



CIVIL LATEST MADEEASY HAND NOTES

1.FLUID MECHANICS SPCL NOTES

HAND WRITTEN NOTS

ESE, GATE, PSU

MADE EASY :-

NAME..... FLUID MECHANICS (CIVIL).....

SUBJECT..... FM. CIVIL..... (GATE/IES)..... RS- 140.....

INSTITUTE..... MADE EASY.....

GATE/IES-2019

MADE-EASY-2018-19

FLUID MECHANICS

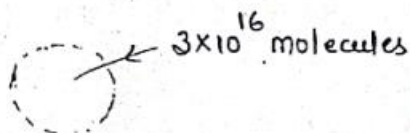
①

Fluids:

Fluids are the substances under the application of tangential force, no matter how small it may be.

Concept of continuum:

In Macro systems when the intermolecular distances are negligible as compared to dimension of system, we can assume that adjacent one molecule there is another molecule without any space. $\therefore$ The entire fluid mass system can be treated as continuous distribution of mass. And such continuous mass of fluid is known as continuum.

ex:For O_2 $T = 20^\circ C$ $P = 1 \text{ atm}$  $V = 1 \text{ mm}^3$ → Fluid Properties

① Density: It is defined as mass per unit volume of a fluid substance:

$$\rho = \frac{M}{V}$$

Units:In MKS kg/m^3 In CGS g/cm^3 SpecificGravity:

It is the ratio of density of a fluid substance to the density of standard fluid.

$$S = \frac{\text{Density of fluid substance}}{\text{Density of standard fluid}}$$

Standard fluid -

For liquids : Water at 4°C

For gases : Air

Ex: $S_{\text{Hg}} = \frac{\rho_{\text{Hg}}}{\rho_{\text{w}}} = \frac{13.6 \times 10^3}{10^3} = 13.6$

③ Specific weight or weight density :

It is defined as weight of fluid substance per unit volume

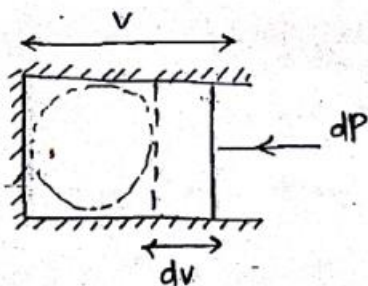
$$\Rightarrow \frac{W}{V} = \rho g$$

④ Compressibility (β) :

It is defined as reciprocal of bulk modulus of elasticity of fluid substance

$$\beta = \frac{1}{K}$$

K = Bulk Modulus of elasticity



$$K = \frac{\text{Stress}}{\text{Strain}}$$

$$\Rightarrow K = \frac{dP}{\left(-\frac{dv}{V}\right)}$$

Now,

$$m = \rho V$$

$$dm = d\rho \cdot V + \rho dV$$

$$-\rho dV = d\rho \cdot V$$

$$\boxed{-\frac{dV}{V} = \frac{d\rho}{\rho}}$$

$$\Rightarrow K = \frac{dP}{d\rho/\rho}$$

$$\boxed{K = \rho \cdot \frac{dP}{d\rho}}$$

$$\boxed{\beta = \frac{1}{\rho} \cdot \frac{d\rho}{dP}}$$

$$\text{If } \frac{d\rho}{dP} = 0 \Rightarrow \boxed{\beta = 0}$$

$$\frac{d\rho}{dP} \neq 0 \Rightarrow \boxed{\beta \neq 0}$$

For liquids :

$$T = 20^\circ\text{C}$$

$$P = 1 \text{ atm}, \rho_w = 998 \text{ kg/m}^3$$

$$P = 100 \text{ atm}, \rho_w = 1003 \text{ kg/m}^3$$

$$\therefore \text{change} = \frac{1003 - 998}{998} \times 100 \approx 0.5\%$$

Hence, liquids are treated as Incompressible.

For Gases :

$$\bullet \text{ } Ma \leq 0.3 \text{ } \tau \text{ } \text{Incompressible.}$$

In gaseous fluids, Mach number ($Ma \leq 0.3$), the flows are considered as incompressible fluids.

(4)

$$Ma = \frac{V}{C}$$

velocity of object
or velocity of the fluid.

velocity of sound in
the medium

⑤ Isothermal compressibility of gases :

$$P = \frac{RT}{V} \leftarrow \text{constant}$$

$$\Rightarrow P V^{-1} = RT = \text{constant}$$

Differentiate it

$$P^{-1} dP - P V^{-2} dV = 0$$

$$P^{-1} \left(dP - \frac{P \cdot dV}{V} \right) = 0$$

$$dP = \frac{P \cdot dV}{V}$$

$$\left(\frac{dP}{dV/V} \right) = P$$

K

$$K_{iso} = P$$

$$\beta_{iso} = \frac{1}{P}$$

⑥ Adiabatic Compressibility of Gases :

$$P \cdot V^\gamma = \text{constant} \quad \text{where, } \gamma = \frac{C_p}{C_v}$$

Note $m = P \cdot V$

$$P \left(\frac{m}{P} \right)^\gamma = \text{constant}$$

$$P \cdot P^{-\gamma} = \text{constant}$$

Differentiate it :

$$\Rightarrow \rho^{-\gamma} \cdot d\rho - \gamma \cdot \rho^{\gamma-1} \cdot P \cdot d\rho = 0$$

(5)

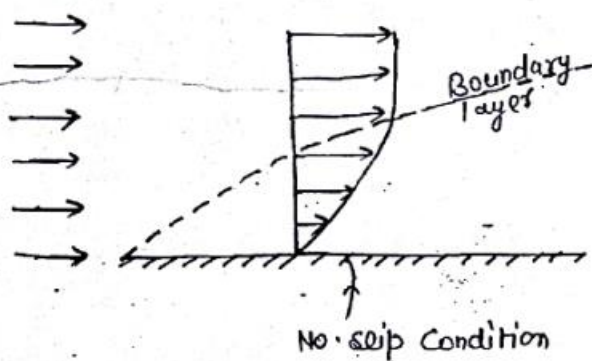
$$\Rightarrow \rho^{-\gamma} \left(d\rho - \gamma \cdot \rho \frac{d\rho}{\rho} \right) = 0$$

$$\Rightarrow d\rho = \gamma \cdot \rho \frac{d\rho}{\rho}$$

$$\Rightarrow \frac{d\rho}{d\rho/\rho} = \gamma \cdot \rho$$

$$\boxed{\text{Kadiabatic} = \gamma \cdot \rho}$$

→ Flow over a Flat plate :

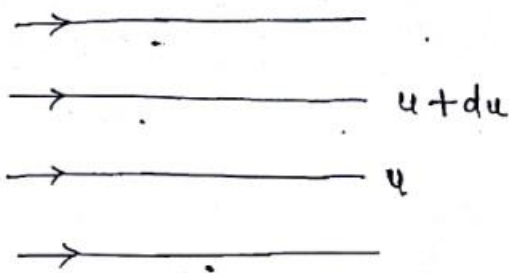


When the real fluid flows over a solid body, the fluid particles on the surface of the body flow with the same velocity as that of the surface to satisfy no-slip condition. So, the relative velocity of fluid particles at the surface of solid body is zero.

Away from the solid body in transverse direction the velocity of fluid particles increases gradually.

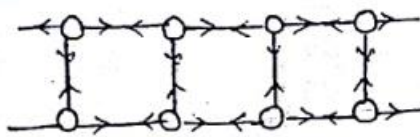
⑦ Viscosity :

The when the 2 adjacent layers of fluid are in relative motion, they resist the motion of each other. Such a fundamental property of fluid is known as viscosity.



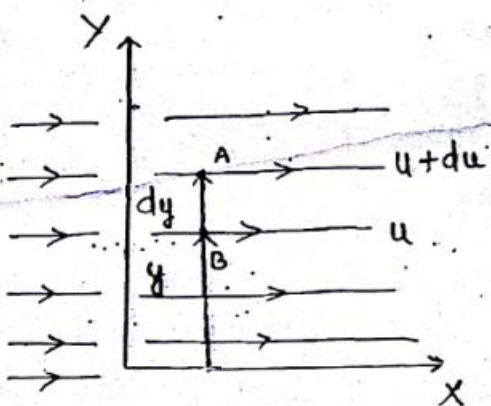
Cause of viscosity

For liquids :

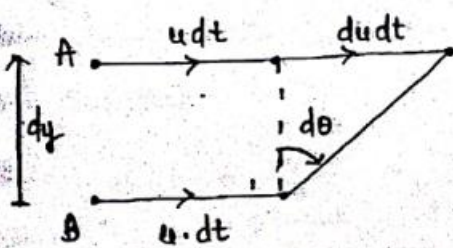


Intermolecular force of attraction $\Rightarrow$ Cohesive force

(a) Angular Deformation



In dt time interval



viscosity $\mu_w > > > \mu_{air}$ (6)

$\mu \Rightarrow$ measurement of internal resistance

For Gases :

Cohesion $\Rightarrow$ Almost Nil

"Random motion of molecules"

$$\tan \theta = \frac{du}{dy}$$

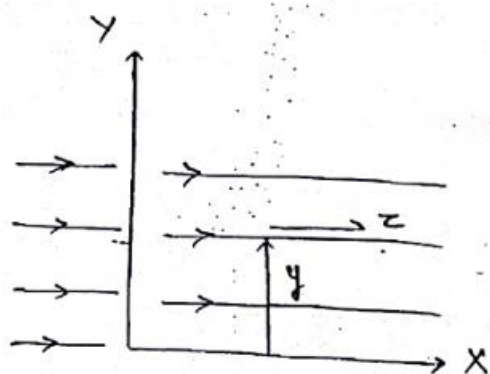
$\because d\theta = \text{very very small}$
 $\Rightarrow d\theta = \tan \theta$

$$d\theta = \frac{du \cdot dt}{dy}$$

$$\Rightarrow \boxed{\frac{d\theta}{dt} = \frac{du}{dy}}$$

Rate of angular deformation.

Newton's Law of Viscosity: According to this, shear stress on a layer at a distance y from the wall is directly proportional to the rate of angular deformation. ①



$$\tau \propto \frac{d\theta}{dt}$$

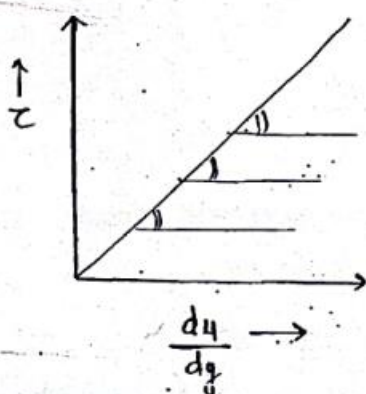
$$\left\{ \frac{d\theta}{dt} = \frac{du}{dy} \right\}$$

$$\Rightarrow \tau \propto \frac{du}{dy}$$

$$\Rightarrow \tau = \mu \frac{du}{dy}$$

where, μ = co-efficient of viscosity
or

Absolute / Dynamic viscosity

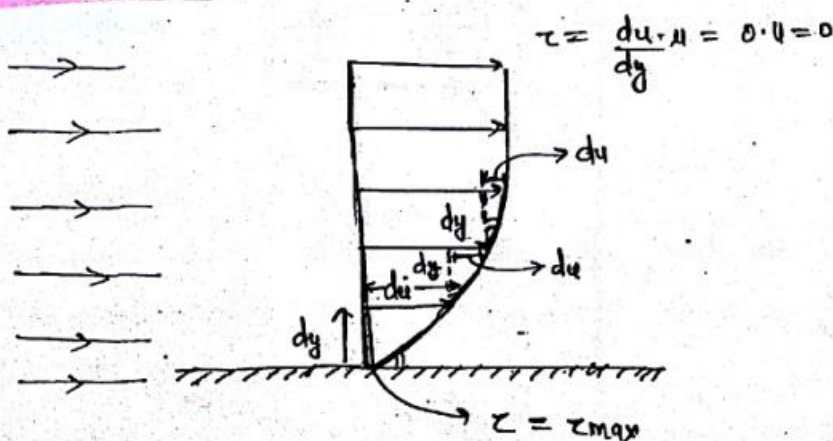


Slope of curve = μ

→ ex:

Air
H₂O
Hg
Kerosene
Petrol

Shear variation:



Since the value of du/dy is maximum at the wall ~~at shear~~
 stress is maximum at the wall and the value of du/dy $\frac{du}{dy}$ (8)
 decreases as we move away from the wall. So, shear stress also
 decreases.

② Kinematic viscosity (ν)

$$\nu = \frac{\mu}{\rho}$$

③ Units :

Dynamic viscosity

In S.I.,

$$\tau = \mu \cdot \frac{du}{dy}$$

$$\frac{N}{m^2} = \mu \cdot \frac{1}{s} \times \frac{1}{m}$$

$$\mu = \frac{N}{m^2} \cdot s$$

$$\mu = Pa \cdot s$$

$$\mu = \frac{N}{m^2} \cdot s = \frac{kg \cdot m}{s^2} \cdot \frac{1}{m^2} \cdot s$$

$$\mu \Rightarrow \frac{kg}{m \cdot s} \rightarrow [ML^{-1}T^{-1}]$$

In C.G.S.,

$$1 \text{ Poise} = \frac{1 g}{cm \cdot sec} = \frac{10^{-3} kg}{10^{-2} m \cdot sec}$$

$$1 \text{ Poise} = \frac{1}{10} \frac{kg}{m \cdot s}$$

Kinematic viscosity (ν)

In M.K.S

$$\nu = \frac{\mu}{\rho} = \frac{kg \cdot s}{m \cdot sec} \cdot \frac{1}{kg/m^3}$$

$$\nu = m^2/sec$$

In C.G.S

$$1 \text{ stoke} = 1 cm^2/sec$$

$$= \frac{10^{-4} m^2}{sec}$$

$$1 \text{ stoke} = \frac{1}{10000} \frac{m^2}{sec}$$

$$1 \text{ centi stoke} = 10^{-2} \text{ stoke} = \frac{m^2}{sec}$$

Effect of Temperature & Pressure on viscosity :

9

1. Effect of Temperature

→ Liquid

- Dynamic Viscosity (μ)

$T \uparrow \Rightarrow$ Intermolecular forces of attraction decreases $\Rightarrow \mu \downarrow$

$$T \uparrow \Rightarrow \mu \downarrow$$

- Kinematic Viscosity

$$\nu = \frac{\mu \downarrow (\text{High})}{\rho \downarrow (\text{slightly})}$$

$$T \uparrow = \nu \downarrow$$

→ Gases

- Dynamic Viscosity (μ)

$T \uparrow \Rightarrow$ Random Motion of molecules increase $\Rightarrow \mu \uparrow$

$$T \uparrow = \mu \uparrow$$

- Kinematic Viscosity

$$\nu = \frac{\mu \uparrow}{\rho \downarrow}$$

$$T \uparrow = \nu \uparrow$$

$$\left\{ \begin{array}{l} P = \rho R T \\ \rho \propto \frac{1}{T} \end{array} \right\} \quad (\because P, R \text{ are constant})$$

2. Effect of Pressure

→ Liquids

- Dynamic Viscosity (μ)

$$P \uparrow \Rightarrow \mu \text{ remains same}$$

($\because$ liquid is incompressible)

- Kinematic Viscosity (ν)

$$\nu = \frac{\mu}{\rho} \quad \left\{ \begin{array}{l} \mu \leftarrow \text{No change} \\ \rho \leftarrow \text{Negligible change} \end{array} \right.$$

$$P \uparrow = \nu \text{ remains same}$$

→ Gases

- Dynamic Viscosity (μ)

$$P \uparrow \Rightarrow \mu \text{ remains same}$$

provided increase in pressure is not very high

- Kinematic Viscosity (ν)

$$\nu = \frac{\mu}{\rho \uparrow} \quad \left\{ \begin{array}{l} \mu \leftarrow \text{No change} \\ \rho \leftarrow \uparrow \end{array} \right.$$

$$P \uparrow = \nu \downarrow$$

$$\left\{ \begin{array}{l} P = \rho R T \\ \rho \propto P \end{array} \right\} \quad (\text{with } T \text{ constant})$$

→ Linearisation of Newton's law of viscosity :

(10)

According to Newton's law of viscosity :

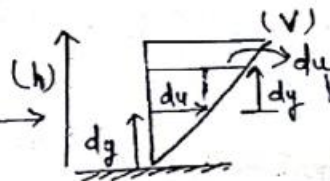
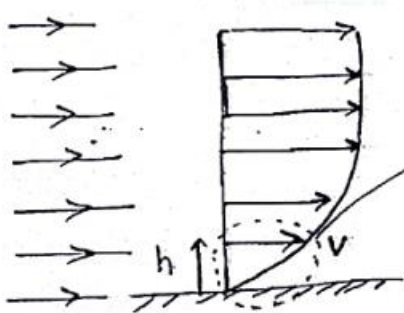
$$\tau = \mu \cdot \frac{du}{dy}$$

Linearised form :

$$\tau = \mu \cdot \frac{V}{h}$$

← Relative velocity
← very - 2 small gap

Determine Shear stress at wall :



{ Slope is same everywhere b/w 0 to h
Also, this is very-very small

$$\text{Slope of tangent at } y=0 = \left. \frac{du}{dy} \right|_{y=0}$$

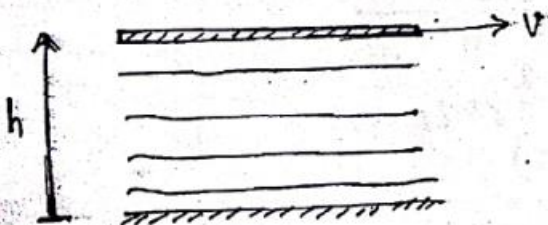
$$\tau_0 = \mu \cdot \left. \frac{du}{dy} \right|_{y=0}$$

← $u = f(y)$
← velocity profile equation

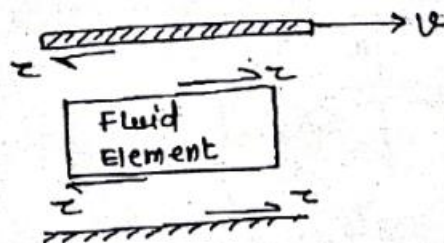
$$= V/h$$

Now, $\tau_0 = \mu \cdot \frac{V}{h}$

→ Conceptual Discussion : Determine the direction of shear force on top and bottom plates.



$h \rightarrow$ very-very small Gap



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Noted-: Above CIVIL MADEEASY 2019 BATCH CLASSROOM
BEST QUALITY Handwriting Notes Unique and Good Handwriting, No
Need other academy Handwriting Notes. Above Notes Enough for your
Preparation.....

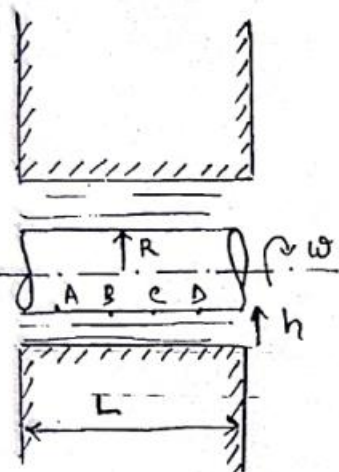
Noted-: Single Source Follow, Revise

Multiple Time Best key of Success

Qb-1

Determine Power consumed ??

(11)



$$\omega_A = \omega_B = \omega_C = \omega_D$$

$$v_A = v_B = v_C = v_D$$

Sol: $P = T \cdot \omega$

$$\Rightarrow F \cdot R \cdot \omega$$

$$\Rightarrow \tau \cdot A \cdot R \cdot \omega$$

$$\Rightarrow \mu \cdot \frac{du}{dy} \cdot A \cdot R \cdot \omega$$

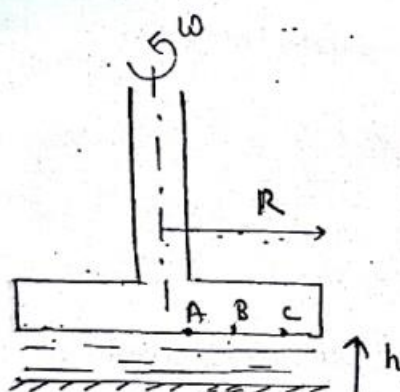
$$\Rightarrow \mu \cdot \frac{v}{h} \cdot A \cdot R \cdot \omega$$

$$\Rightarrow \mu \cdot \frac{v}{h} \cdot (2\pi R \times L) \cdot R \cdot \omega$$

$$\Rightarrow \boxed{P = \frac{2\pi \mu R^3 \omega^2 L}{h}}$$

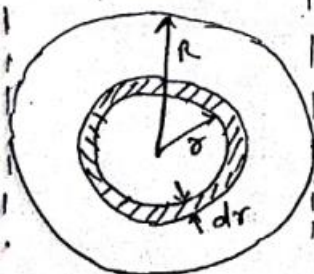
Qb-2

Determine Torque required ??



$$\omega_A = \omega_B = \omega_C$$

$$v_A \neq v_B \neq v_C$$



$$dT = d\tau \cdot r$$

$$\Rightarrow \tau \cdot dA \cdot r$$

$$\Rightarrow \mu \cdot \frac{du}{dy} \cdot (2\pi r dr) \cdot r$$

$$\Rightarrow \mu \cdot \frac{v}{h} \cdot (2\pi r dr) \cdot r \quad \{ \because v = r\omega \}$$

$$\Rightarrow \mu \cdot \left(\frac{\omega r}{h} \right) \cdot 2\pi r^2 \cdot dr$$

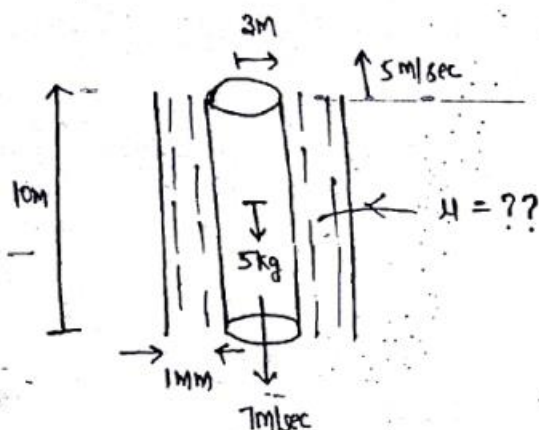
Integrating it:

$$\int dT = \int \frac{2\pi \mu \omega r^3 \cdot dr}{h}$$

$$T = \frac{2\pi \mu \omega}{h} \int_0^R r^3 \cdot dr$$

$$\boxed{T = \frac{\pi \mu \omega R^4}{2h}}$$

Pb-3



Wt = Resistive Force
 $5 \times 9.81 = \tau \cdot A$

$$\Rightarrow \mu = \frac{5 \times 10}{2\pi \times 3 \times 10 \times 12 \times 10^{-3}}$$

Sol: $\Rightarrow$ Relative velocity = $5 + 7 = 12 \text{ m/sec}$ (12)

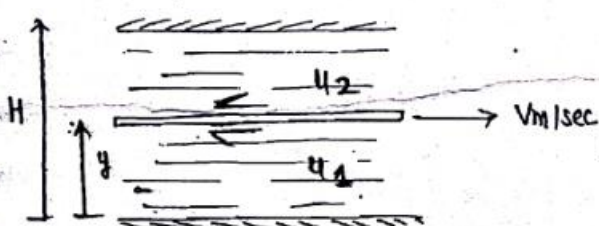
$$\Rightarrow \tau = \frac{\mu \cdot v}{h}$$

$$\tau = \frac{\mu \cdot 12}{1 \text{ mm} \times 10^{-3}}$$

$$5 \times 10 \times 2\pi \times 3 \times 10 = \mu \times 12 \times 10^3$$

$$\mu = \frac{5 \times 10 \times 2\pi \times 3 \times 10}{12 \times 10^3}$$

Pb-4



H = very - very small gap

Determine y so that:

- ① the drag force is equal on both the sides of plate
- ② The Total drag force is minimum

Sol: $\tau(F_1) = \frac{v}{y} \cdot \mu_1$

$$\tau(F_2) = \frac{v}{(H-y)} \cdot \mu_2$$

①

$$\Rightarrow \frac{\mu_1}{y} = \frac{\mu_2}{(H-y)}$$

$$(y\mu_2 + y\mu_1) = \mu_1 H$$

$$y = \frac{\mu_1 H}{\mu_1 + \mu_2}$$

$$\textcircled{2} \times \frac{\mu_1}{y} + \frac{\mu_2}{H-y} = 0$$

$$\Rightarrow \mu_1(H-y) + \mu_2(y) = 0$$

$$\Rightarrow (\mu_1 + \mu_2)y = \mu_1 H$$

$$\Rightarrow y = \frac{\mu_1 H}{\mu_1 + \mu_2}$$

$$\Rightarrow (\mu_2 - \mu_1)y = -\mu_1 H$$

$$y = \frac{\mu_1 H}{\mu_1 - \mu_2}$$

$$[\textcircled{b} \quad F_{DT} = F_{D1} + F_{D2}$$

(13)

$$\Rightarrow \tau_1 A + \tau_2 A$$

$$\Rightarrow \left(\mu_1 \cdot \frac{V}{y} + \mu_2 \cdot \frac{V}{H-y} \right) \cdot A$$

$$\Rightarrow V \cdot A \left(\frac{\mu_1}{y} + \frac{\mu_2}{H-y} \right)$$

For minimum, $\frac{dF_{DT}}{dy} = 0$

$$V \cdot A \left(-\frac{\mu_1}{y^2} - \frac{\mu_2 \cdot (-1)}{(H-y)^2} \right) = 0$$

$$\Rightarrow -\frac{\mu_1}{y^2} + \frac{\mu_2}{(H-y)^2} = 0$$

$$\Rightarrow \frac{\sqrt{\mu_2}}{H-y} = \frac{\sqrt{\mu_1}}{y}$$

$$\Rightarrow y \cdot \sqrt{\mu_2} = H \sqrt{\mu_1} - y \sqrt{\mu_1}$$

$$\Rightarrow y (\sqrt{\mu_1} + \sqrt{\mu_2}) = H \sqrt{\mu_1}$$

$$y = \frac{H \sqrt{\mu_1}}{\sqrt{\mu_1} + \sqrt{\mu_2}}$$

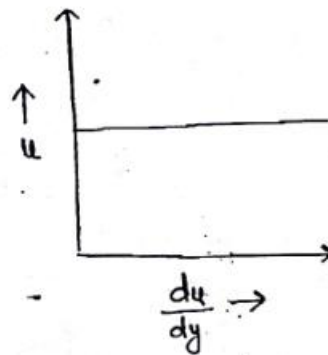
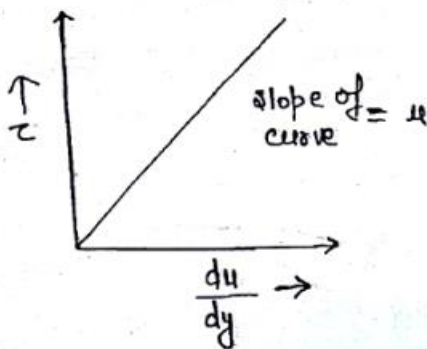
→ Rheology: Branch of science which deals with study of different types of fluid in flow. (14)

① Newtonian's Fluid

Obeys Newton's Law of viscosity

$$\tau = \mu \frac{du}{dy}$$

or $y = mx$



Ex: Air
Water
Kerosene
Hg, Petrol

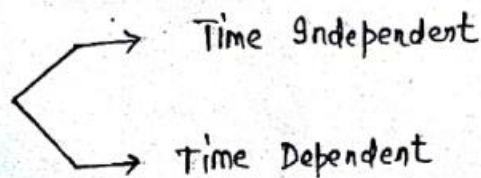
② Non-Newtonian fluid

Don't obey Newton's Law of viscosity

$$\tau = A \left(\frac{du}{dy} \right)^n$$

$A \Rightarrow$ Consistency Index

$n \Rightarrow$ Flow Behaviour Index



Time Independent

Pseudo Plastic

$(n < 1)$

$$\tau = A \left(\frac{du}{dy} \right)^n$$

Dilatant

$(n > 1)$

$$\tau = A \left(\frac{du}{dy} \right)^n$$

(58)

⑧

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Ex:

Ex



→

$$z = A \left(\frac{dy}{dx} \right)^n + B$$

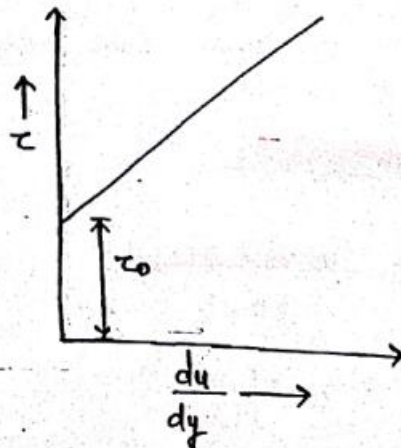
Here,

$$n = 1$$

$$B = \tau_0$$

$$\tau = \mu \frac{du}{dy} + \tau_0$$

$$y = mx + c$$



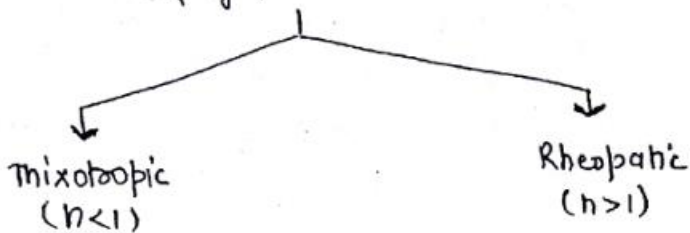
2. Time Dependent Fluids

linear stress $\Rightarrow$ const
Time $\rightarrow$ $\Rightarrow$ Time dependent fluid

Time Dependent

(16)

$$\tau = A \left(\frac{du}{dy} \right)^n + B, \quad [B > 0]$$

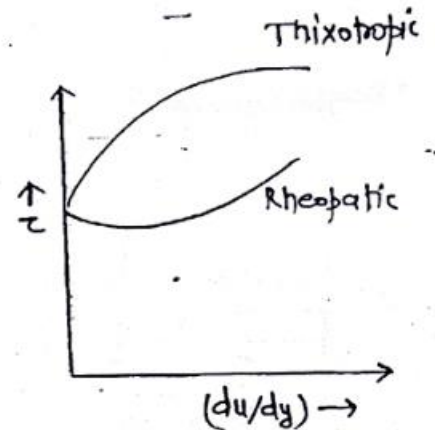


μ app. decreases with respect to time

ex: some paints

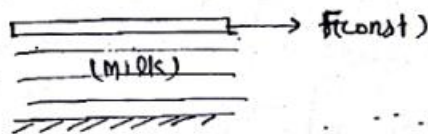
μ app. increases with respect to time

ex: Gypsum paste in H_2O



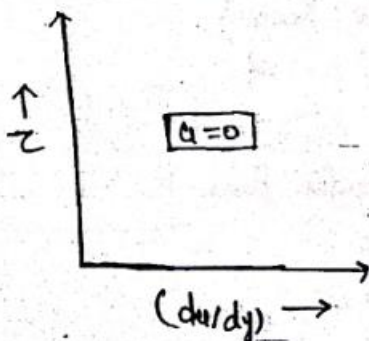
Note: If τ not const $\Rightarrow \mu$ changes but it doesn't mean implies it is time independent or dependent

To check that τ is kept constant & then μ is determined using mixer



3. Ideal Fluid:

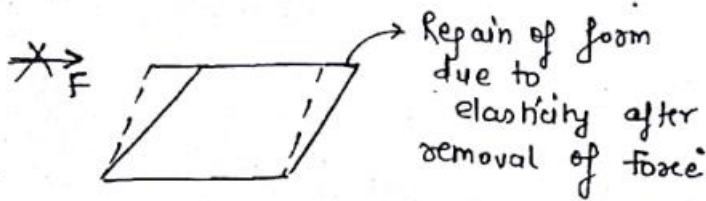
- Inviscid or Frictionless ($\mu = 0$)
- Incompressible
- No Surface Tension effect



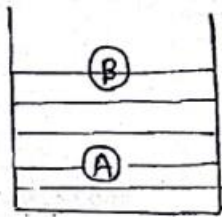
Visco-Elastic Fluid :

These types of fluids having the property of elasticity upto certain extent.

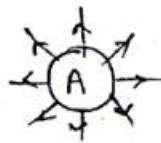
(17)



→ Surface Tension :

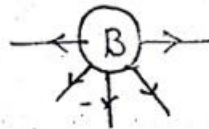


Molecule - A (Inside the liquid)



$$\Rightarrow F_A = 0$$

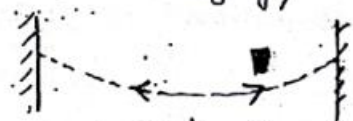
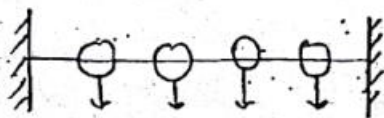
Molecule - B



$$\Rightarrow F_B \downarrow$$

Molecule A
(Inside the liquid)
Molecule B
(At the surface)

(Like a rope with clothes hanging)



(Tensile force generated at the surface due to downward cohesive force)

Mathematically,

$$\gamma = \frac{F}{L} \text{ N/m}$$

$F \Rightarrow$ Tensile force { i.e. the effect of cohesive force at the surface }