

Hcampus

be reasonable, to be successful

ለተጨማሪ ፈተናዎች በቴሌግራም ቻናላችን ይከታተሉን /https//t.me/z_campus ! ይከታተሉ !

University of Gondar
College of Natural & Computational Sciences
Basic Mathematics for Natural Sciences
Worksheet-I

1. Which of the following sentences are propositions? For those that are, indicate the truth value.

   (a) 123 is a prime number.
   (b) 0 is an even number.
   (c) $x^2 - 4 = 0$.
   (d) Multiply $5x + 2$ by 3.
   (e) What an impossible question!
   (f) Every Ethiopian has a dream.
   (g) $(\forall n \in \mathbb{N})(n^2$ is odd)
   (h) $|x| \geq 0$, for all real number x.

2. If $p \Rightarrow q$ is true, what can be deduced about the truth values of the following?

   (a) $p \vee r \Rightarrow q \vee r$
   (b) $p \wedge r \Rightarrow q \wedge r$
   (c) $\neg p \wedge q \Leftrightarrow p \vee q$

3. Determine whether the following statments are tautology, contradiction, or not.

   (a) $p \Rightarrow \neg q$
   (b) $p \Rightarrow (q \Rightarrow p)$
   (c) $(p \vee q) \Rightarrow (p \wedge q)$

4. Determine the converse, inverse, contrapositives and negations of the following conditional statments

   (a) If you don't do your work, then you will fail in Mathematics.
   (b) Two angles are supplementary if they measure 180°.
   (c) It is raining is sufficient condition for my not going to town.

5. If $p \Rightarrow q$ is true, determine the truth value of the following compound propositions using truth table.

   (a) $p \vee r \Rightarrow q \wedge r$
   (b) $\neg p \wedge q \Leftrightarrow p \vee q$
   (c) $\neg p \vee q$

6. Translate the following statments using open propositions and quantifiers

   (a) Everybody loves someone.
   (b) For any two people, if they are Ethiopian, then they are brothers.
   (c) For all positive integers, their sum is positive.
   (d) There is a student who has taken mathematics test.

7. Determine the truth-values of the following quantified propositions

   (a) $(\forall x)P(x) \Rightarrow (\exists x)P(x)$
   (b) $\neg[(\forall x)P(x) \Rightarrow Q(x)] \Rightarrow (\exists x)[P(x) \wedge \neg Q(x)]$
   (c) $(\forall x)[P(x) \Rightarrow Q(x)] \Rightarrow [(\forall x)P(x) \Rightarrow (\forall x)Q(x)]$
   (d) $(\exists x)P(x) \wedge (\exists x)Q(x) \Rightarrow (\exists x)[P(x) \wedge Q(x)]$

8. Show that the following paired propositions are logical equivalent

   (a) $p \vee q$ and $\neg p \Rightarrow q$
   (b) $p \wedge q$ and $\neg(p \Rightarrow \neg q)$
   (c) $p \wedge q \Rightarrow r$ and $p \Rightarrow (q \Rightarrow r)$
   (d) $\neg(p \vee (\neg p \wedge q))$ and $\neg(p \vee q)$, without truth-table
   (e) $\neg p \vee (q \vee r)$ and $(\neg r \wedge p) \Rightarrow q$, without truth-table
   (f) $p \Rightarrow (q \wedge \neg q)$ and $\neg p$
   (g) $p \Leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$

9. Show that the following compound propositions are tautology or contradiction without truth-table.

   (a) $\neg(\neg p \vee (p \vee q)) \Rightarrow q$
   (b) $((p \Rightarrow q) \wedge q) \Rightarrow q$
   (c) $((p \Rightarrow q) \wedge (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)$
   (d) $\neg(p \vee \neg(p \wedge q))$
   (e) $p \Rightarrow (q \Rightarrow p)$
   (f) $((p \Rightarrow q) \wedge \neg q) \Rightarrow \neg p$

10. Determine the truth-value of the following quantified propositions with the given corresponding domains.

(a) $(\forall x \in \mathbb{N})(x - x! = 0)$

(b) $(\exists x > 0)(x^2 + x = 0)$

(c) $(\forall x)(x \text{ is mortal})$, where the domain of x is the set of all people.

(d) $(\exists x \in \mathbb{C})(x^4 = -1)$

(e) $(\forall x \in \mathbb{Z})$(If x is even, then x^2 is even.)

(f) $(\exists x)(x$ is prime and even.)

(g) $(\forall x \in \mathbb{Z})(\exists y \in \mathbb{N})(x < y)$

11. Check the validity of the following arguments.

(a) If I quit school, I will play football. I did quit school. Therefore, I play football.

(b) I have my keys or I am locked out. I am not locked out. Therefore, I have my keys.

(c) I walk or I chew gum. I am walking. Therefore, I am not chewing gum.

(d) If I get elected, I will reduce taxes. If I reduce taxes, the economy will prosper. Thus, If I get elected, the economy will prosper.

(e) I got scolarship and I got an "A" in math. I am not good at logic or I got an "A" in math. Therefore, I am good at logic or I don't get a scholarship.

12. Find the relation between set A and B that make the following propositions true.

(a) $A \cup B = B$
(b) $A \cap B = B$
(c) $A \cup B = A \cap B$
(d) $A - B = \emptyset$
(e) $A - B = A$
(f) $A - B = B$
(g) $A - B = B - A$
(h) $(A - B) \cap (A \cap B) = \emptyset$
(i) $A \cap B = B$ and $A \cup B = B$
(j) $A \Delta B = A$

13. For any sets A, B and C prove the following properties of set operations

(a) $(A \cup B)' = A' \cap B'$
(b) $(A \cup B) - (C - A) = A \cup (B - C)$
(c) $(A - B) \cap (A \cap B) = \emptyset$
(d) $A \Delta B = (A \cup B) - (A \cap B)$

14. There were 100 university students interviewed about their dail use of social media

- ☐ 18 students use telegram.
- ☐ 29 students use instagram and facebook.
- ☐ 15 students use facebook, instagram and telegram.
- ☐ 40 students use facebook and telegram.
- ☐ 20 students use instagram and telegram.
- ☐ 60 students use facebook.
- ☐ 7 students do not use instagram, nor facebook, nor telegram.

Based on the above information answer the following questions

(a) Represent the given information using Venn diagram.

(b) How many students use instagram?

(c) How many students use facebook or telegram?

15. How many elements does A contain if it has:

(a) 64 subsets?
(b) 31 proper subsets?
(c) No proper subset?
(d) 255 proper subsets?

16. Is there a set A with exactly the following indicated property?

(a) Only one subset
(b) Only one proper subset
(c) Exactly 3 proper subsets
(d) Exactly 4 subsets
(e) Exactly 6 proper subsets
(f) Exactly 30 subsets

University of Gondar
College of Natural & Computational Sciences
Basic Mathematics for Natural Sciences
Worksheet-II

17. Prove the following propositions using principle of mathematical induction

(a) $1+5+9+13+\ldots+(4n-3)=\dfrac{n(4n-2)}{2}$

(b) $\sum_{k=1}^{n} k^3=\dfrac{n^2(n+1)^2}{4}$

(c) $\sum_{r=1}^{n} r(r+1)=\dfrac{n(n+1)(n+2)}{3}$

(d) $n!>3^n$ for $n\geq 7$

(e) $3^n<(n+1)!$ for $n\geq 4$

(f) $2^n\geq 1+n$ for $n\geq 1$

(g) 8^n-3^n is divisible by 5 for all $n\in\mathbb{N}$

(h) n^3+2n is divisible by 3 for all $n\in\mathbb{N}$

18. Consider the sequence of real numbers defined by the recurrence relation

$$x_1=1 \quad \text{and} \quad x_{n+1}=\sqrt{1+2x_n} \quad \text{for } n\geq 1$$

Use PMI to show that $x_n<4$ for all $n\geq 1$.

19. Identify all properties of real numbers in steps of solving the following equations or inequalities

(a) $3x-5=2$

(b) $ax-b=c,\ \forall a,b,c\in\mathbb{R}$ and $a\neq 0$.

(c) $\frac{2}{3}x-2\leq 1$

20. Prove the following statements using properties of real number.

(a) $(-1)a=-a$

(b) If $ab=ac,\ a\neq 0$, then $b=c,\ \forall a,b,c\in\mathbb{R}$

(c) $\sqrt{3}$ is irrational number.

21. Does the set of rational numbers hold properties of completnes? Justify your answer.

22. Find the *lub* and the *glb* of the following sets, in the set of real number, if exists.

(a) $A=\{x\in\mathbb{Q}: x^2<2\}$

(b) $B=\left\{1-\frac{1}{n}: n\in\mathbb{N}\right\}$

(c) $C=\left\{\frac{m}{m+n}: m,n\in\mathbb{N}\right\}$

(d) $D=\left\{(-1)^n+\frac{1}{n}: n\in\mathbb{N}\right\}$

(e) $E=\{0.2,0.23,0.234,0.2343,0.23434,0.234343,\ldots\}$

(f) $F=[[-3,0)-(-1,1)]\cap[[-1,2)\cup(0,3)]$

23. Write the following complex numbers in $x+iy$ form

(a) $\dfrac{(2+i)(3-2i)}{1+i}$

(b) $\dfrac{(1-i)^3}{(2+i)^2}$

(c) $\dfrac{1}{3+i}-\dfrac{1}{i-3}$

(d) $(1-i)^3$

(e) $(3+2i)\dfrac{(1-i)}{i^3}$

(f) $(\sqrt{3}-i)^{10}$

24. Find the values of the real numbers x and y in each of the following equations

(a) $\dfrac{x}{1+i}+\dfrac{y}{1-2i}=1$

(b) $\dfrac{x}{2-i}+\dfrac{iy}{3+i}=\dfrac{2}{1+i}$

25. Solve for z

(a) $z(2+i) = 3-2i$

(b) $(z+i)(1-i) = 3i$

(c) $\dfrac{1}{z} + \dfrac{1}{2-i} = \dfrac{3}{1+i}$

26. Given that a and b are real and that $1+2i$ is a root of the equation

$$z^2 + (m+5i)z + n(2-i) = 0$$

Determine

(a) The value of m and n

(b) The roots of the equation

27. Show that the statment is always true (prove) or give a counter example that the statment is not always true (disprove). For any two complex numbers z_1 and z_2

(a) $\mathbf{Re}(iz_1) = \mathbf{Im}(z_1)$

(b) $\mathbf{Re}(iz_1) = i\mathbf{Im}(z_1)$

(c) $\mathbf{Re}(z_1) = \mathbf{Im}(iz_1)$

(d) $\mathbf{Re}(z_1)\mathbf{Re}(z_2) = \mathbf{Re}(z_1 z_2)$

(e) $\mathbf{Im}(z_1) + \mathbf{Im}(z_2) = \mathbf{Im}(z_1 + z_2)$

28. The points A,B,C and D represent the complex numbers z_1, z_2, z_3 and z_4 and O is the origin.

(a) If OABC is a parallelogram, and $z_1 = 1+i$ and $z_2 = 4+5i$, find z_3

(b) Find z_2 and z_4 when ABCD is a square where $z_1 = 2+i$ and $z_2 = 6+7i$.

29. Prove the following using De'Moivre's theorem

(a) $\dfrac{\cos 2\theta + i\sin 2\theta}{\cos 3\theta + i\sin 3\theta} = \cos\theta - i\sin\theta$

(b) $\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$

(c) $z^n + \dfrac{1}{z^n} = 2\cos n\theta$, where $z = \cos\theta + i\sin\theta$.

30. Given $z_1 = 1+i$ and $z_2 = 1-\sqrt{3}i$, then find the modulus and arguments of the following, where $r > 0$ and $-\pi < \theta \leq \pi$.

(a) $z_1 z_2$

(b) $\dfrac{z_1}{z_2}$

(c) z_2^3

(d) $\dfrac{z_1^2}{z_2^4}$

31. Find the n^{th} roots of the following complex numbers for the specified value of n.

(a) $z = 1+i,\ n = 4$

(b) $z = 1-i,\ n = 6$

(c) $z = 2+2i,\ n = 3$

(d) $z = -2i,\ n = 6$

(e) $z = 8,\ n = 4$

(f) $z = 1,\ n = 8$

32. Show that the three roots of $z^3 = 1$ can be expressed in the form $1, \omega, \omega^2$ and $1 + \omega + \omega^2 = 0$

33. Write clear and complete prove for the following mathematical statements

(a) If $x \mid a$ and $x \mid b$, then $x \mid (a-b)$.

(b) If m is even and n is odd, then $m+n-2$ is odd.

(c) If $x \mid a$ and $x \mid b$, then $x \mid (a^2 - b^2)$.

(d) If $x \mid a$ and $y \mid b$, then $xy \mid ab$.

(e) If m is even and n is odd, then $mn+3n$ is odd.

(f) If m is odd, then m^2+1 is even.

(g) If a and b are real numbers, where $b \neq 0$, then $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}$.

(h) The expression $2m^2 - 1$ is odd for all integers m.

(i) If $n^3 + 5n + 6$ is divisible by 3 for some integer n, then $(n+1)^3 + 5(n+1) + 6$ is divisible by 3.