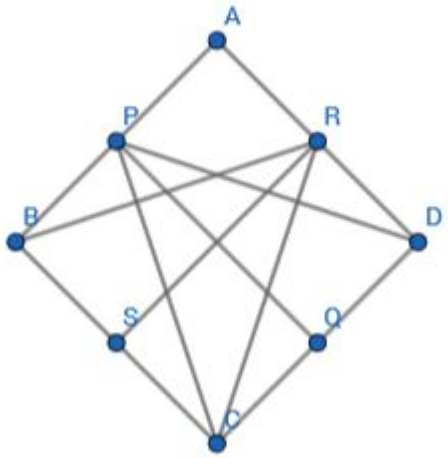
Opposite sides of rhombus are parallel

And all sides of are equal in rhombus

Area of triangle = $\frac{1}{2}\times$ Base × Height

Draw Δ DPC by joining DP and PC and

Δ CRB by joining CR and RB in rhombus



In triangle DPC and rhombus ABCD

Both lies on same base CD

And lies between parallel lines AB || CD

⇒ Because opposite sides of rhombus are parallel

∴ Area of Rhombus ABCD = $\frac{1}{2}\times$ Area of triangle DPC

In triangle CRB and rhombus ABCD

Both lies on same base BC

And lies between parallel lines BC || AD

⇒ Because opposite sides of rhombus are parallel

∴ Area of Rhombus ABCD = $\frac{1}{2}$ × Area of triangle CRB

By equating both

Area of triangle DPC = Area of triangle CRB

$\frac{1}{2}$ × PQ × CD = $\frac{1}{2}$ × RS × BC

As all sides of rhombus are equal

⇒ CD = BC

$\frac{1}{2}$ × PQ × CD = $\frac{1}{2}$ × RS × CD

∴ PQ = RS

Hence proved ;

3. Question

P and Q are the mid points of sides AB and DC of parallelogram ABCD respectively. Let's prove that PBQD is a parallelogram and triangle PBC = $\frac{1}{2}$ parallelogram PBQD.

Answer

Given.

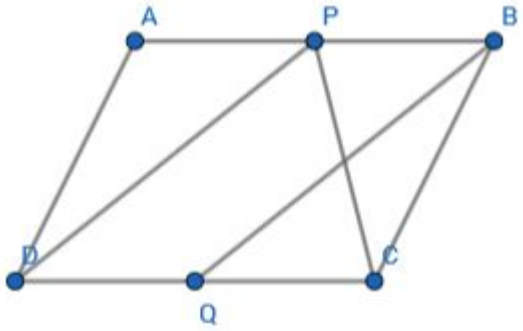
P and Q are the mid points of sides AB and DC of parallelogram ABCD

Formula used.

Formula used.

Area of parallelogram = Base × Perpendicular

Area of triangle = $\frac{1}{2}$× Base × Height



In PBQD

As P and Q are the mid points of sides AB and DC of parallelogram ABCD

PB = QD and PB || QD

∴ PBQD is a parallelogram

In triangle PBC and Parallelogram PBQD

Both are on same base PB which lies on line AB

And AB||CD ∵ Opposite sides of parallelogram are parallel

⇒ Both parallelogram and triangle are on same base and lies between parallel lines AB and CD

∴ triangle PBC = $\frac{1}{2}$ × parallelogram PBQD

Hence proved;

4. Question

In an isosceles triangle ABC, AB = AC and P is any point on produced side BC. PQ and PR are perpendicular on sides AB and AC from the point P respectively. BS is perpendicular on side AC from point B; let's prove that PQ – PR = BS.

Answer

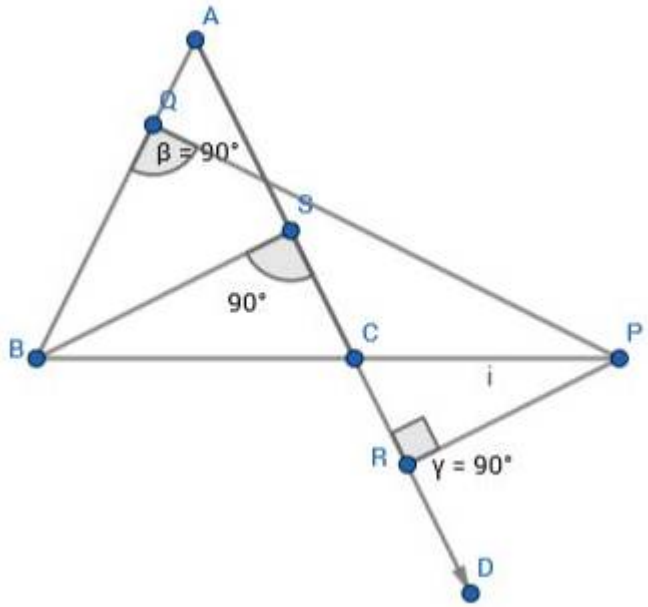
Given.

AB= AC;

PQ and PR are perpendicular on sides AB and AC from the point P respectively. BS is perpendicular on side AC from point B

Formula used.

Area of triangle = $\frac{1}{2}$ × Base × Height



As triangle ABC and triangle ACP combines to form triangle ABP

Area of triangle ABP = Area of triangle ABC + Area of triangle ACP

Area of triangle ABP = $\frac{1}{2}$ × Base × Height

$\frac{1}{2}$ × AB × PQ

Area of triangle ABC = $\frac{1}{2}$ × Base × Height

$\frac{1}{2}$ × AC × BS

Area of triangle ACP = $\frac{1}{2}$ × Base × Height

$\frac{1}{2}$ × AC × PR

$\Rightarrow \frac{1}{2} \times AB \times PQ = \frac{1}{2} \times AC \times BS + \frac{1}{2} \times AC \times PR$

As AB = AC (Given)

$\Rightarrow \frac{1}{2} \times AC \times PQ = \frac{1}{2} \times AC \times BS + \frac{1}{2} \times AC \times PR$

Taking common $\frac{1}{2}$×AC get removed

$\Rightarrow$ PQ = BS + PR

$\therefore$ PQ – PR = BS

Hence proved;

6. Question

O is any point outside the equilateral triangle ABC and within the angular region on ABC; OP, OQ and OR are the perpendicular distance of AB, BC and CA respectively from the point O. Let us prove that the altitude of the triangle = OP + OQ – OR.

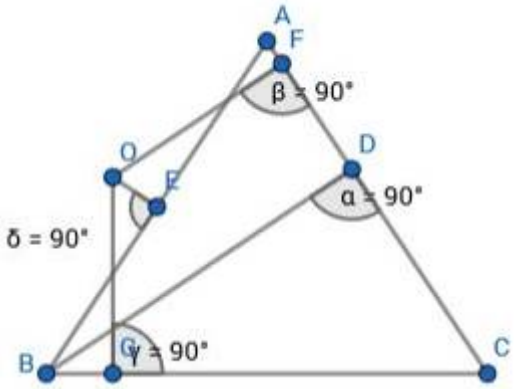
Answer

Given.

OP, OQ and OR are the perpendicular distance of AB, BC and CA respectively from the point O

Formula used.

Area of triangle = $\frac{1}{2}$ × Base × Height



As triangle ABC, triangle OAB, triangle OBC combines to form triangle OAC

Area of triangle ABC = Area of triangle OCB + triangle OAB – triangle OAC

Area of triangle ABC = $\frac{1}{2}$ × Base × Height

$\frac{1}{2}$ × AC × BD

Area of triangle OAC = $\frac{1}{2}$ × Base × Height

$\frac{1}{2}$ × AC × OR

Area of triangle OAB = $\frac{1}{2}$ × Base × Height

$\frac{1}{2}$ × AB × OP

Area of triangle OCB = $\frac{1}{2}$ × Base × Height

$\frac{1}{2}$ × BC × OQ

$\Rightarrow \frac{1}{2} \times AC \times BD = \frac{1}{2} \times BC \times OQ + \frac{1}{2} \times AB \times OP - \frac{1}{2} \times AC \times OR$

As AB = AC = BC (Given)

$\Rightarrow \frac{1}{2} \times AC \times BD = \frac{1}{2} \times BC \times OQ + \frac{1}{2} \times AC \times OP - \frac{1}{2} \times AC \times OR$

Taking common $\frac{1}{2} \times AC$ get removed

$\therefore BD = OQ + OP - OR$

Hence proved;

7. Question

E is any point on side DC of parallelogram ABCD, produced AE meets produced BC at the point F. Joined D, F. Let's prove that

(i) triangle ADF = triangle ABE

(ii) triangle DEF = triangle BEC.

Answer

Given.

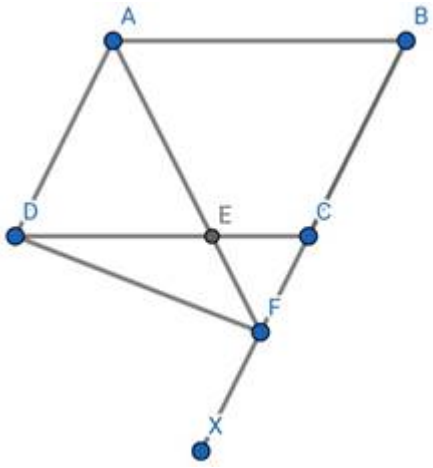
E is any point on side DC of parallelogram ABCD, produced AE meets produced BC at the point F

Formula used.

If 2 triangles are on same base And lies between 2 parallel lines then both triangles are equal.

If triangle and parallelogram are on same base And lies between 2 parallel lines then triangle gets half of parallelogram.

Draw the diagonal BD of parallelogram ABCD



In triangle ADF and triangle ADB

Both lies on same base AD

And AD || BC ∵ opposite sides of parallelogram are parallel

As BF is extended part of BC

Then AD || BF

∴ triangle ADF = triangle ADB = $\frac{1}{2}$ of parallelogram ABCD

∵ Diagonal of parallelogram gives 2 congruent triangles

In triangle ABE and parallelogram ABCD

Both lies on same base AB

And AB || DC ∵ opposite sides of parallelogram are parallel

∴ triangle ABE = $\frac{1}{2}$ of parallelogram ABCD

Both triangle ADF and triangle ABE are half of parallelogram

∴ triangle ADF = triangle ABE

If ∴ triangle ABE = $\frac{1}{2}$ of parallelogram ABCD

⇒ triangle ADE + triangle EBC = Another $\frac{1}{2}$ of parallelogram ABCD

∴ triangle ADF = $\frac{1}{2}$ of parallelogram ABCD

⇒ triangle ADF= triangle ADE+ triangle DEF

→ triangle ADE+ triangle DEF = triangle ADE+ triangle ABE

∴ triangle DEF = triangle ABE

8. Question

Two triangles ABC and ABD with equal area and stand on the opposite side of AB let's prove that AB bisects CD.

Answer

Given.

Two triangles ABC and ABD with equal area and stand on the opposite side of AB

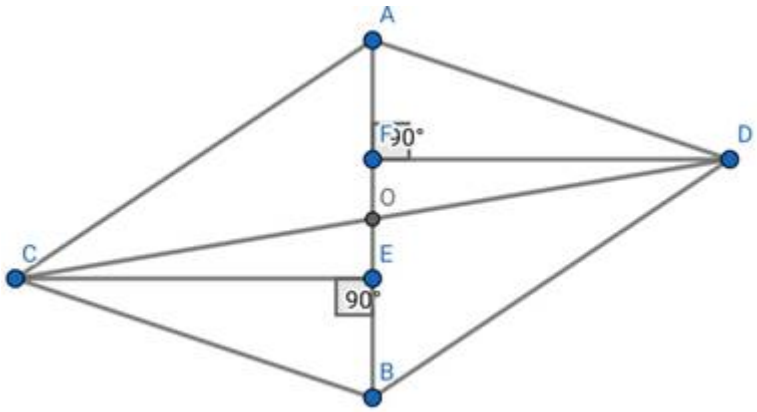
Formula used.

Area of triangle = $\frac{1}{2}$ × Base × Height

AAS congruency rule = If 2 angles and one side of both triangles are equal then both triangle are congruent.

Make the perpendicular CE and DF on triangle ABC and triangle ABD

Mark intersection point of AB and CD as O



If Area of triangle ABC = Area of triangle ABD

$\frac{1}{2} \times AB \times CE = \frac{1}{2} \times AB \times DF$

CE = DF

In triangle CEO and triangle DFO

$\angle DFO = \angle CEO$ ∵ Both are perpendicular 90°

$\angle DOF = \angle COE$ ∵ vertically opposite angle

CE = DF ∵ proved above

triangle CEO ≅ triangle DFO ∵ AAS congruency rule

∴ CO = DO

Hence AB bisect CD

9. Question

D is the midpoint of side BC of triangle ABC Parallelogram CDEF stands between side BC and parallel to BC through the point A. Let's prove that triangle ABC = parallelogram CDEF.

Answer

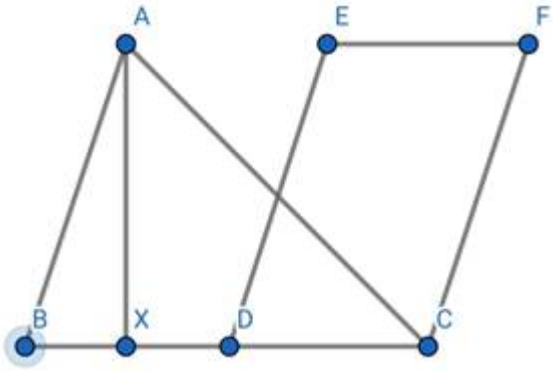
Given.

D is the midpoint of side BC of triangle ABC Parallelogram CDEF stands between side BC and parallel to BC through the point A

Formula used.

Area of triangle = $\frac{1}{2} \times$ Base × Height

Area of parallelogram = Base × Height



In triangle ABC

Area of triangle ABC = $\frac{1}{2}$ × Base × Height

Area of triangle ABC = $\frac{1}{2}$ × BC × AX

In parallelogram CDEF

Area of parallelogram CDEF = Base × Height

Area of parallelogram CDEF = DC × AX

⇒ height of both parallelogram and triangle will be same

Because of stands on BC and parallel through point A

As D is mid-point of BC

DC = $\frac{1}{2}$ × BC

⇒ 2×DC = BC

Area of triangle ABC = $\frac{1}{2}$ × BC × AX

= $\frac{1}{2}$ × 2 × DC × AX

= DC × AX

= Area of parallelogram CDEF

∴ triangle ABC = parallelogram CDEF

10. Question

P is any a point on diagonal BD of parallelogram ABCD. Let's prove that triangle APD = triangle CPD.

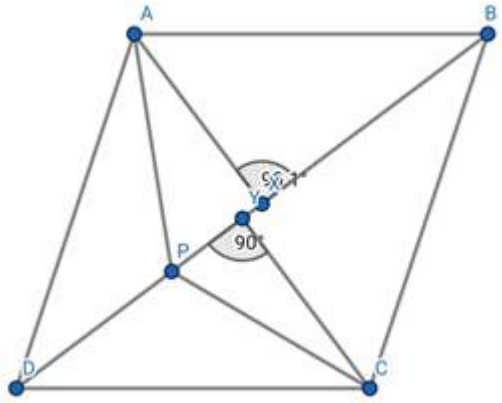
Answer

Given.

P is any a point on diagonal BD of parallelogram ABCD

Formula used.

Area of triangle = $\frac{1}{2}$ × Base × Height



In triangle APD and triangle CPD

As ABCD is a parallelogram

And BD is the diagonal

∴ it divides both congruent triangles

Hence perpendicular of both the triangles are same

AX= CY

For any place of P on BD

The perpendicular of triangle will be same of as of triangle ADB and triangle CBD

Area of triangle APD = $\frac{1}{2}$ × AX × DP

Area of triangle CPD = $\frac{1}{2}$ × CY × DP

= $\frac{1}{2}$ × AX × DP

Area of triangle APD = Area of triangle CPD

∴ triangle APD = triangle APD

11. Question

AD and BE are the medians of triangle ABC. Let's prove that triangle ACD = triangle BCE.

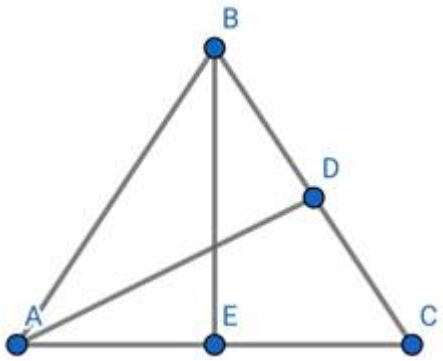
Answer

Given.

AD and Be are medians of triangle ABC

Formula used.

Median divides the triangle into 2 equal parts



As AD is median of BC of triangle ABC

triangle ADC = triangle ADB

Δ ADC + triangle ADB = triangle ABC

2 × triangle ADC = triangle ABC

triangle ADC $=\frac{1}{2}$ × triangle ABC

As BE is median of AC of triangle ABC

triangle BCE = triangle BAE

triangle BCE + triangle BAE = triangle ABC

2 × triangle BCE = triangle ABC

triangle BCE $=\frac{1}{2}$ × triangle ABC

By concluding both;

triangle ADC = triangle BCE

12. Question

A line parallel to BC of triangle ABC intersects sides AB and AC at the points P and Q respectively. CQ and BQ intersect each other at the point X. Let's prove that.

(i) ΔBPQ = triangle CPQ

(ii) ΔBCP = triangle BCQ

(iii) ΔACP = triangle ABQ

(iv) ΔBXP = triangle CXQ

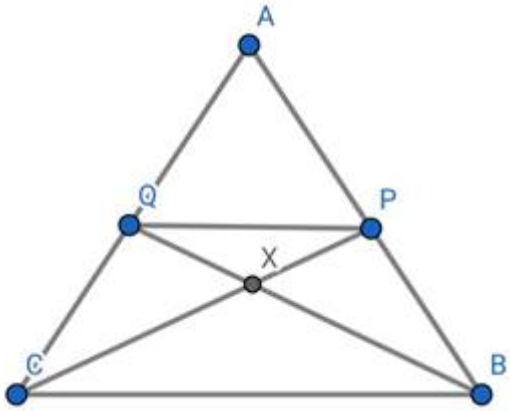
Answer

Given.

A line parallel to BC of triangle ABC intersects sides AB and AC at the points P and Q respectively. CQ and BQ intersect each other at the point X

Formula used.

If 2 triangles are on same base And lies between 2 parallel lines then both triangles are equal.



⇒ In triangle PQB and triangle PQC

As both triangles lies on same base PQ

And both are between parallel lines as PQ || BC

∴ triangle PQB = triangle PQC

⇒ In triangle BCP and triangle BCQ

As both triangles lies on same base BC

And both are between parallel lines as PQ || BC

∴ triangle BCP = triangle BCQ

⇒ In triangle ACP and triangle ABX

As triangle PQB = triangle PQC ∵ proved above

Add triangle APQ in both triangles

triangle PQB + triangle APQ = triangle PQC + triangle APQ

triangle ACP = triangle ABX

⇒ In triangle BXP and triangle CXQ

As triangle PQB = triangle PQC ∵ proved above

Subtract triangle PXQ in both triangles

triangle PQB – triangle PXQ = triangle PQC – triangle PXQ

triangle BXP = triangle CXQ

13. Question

D is the midpoint of BC of triangle ABC and P is any a point on BC. Join P, A- Through the point D a straight line parallel to line segment PA meets AB at point Q. Let's prove that,

(i) ΔADQ = triangle PDQ

(ii) ΔBPQ = $\frac{1}{2}$ ΔABC

Answer

Given.

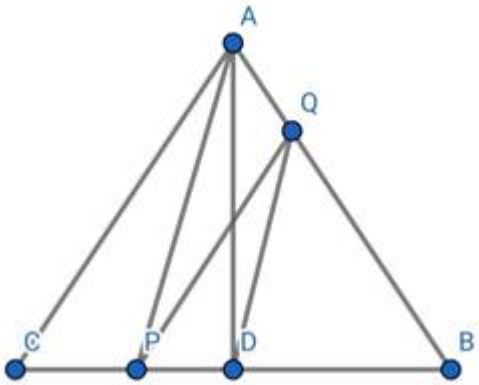
D is the midpoint of BC of triangle ABC

From point D a straight line parallel to line segment PA meets AB at point Q

Formula used.

If 2 triangles are on same base And lies between 2 parallel lines then both triangles are equal.

Join DA



In triangle PDQ and triangle ADQ

As both triangles lies on same base DQ

And both are between parallel lines as PA || DQ

∴ triangle PDQ = triangle ADQ

As AD is median of triangle ABC

triangle ADC = triangle ABD = $\frac{1}{2}$ × triangle ABC

triangle ABD = triangle BDQ + triangle ADQ

triangle ABD = triangle BDQ + triangle PDQ ∵ proved above

$\frac{1}{2}$ × triangle ABC = triangle BPQ

14. Question

In triangle ABC of which AB = AC; perpendiculars through the points B and C on sides AC and AB intersect sides AC and AB at the points E and F. Let's prove that, FE || BC.

Answer

Given.

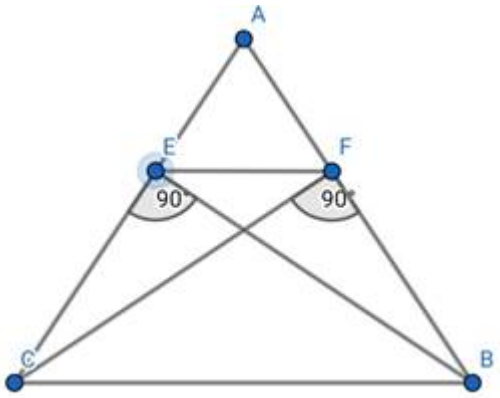
triangle ABC is isosceles triangle as AB = AC

BE and CF are perpendicular on AB and AC

Formula used.

AAS congruency rule = If 2 angles and one side of both triangles are equal then both triangle are congruent.

If 2 triangles are on same base And having equal area then they lies between 2 parallel lines.



In triangle AEB and triangle AFC

AB = AC ∵ Given

∠ AEB = ∠ AFC ∵ Both are perpendicular (90°)

∠ A = ∠ A ∵ common

triangle AEB ≅ triangle AFC [By AAS property]

If triangle AEB = triangle AFC

Then opening the triangles we get,

In triangle FEB + triangle AFE = triangle FEC + triangle AFE

On subtracting we get

triangle FEB = triangle FEC

In triangle FEB and triangle FEC

triangle FEB = triangle FEC

And they are on same base FE

Hence; they are between parallel lines

FE || BC

15. Question

In triangle ABC ∠ABC = ∠ACB; bisectors of an angle ∠ABC and ∠ACB intersect the side AC and AB at the points E and F respectively. Let's prove that, FE || BC.

Answer

Given.

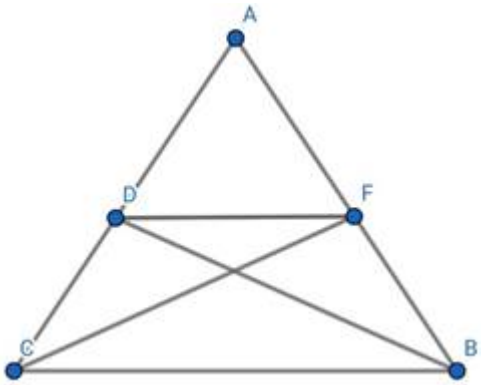
∠ABC = ∠ACB

BE and CF are the bisectors of ∠ABC and ∠ACB respectively

Formula used.

ASA congruency rule = If 2 angles and one side between them of both triangles are equal then both triangle are congruent.

If 2 triangles are on same base And having equal area then they lies between 2 parallel lines.



As ∠ABC = ∠ACB

Triangle ABC is isosceles triangle

∴ AB = AC

If BE and CF are the bisectors of ∠ABC and ∠ACB

∠ ABE = ∠ CBE and ∠ ACF = ∠ BCF

As ∠ABC = ∠ACB

Then

∠ ABE = ∠ CBE = ∠ ACF = ∠ BCF

In triangle AEB and triangle AFC

AB = AC ∵ Proved above

∠ ABE = ∠ ACF ∵ Proved above

∠ A = ∠ A ∵ common

Triangle AEB ≅ triangle AFC [By ASA property]

If triangle AEB = triangle AFC

Then opening the triangles we get,

In triangle FEB + triangle AFE = triangle FEC + triangle AFE

On subtracting we get

triangle FEB = triangle FEC

In triangle FEB and triangle FEC

triangle FEB = triangle FEC

And they are on same base FE

Hence; they are between parallel lines

FE || BC

16. Question

The two parallelogram shaped regions ABCD and AEFG of which ∠A is common are equal in area and E lies on AB. Let's prove that DE || FC.

Answer

Given.

2 parallelogram ABCD and AEFG equal in area with common ∠ A

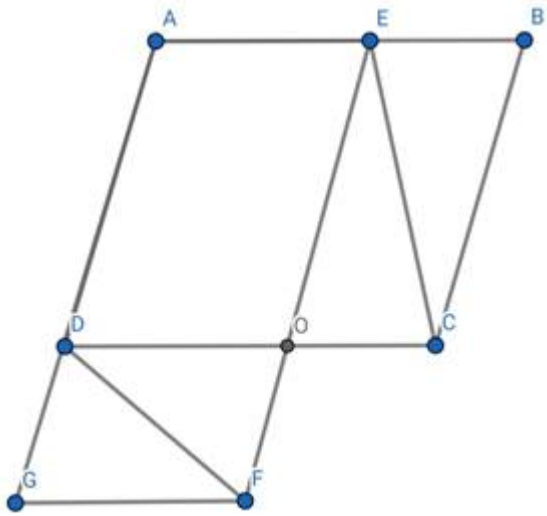
Formula used.

If 2 triangles are on same base And having equal area then they lies between 2 parallel lines.

If both pair of opposite sides parallel then quadrilateral is parallelogram.

Join DF and EC

Mark intersection point of both parallelogram as O



In quadrilateral AEOD

AE || DO ∵ AB || CD [opposite sides of parallelogram]

AD || EO ∵ AG || GH [opposite sides of parallelogram]

∴ AEOD is a parallelogram

If ABCD is a parallelogram and AEOD is also a parallelogram then EBCO is also a parallelogram

If AEFG is a parallelogram and AEOD is also a parallelogram then DOFG is also a parallelogram

Parallelogram ABCD = Parallelogram AEFG

Parallelogram [AEOD+EBCO]=Parallelogram [AEOD+DOFG]

Cutting both sides parallelogram AEOD

We get ;

Parallelogram EBCO = Parallelogram DOFG

As DF , EC are diagonals of parallelogram EBCO and DOFG

⇒ Diagonal of parallelogram bisect in 2 equal triangles

Hence we get

2× triangle EOC = 2× triangle DOF

Triangle EOC = triangle DOF

Add triangle FOC on both side

Triangle EOC + triangle FOC = triangle DOF + triangle FOC

Triangle ECF= triangle DFC

2 triangles triangle ECF, triangle DFC are equal

And they are on same base FC

Hence they are between parallel lines FC and DE

∴ FC || DE

17. Question

ABCD is a parallelogram and ABCE is a quadrilateral shaped regions. Diagonal AC divides the quadrilateral shaped region ABCE into two equal parts. Let's prove that AC || DE.

Answer

Given.

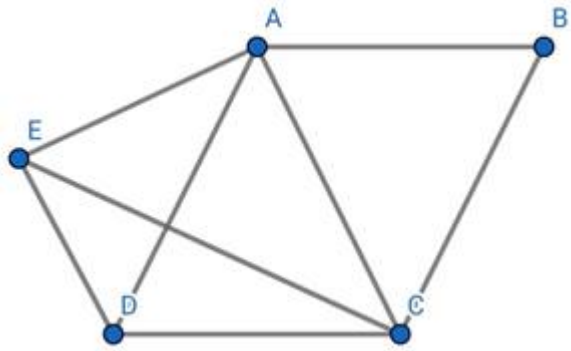
ABCD is a parallelogram and ABCE is a quadrilateral

Diagonal AC divides the quadrilateral shaped region ABCE into two equal parts

Formula used.

If 2 triangles are on same base And having equal area then they lies between 2 parallel lines.

Diagonal of parallelogram divide it into 2 congruent triangles



In quadrilateral ABCE

As AC divides quadrilateral into 2 equal parts

Triangle ABC = triangle AEC

In parallelogram ABCD

As AC is diagonal which divides parallelogram into 2 equal triangles

Triangle ABC = triangle ADC

Comparing both

We get

Triangle AEC = triangle ADC

As both triangles triangle AEC ,Δ ADC are equal

And both lies on same base AC

Hence both triangles comes between parallel lines AC , DE

∴ AC || DE

18. Question

D is the midpoint of side BC of triangle ABC; P and Q lie on sides BC and BA in such a way that triangle BPQ = $\frac{1}{2}$ ΔABC. Let's prove that, DQ || PA.

Answer

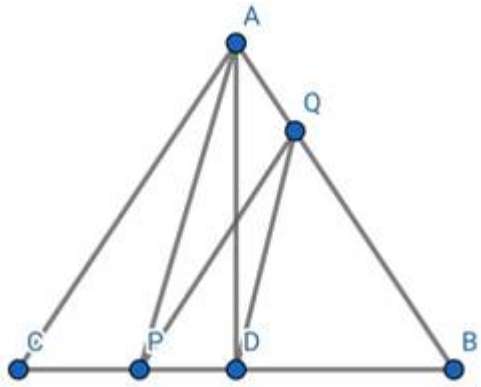
Given.

D is the midpoint of side BC of triangle ABC; P and Q lie on sides BC and BA in such a way that triangle BPQ = $\frac{1}{2}$ΔABC

Formula used.

If 2 triangles are on same base And having equal area then they lies between 2 parallel lines.

Median divides triangle in 2 equal parts



As D is mid-point of side BC

AD is median

Triangle ADC = triangle ADB = $\frac{1}{2}$× triangle ABC

As we have given with

Triangle BPQ = $\frac{1}{2}$ triangle ABC

Triangle BPQ = triangle ADB

Triangle BPQ = triangle BQD + triangle DQP

Triangle ADB = triangle BQD + triangle DQA

By subtracting triangle BQD from both sides

triangle DQP = triangle DQA

As both triangles triangle DQP , triangle DQA are equal

And both lies on same base DQ

Hence both triangle 's lies between parallel lines DQ and PA

∴ DQ || PA

19. Question

Parallelogram ABCD of which midpoints are E, F, G and H of sides AB, BC, CD and DA respectively. Let's prove that

(i) EFGH is a parallelogram.

(ii) Area of parallelogram shaped region EFGH is half of the area of parallelogram shaped region ABCD.

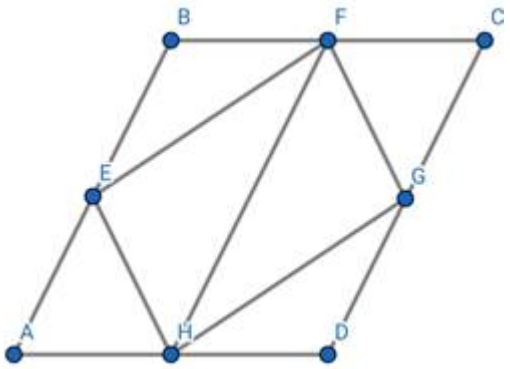
Answer

Given.

Parallelogram ABCD of which midpoints are E, F, G and H of sides AB, BC, CD and DA respectively

Formula used.

SAS congruency rule = If 2 sides and angle between them are equal in both triangles then both triangles are congruent



In triangle DGH and triangle BEF

DG= EB ∵ AB= CD then their half also be equal

DH= FB ∵ BC= DA then their half also be equal

$\angle D = \angle B$ ∵ opposite angles of parallelogram are equal

triangle DGH ≅ triangle BEF [By SAS congruency]

∴ GH = EF

In triangle AEH and triangle CGF

GC= AE ∵ AB= CD then their half also be equal

AH= FC ∵ BC= DA then their half also be equal

$\angle C = \angle A$ ∵ opposite angles of parallelogram are equal

triangle DGH ≅ triangle BEF [By SAS congruency]

∴ HE = GF

If both pair of sides are equal in quadrilateral then this is called parallelogram

∴ EFGH is a parallelogram

Join the mid-points FH

Parallelogram divides in 2 equal parallelograms

In parallelogram ABFH and triangle EFH

Both lies on same base FH

And both lies on parallel lines AB and FH

AB || FH ∵ opposite sides of parallelogram are parallel

Then

Area of triangle EFH = $\frac{1}{2}$ × Area of parallelogram ABFH

In parallelogram FHDC and triangle EFG

Both lies on same base FH

And both lies on parallel lines CD and FH

CD || FH ∵ opposite sides of parallelogram are parallel

Then

Area of triangle GFH = $\frac{1}{2}$ × Area of parallelogram FHDC

Adding both we get;

Area of [Δ EFG+Δ GFH]=$\frac{1}{2}$×Area of parallelogram [ABFH+FHDC]

Area of parallelogram EFGH = $\frac{1}{2}$×Area of parallelogram ABCD

20. Question

AB || DC of a trapezium ABCD and E is midpoint of BC. Let's prove that area of triangular region AED = $\frac{1}{2}$ × area of trapezium shaped region ABCD.

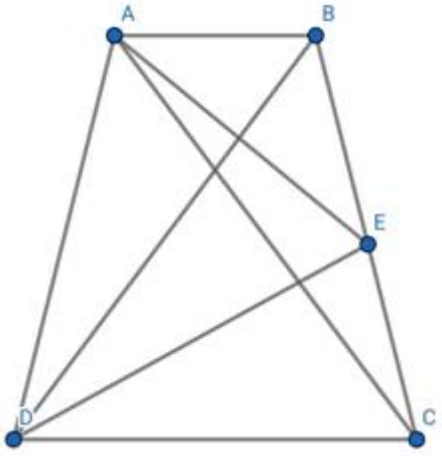
Answer

Given.

AB || DC of a trapezium ABCD and E is midpoint of BC

Formula used.

Median of triangle divides it into 2 equal parts



As E is midpoint of BC

In triangle ABC

AE is median

Triangle ABE = $\frac{1}{2}$× triangle ABC

2 × triangle ABE = triangle ABC ……eq 1

In triangle BDC

DE is median

Triangle DEC = $\frac{1}{2}$× triangle DBC

2 × triangle DEC = triangle DBC ……eq 2

In triangle ADC and triangle DBC

Both are on same base DC

And AB || CD

Triangle ADC = triangle DBC

Putting value from eq 2

Triangle ADC = 2 × triangle DEC ……eq 3

Add Eq1 and Eq 3

We get ;

triangle ADC + triangle ABC = 2 × [Δ ABE + triangle DEC]

Trapezium ABCD = 2 × [trapezium ABCD – triangle AED]

2 × triangle AED =trapezium ABCD

Area of triangle AED = $\frac{1}{2}$ × trapezium ABCD

21 A. Question

D, E and F are midpoints of sides BC, CA and AB respectively of a triangle ABC. If triangle ABC = 16 sq. cm., then the area of the trapezium shaped region FBCE is

A. 40 sq. cm

B. 8 sq. cm

C. 12 sq. cm

D. 100 sq. cm

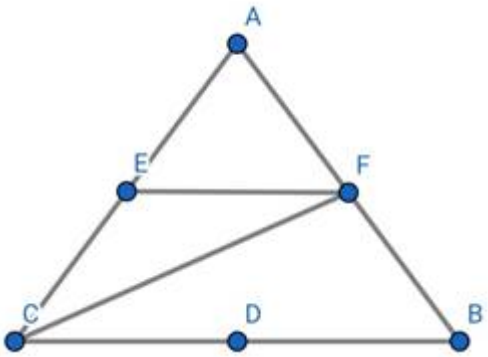
Answer

Given.

D, E and F are midpoints of sides BC, CA and AB respectively of a triangle ABC. If triangle ABC = 16 sq. cm

Formula used .

Median of triangle divides triangle in 2 equal parts



As F is midpoint of AB

CF is median of triangle ABC

∴ Area of triangle ABC = Area of triangle CFB + Area of triangle CFA

Area of triangle CFB = Area of triangle CFA=$\frac{1}{2}$× Area of triangle ABC

Area of triangle CFB = Area of triangle CFA=8 cm^2

In triangle AFC

FE is acting as median

∴ Area of triangle CFA = Area of triangle CEF + Area of triangle AFE

Area of triangle CEF = Area of triangle AFE =$\frac{1}{2}$× Area of triangle CFA

Area of triangle CEF = Area of triangle AFE=4 cm^2

Area of trapezium FBCE = Area of triangle ABC – Area of triangle AFE

= 16 cm^2 – 4 cm^2

= 12 cm^2

21 B. Question

A, B, C, D are the mid points of sides PQ, QR, RS and SP respectively of parallelogram PQRS. If area of the parallelogram shaped region PQRS = 36 sq.cm then area of the region ABCD is

A. 24 sq. cm

B. 18 sq. cm

C. 30 sq. cm

D. 36 sq. cm

Answer

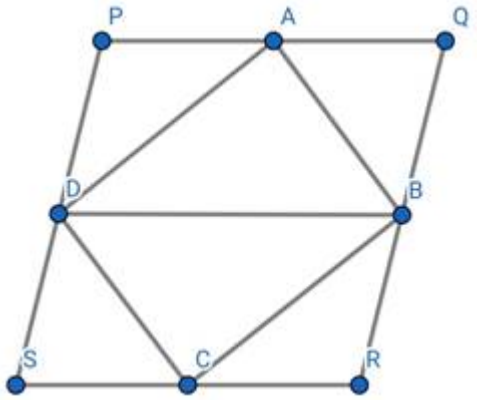
Given

A, B, C, D are the mid points of sides PQ, QR, RS and SP respectively of parallelogram PQRS. If area of the parallelogram shaped region PQRS = 36 sq.cm

Formula used.

If one triangle and one parallelogram are on same base and both are between 2 parallel lines then Area of triangle gets half of parallelogram

Solution



Join BD

Parallelogram BDPQ and triangle ADB are on same base BD

And PQ || BD

Thus;

Area of triangle ABD = $\frac{1}{2}$× Area of parallelogram BDPQ

Parallelogram BDSR and triangle CDB are on same base BD

And SR || BD

Thus;

Area of triangle CBD = $\frac{1}{2}$× Area of parallelogram BDSR

Adding both we get

Area of[Δ ABD+Δ CBD]= $\frac{1}{2}$× Area of parallelogram [BDPQ+BDSR]

Area of ABCD parallelogram = $\frac{1}{2}$× Area of PQSR parallelogram

Area of ABCD parallelogram = $\frac{1}{2}$× 36 cm^2 = 18 cm^2

21 C. Question

O is any a point inside parallelogram ABCD. If triangle AOB + triangle COD = 16 sq.cm, then area of the parallelogram shaped region ABCD is

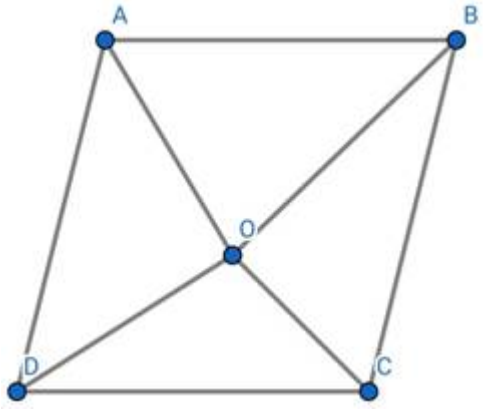
A. 8 sq. cm

B. 4 sq. cm

C. 32 sq. cm

D. 64 sq. cm

Answer



If point O present inside parallelogram

Triangle AOB + triangle COD is half of the parallelogram

Then;

Triangle BOC + triangle DOA is another half of parallelogram

If triangle AOB + triangle COD = 16 sq. cm

Then area of parallelogram = 2×[triangle AOB + triangle COD]

= 2× 16 sq. cm

= 32 sq. cm

21 D. Question

D is the midpoint of side BC of triangle ABC. E is the midpoint of side BD and O is the midpoint of AE; area of triangular field BOE is

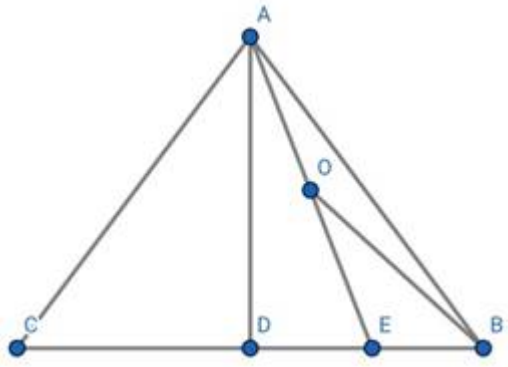
A. $\frac{1}{3}$ × Area of triangle ABC

B. $\frac{1}{4}$ × Area of triangle ABC

C. $\frac{1}{6}$ × Area of triangle ABC

D. $\frac{1}{8}$ × Area of triangle ABC

Answer



As D is midpoint of BC

AD median divides triangle ABC in 2 equal parts

Area of triangle ABD = $\frac{1}{2}$ × Area of triangle ABC

As E is midpoint of BD

AE median divides triangle ABD in 2 equal parts

Area of triangle ABE = $\frac{1}{2}$ × Area of triangle ABD

= $\frac{1}{2} \times \frac{1}{2}$ × Area of triangle ABC

= $\frac{1}{4}$ × Area of triangle ABC

As O is midpoint of AE

AO median divides triangle ABE in 2 equal parts

Area of triangle BOE = $\frac{1}{2}$ × Area of triangle ABE

= $\frac{1}{2} \times \frac{1}{4}$ × Area of triangle ABC

= $\frac{1}{8}$ × Area of triangle ABC

21 E. Question

A parallelogram, a rectangle and a triangular region stand on same base and between same parallel and if their area are P, R and T respectively then

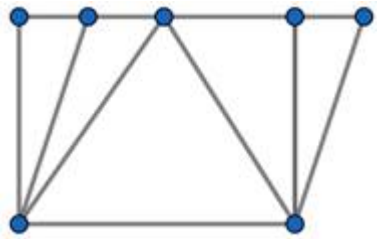
A. P = R = 2T

B. $P = R = \frac{T}{2}$

C. 2P = 2R = T

D. P = R = T

Answer



When an parallelogram and a triangle lies on same base and are between 2 parallel lines

The area of triangle is half area of parallelogram

If Area of parallelogram = P

And Area of triangle = T

Then ,

P = 2T

Rectangle is also type of parallelogram

Hence area of rectangle is

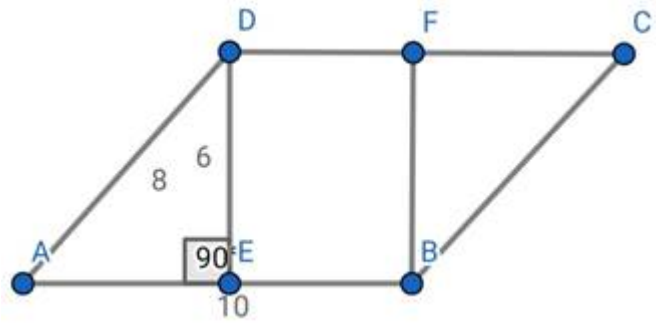
R = 2T

∴ P = 2T = R

22 A. Question

DE is perpendicular on side AB from point D of parallelogram ABCD and BF is perpendicular on side AD from the point B; if AB = 10 cm, AD = 8 cm and DE = 6 cm, let us write how much length of BF is

Answer



Area of parallelogram ABCD = Base × Height

= DE × AB

= 6 cm × 10 cm = 60 cm^2

Also area of parallelogram ABCD = Base × Height

= BF × DC

DC = AB ∵ opposite sides of parallelogram

= BF × 10 cm

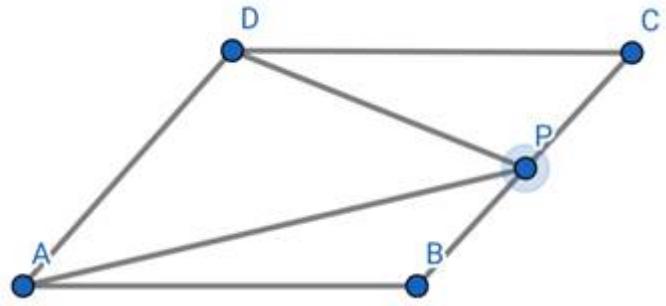
= 60 cm^2

BF = $\frac{60\ cm^2}{10cm}$ = 6 cm

22 B. Question

The area of the parallelogram shaped region ABCD is 100 sq units. P is midpoint of side BC; let us write how much area of triangular region ABP is.

Answer



As the triangle ABP and parallelogram ABCD

Both are on same base AB

And between parallel lines Because P is on BC while opposite sides of parallelogram are parallel

∴ Area of triangle ABP = $\frac{1}{2}$ × Area of parallelogram ABCD

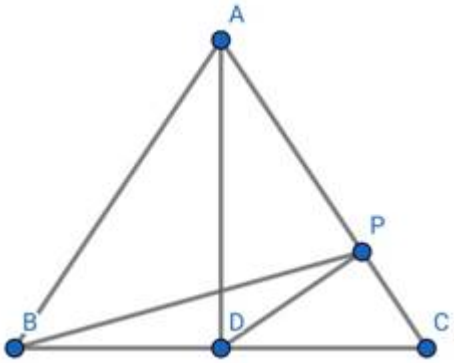
Area of triangle ABP = $\frac{1}{2}$ × 100 sq. unit

= 50 sq. unit

22 C. Question

AD is the median of triangle ABC and P is any point on side AC in such a way that area of triangle ADP : area of triangle ABD = 2 : 3. Let us write area of triangle PDC : area of triangle ABC.

Answer



Area of triangle ADP : area of triangle ABD = 2 : 3

As AD is median

area of triangle ADB = area of triangle ADC = $\frac{1}{2}$ × area of triangle ABC

Let triangle ADP be 2x

And triangle ABD be 3x

Then Area of triangle ABC = 2 × Area of triangle ABD

= 2 × 3x

= 6x

area of triangle ADB = area of triangle ADC

area of triangle ADB = area of triangle ADP + area of triangle PDC

3x = 2x + area of triangle PDC

Area of triangle PDC = x

area of triangle PDC : area of triangle ABC

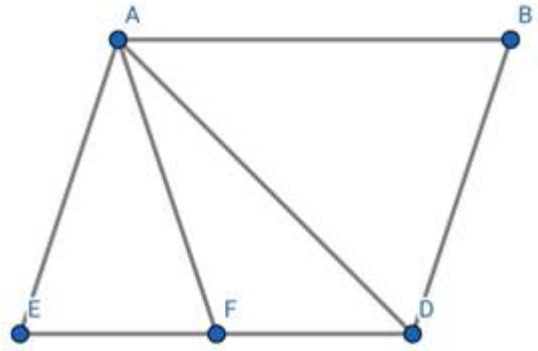
x : 6x

1 : 6

22 D. Question

ABDE is a parallelogram. F is midpoint of side ED. If area of triangular region ABD is 20 sq. unit, then let us write how much area of triangular region AEF is.

Answer



As the triangle ABD is forms by diagonal of parallelogram ABDE

Triangle ABD = triangle AED = 20 sq. unit

As F is midpoint of ED

AF is median of triangle AED

Triangle AEF = triangle AFD = $\frac{1}{2}$ × triangle AED

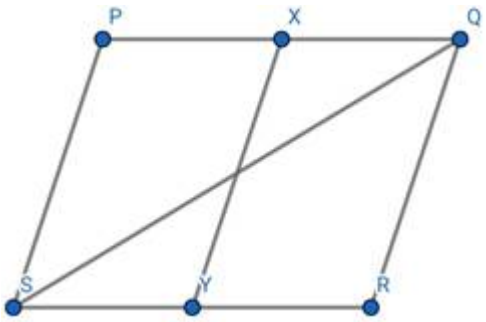
Triangle AEF = triangle AFD = $\frac{1}{2}$ × 20 sq. unit

Triangle AEF = 10 sq. unit

22 E. Question

PQRS is a parallelogram X and Y are the mid points of side PQ and SR respectively. Join diagonal SQ. Let us write area of the parallelogram shaped region XQRY : area of triangular region QSR.

Answer



In parallelogram PQRS and XQRY

Area of PQRS = SR × Height

Area of XQRY = YR × Height

YR = $\frac{1}{2}$× SR

Area of parallelogram XQRY = $\frac{1}{2}$× SR × Height

$= \frac{1}{2} \times$ Area of parallelogram PQRS

In parallelogram PQRS and triangle QSR

Both lies on same base SR

And both are between parallel lines

$\therefore$ Area of triangle QSR $= \frac{1}{2} \times$ Area of parallelogram PQRS

Area of triangle QSR = Area of parallelogram XQRY

Area of triangle QSR : Area of parallelogram XQRY

1 : 1