Chapter 1

Statics and Translations

one

We begin our study of physics with the subject of **mechanics**: the study of motion and its causes. This is a natural starting point, everyday experience offers abundant examples of mechanical principles, more than for any other area of physics.



1.1 Statics

1.1.1 Measurement

Physics is based on measurement. Much of physics deals with measurements of physical quantities such as length, time, frequency, velocity, area, volume, mass, density, charge, temperature and energy. Many of these quantities are interrelated. For example, velocity is length divided by time. Density is mass divided by volume, and volume is a length times a second length times a third length. Most of the physical quantities are related to length, time and mass.

There are two basic types of physical quantities-fundamental quantities and derived quantities. **Fundamental quantities** are those quantities that cannot be defined in terms of other quantities. **Derived quantities**, on the other hand, are defined in terms of fundamental quantities by means of a defining relationship that is normally an equation. The present metric system, called the **International System** of Units, or SI for short, recognizes seven fundamental physical quantities. There are length (measured in meters), time (measured in seconds), mass (measured in kilograms), temperature (measured in kelvins), electric current (measured in amperes), amount of substance (measured in moles), luminous intensity (measured in candelas). We will consider five of these fundamental quantities-length, time, mass, temperature and amount of substance in the early chapters of this book.

1.1.2 Force

Force is a central concept in all of physics. When we push or pull on a body, we are said to exert a **force** on it. Forces can also be exerted by inanimate objects: a stretched spring exerts forces on the bodies to which its end are attached; compressed air exerts a force on the walls of its container; a locomotive exerts a force on the train it is pulling or pushing. The force of gravitational attraction exerted on every physical body by the earth is called the **weight** of the body. Gravitational forces (and electrical and magnetic forces also) can act through empty space without contact. A force on an object resulting from direct contact with another object is called a **contact force**; viewed on an atomic scale, contact forces arise chiefly from electrical attraction

and repulsion of the electrons and nuclei making up the atoms of material.

Force is a vector quantity. To describe a force, we need to describe the **direction** in which it acts, as well as its **magnitude** in terms of a standard unit of force. The SI unit of force is the **Newton**, abbreviated N.

Suppose we slide a box along the floor by pulling it with a string or pushing it with a stick, as in Fig. 1-1. Our point of view is that the motion of the box is caused not by the **objects** that push or pull on it, but by the **forces** these objects exert. The forces in the two cases can be represented as in Fig. 1-2; the labels indicate the magnitude and direction of the force, and the length of the arrow, to some chosen scale, also shows the magnitude.

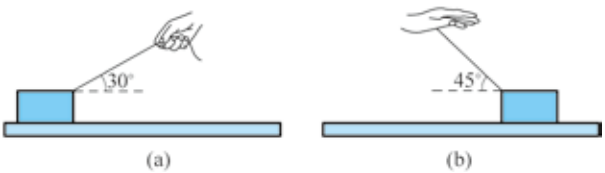


Fig. 1-1 Force may be exerted on the box by either pulling it (a), or pushing it (b).



Fig. 1-2 Force diagram for the forces acting on the box in Fig. 1-1.

Some vector quantities, of which force is one, are not **completely** specified by their magnitude and direction alone. The effect of a force depends also on the position of the point at which the force is applied. For example, when one pushes horizontally against a door, the effectiveness of a given force in setting the door in motion depends on the distance of the point of application from the line of the hinges, about which the door rotates. We return to this consideration in Chapter 2.

Now consider the following physical problem. Two forces, represented by the vectors $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ in Fig. 1-3, are applied simultaneously at the same point A of a body. Is it possible to produce the same effect by applying a single force at A, and if so, what should be its magnitude and direction? The question can be answered only by experiment; investigation shows that a single force, represented in magnitude, direction, and line of action by the vector sum $\boldsymbol{R}$ of the original forces, is in all respects equivalent to them. This single force is called the resultant of the original forces. Hence the mathematical process of **vector addition** of two force vectors corresponds to the physical operation of finding the resultant of two forces, simultaneously applied at a given point.

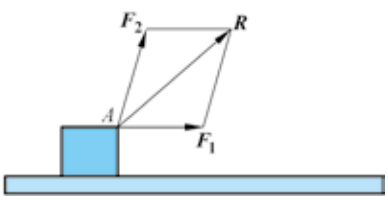


Fig. 1-3 A force represented by the vector $\boldsymbol{R}$, equal to the vector sum of $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$, produces the same effect as the forces $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ acting simultaneously.

The fact that forces can be combined by vector addition is of the utmost importance, as we shall see in the following sections. Furthermore, this fact also permits a force to be represented by means of

components. In Fig. 1-4 (a), force $\boldsymbol{F}$ is exerted on a body at point O. The rectangular components of $\boldsymbol{F}$ in the directions ox and oy are $\boldsymbol{F}_x = F_x\,\boldsymbol{i}$ and $\boldsymbol{F}_y = F_y\,\boldsymbol{j}$ ($\boldsymbol{i}$ and $\boldsymbol{j}$ are the unit vectors along ox and oy, respectively), and it is found that simultaneous application of the forces $\boldsymbol{F}_x$ and $\boldsymbol{F}_y$ as in Fig. 1-4 (b), is equivalent in all respects to the effect of the original force. **Any force can be replaced by its rectangular components, acting at the same point.**

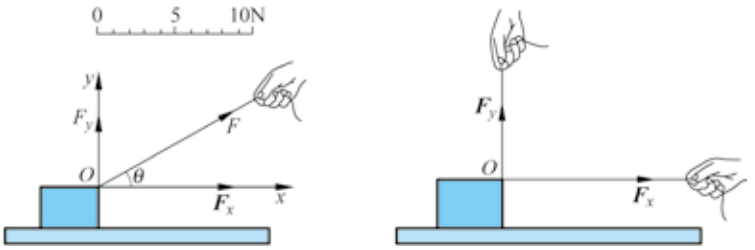


Fig. 1-4 The inclined force $\boldsymbol{F}$ may be replaced by its rectangular components $\boldsymbol{F}_x$ and $\boldsymbol{F}_y$.

As a numerical example, let

$$F = 10.0\text{N},\ \theta = 30°$$

Then

$$F_x = F\cos\theta = (10.0\text{N}) \times 0.866 = 8.66\text{N}$$
$$F_y = F\sin\theta = (10.0\text{N}) \times 0.500 = 5.00\text{N}$$

and the effect of the original 10.0N force is equivalent to the simultaneous application of a horizontal force, to the right, of 8.66N, and a lifting force of 5.00N.

The axes used to obtain rectangular components of a vector need not be vertical and horizontal. For example, Fig. 1-5 shows a block being pulled up an inclined plane by a force $\boldsymbol{F}$, represented by its components $\boldsymbol{F}_x$ and $\boldsymbol{F}_y$, parallel and perpendicular to the sloping surface of the plane .

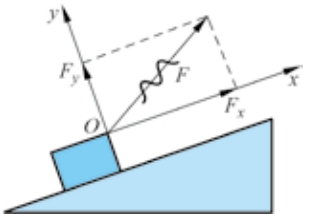


Fig. 1-5 $\boldsymbol{F}_x$ and $\boldsymbol{F}_y$ are the rectangular components of $\boldsymbol{F}$, parallel and perpendicular to the sloping surface of the inclined plane.

We shall often have occasion to find the vector sum of several forces acting on a body, which again may be called their **resultant**. The Greek letter $\sum$ is often used in a shorthand notation for this sum. Thus if the forces and labeled $\boldsymbol{F}_1$, $\boldsymbol{F}_2$, $\boldsymbol{F}_3$, and so on, and their resultant is $\boldsymbol{R}$, then the operation

$$\boldsymbol{R} = \boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_3 + \cdots$$

is often abbreviated

$$\boldsymbol{R} = \sum \boldsymbol{F} \qquad (1\text{-}1)$$

where $\sum \boldsymbol{F}$ is read as the "sum of the forces". This means, of course, the vector sum. In terms of components, we may write

$$R_x = \sum F_x, \qquad R_y = \sum F_y \qquad (1\text{-}2)$$

in which $\sum F_x$ is read as the sum of the x-com ponents of the forces and $\sum F_y$ is read as the sum of the y-components of the forces.

If you try to slide a book across a tabletop and you push too gently, the book won't budge. All solid surfaces, even the smoothest, have microscopic bumps and dips. Press two surfaces together and those irregularities catch on each other and resist any motion. If you push harder,

the growing force from your hand will finally break the holds the irregularities have on each other and the book will begin to move. Then there is less resistance—in other words, less friction—because the surfaces are skimming over each other and fewer of the "bumps and dips" can "catch and grab". Even if the surfaces are smooth at the atomic level, the individual atoms attract and cause friction. If two extremely flat surfaces of the same metal are pressed together, they "cold weld"and the two pieces of metal become one.

The force of friction between the table and book before the book moves is called **static friction**. As you push harder, the static friction force grows, matching your push as you press harder—up to some maximum force, at which point the surfaces slip and begin to move past each other. The smallest frictional force that remains between them once motion begins is called **kinetic friction**.

1.1.3 Newton's Laws of Motion

To get a book to slide across a desk, you have to push or pull it. When you stop pushing or pulling, the book stops moving. Aristotle (384 B. C.-322 B. C.) described this action by saying that rest is the natural state of matter and, if no force acts, there is no motion. But he missed an important point. Forces can **stop** motion as well as start it, and in this case a frictional force from the desk stops the book.

Galileo (1564—1642) realized that when there is little frictional resistance, a rolling ball, a sliding book, or any other moving body travels much farther before stopping. He concluded that if there was no friction at all, an object in motion should continue to travel without any change in speed or direction—but he failed to state his idea convincingly. Isaac Newton (1643—1727) ,who was born the next year Galileo died, made clear what Galileo had discovered in what is now called

Newton's first law of motion:

Any object remains at rest or in motion along a straight line with constant speed unless acted upon by a net force (or resultant force $\sum \boldsymbol{F}$ *).*

Only a net push or pull can change something's speed or direction. This property of matter, that its motion will not change unless a net force acts on it, is what we call **inertia**.

Newton's second law of motion:

The product of the mass (m) *of any object times its acceleration* **(*a*)** *is equal to the net force* **(**$\boldsymbol{F}_{net}$**)** acting on the object.

$$\boldsymbol{F}_{net} = \sum \boldsymbol{F} = m\boldsymbol{a} \tag{1-3}$$

Notice that this equation says the acceleration is always in the same direction as the net force, although they are very different quantities. When forces and acceleration are along a straight line we will usually drop the boldface notation.

Newton's second law works for baseballs and baby carriages, moons and planets, simple motions and complex motions. As long as you know all the forces acting on any object, you can add them to find the net force. The direction of the net force gives you the direction of the object's acceleration, and the strength of the force $\boldsymbol{F}_{net}$ divided by the object's mass m tells you the acceleration $\boldsymbol{a}$. In short, this law lets us predict changes in an object's motion.

If you toss a book across the room to a friend, your hand exerts a force on the book. You can **feel** the book resists the change in its motion as you push it, that is, the book exerts a force back against your hand. Newton realized that forces always occur in pairs. When two children collide on a playground, **both** get a push, not just one. Newton studied the actions of colliding

objects and came to the conclusion now known as **Newton's third law of motion:**

For every force, or action, there is an equal but opposite force, or reaction.

The important thing to realize about this law is that the "action" force is on one object and the "reaction" force is on the other. These two forces always **act on different objects**, so they can never balance each other, or cancel. Only when equal and opposite forces act on the **same** object can you add them together and then they do balance one another. So in a playground collision, the force on one child can't cancel the force on the other.

Suppose you push a bowling ball. As the ball accelerates, you feel a resistance from it, and this resistance is its reaction force on you. As you push the ball, it presses against your hand with the same amount of force. The bowling ball's reaction force would cause you to accelerate backward except for the friction between the ground and your feet. If you stand on a frozen pond on ice skates and push that ball, you will accelerate backward. You won't accelerate as fast as the ball does because $\boldsymbol{a} = \boldsymbol{F}_{net} / m$, and this equation shows us that if two objects get the same amount of force, the one with the most mass will have less acceleration.

1.1.4 Idealized Models

The phenomena of nature are seldom simple; often a phenomenon involves several interrelated principles, and the relationships can be extremely complex. Often one must make simplifying assumptions and approximations in order to facilitate analysis of such a phenomenon and to focus attention on its most significant aspects. This process of simplifying and idealizing is called making a **model**, and models play an essential role in applications of principles of physics.

Here is an example. A baseball is thrown into the air, and we want to calculate where it lands and with what velocity. The ball has a somewhat irregular surface; it is spinning as well as moving through the air. It is affected by the force of gravity, which decreases slightly as the ball ascends. Additional effects are caused by air resistance (friction) and by buoyancy in the air. There may be variable air currents that complicate these effects further.

Clearly, the analysis is hopelessly complicated if we try to include all these effects; we need to make a simplified model. If we approximate the ball as a uniform sphere and assume it moves through still air, we can calculate the air resistance and buoyant force, although these are still complex. The buoyant force is usually much smaller than the force of gravity, and if the ball moves slowly enough the friction force is also small, so we may wish to omit these completely. We may feel that the shape and spinning motion are irrelevant for an overall description of motion; if so, we can pretend that the ball is a **particle** (a point mass without any size, shape, or spin).

The concept of an idealized model plays an important role in physics. There are different idealized models in different areas of physics. **Particle** is just one of them. In later chapters of this book, we will meet other idealized models such as **rigid body** (a body with a perfectly definite and unchanging shape), **ideal gas**, **ideal fluid**, and so on.

1.1.5 Equilibrium

One effect of a force is to alter the state of motion of the body. This motion can be considered as made up of its motion as a whole, or its **translational** motion, together with any **rotational** motion the body may have. In the most general case, a single force acting on a body

produces a change in both its translational and rotational motion. However, when several forces act on a body simultaneously, their effects can compensate one another, with the result that there is no change in either the translational or rotational motion. When this is the case, the body is said to be in **equilibrium**.

Consider four examples: ① a book resting on a table, ② a hockey puck sliding across the ice with constant velocity, ③ the uniformly rotated blades of a ceiling fan, and ④ the wheel of a bicycle that is traveling along a straight path at constant speed. We say that such bodies are in equilibrium. Of the examples mentioned above, only the first one—the book resting on the table—is in **static equilibrium**. The reason is that the book is not moving in any way—either in translation or in rotation—in the reference frame from which we observe them. Such bodies are in **static equilibrium**. We can see that there are two conditions to be satisfied for **static equilibrium**:

1. The vector sum of all the external forces that act on the body must be zero.

$$\sum \boldsymbol{F} = 0 \tag{1-4}$$

2. The vector sum of all the external torques that act on the body must also be zero.

$$\sum \boldsymbol{\tau} = 0 \tag{1-5}$$

The analysis of static equilibrium is very important in engineering practice. The design engineer must isolate and identify all the external forces and torques that act on the structure. By good design and wise choice of materials, the engineer must make sure that the structure will tolerate the loads that bear on it. Such analyses are necessary to make sure that bridges do not collapse under their traffic and wind loads, that the landing gear of aircraft will survive the shocks of rough landings, and so on.

Example 1. In Fig. 1-6 (a), a block of weight w hangs from a cord, which is knotted at point O to two other cords, one fastened to the ceiling, the other to the wall. We wish to find the tensions in these three cords, assuming the weights of the cords to be negligible.

Solution: To use the conditions of equilibrium to determine an unknown force, we must consider some body that is in equilibrium and on which the desired force acts. The tension T_1 in the vertical cord supporting the block is equal in magnitude to the weight of the block. The other cords do not exert forces on the block (because they are not attached directly to it) but they do act on the knot at O. Hence we consider the knot as a particle in equilibrium, considering the weight of the knot itself as negligible.

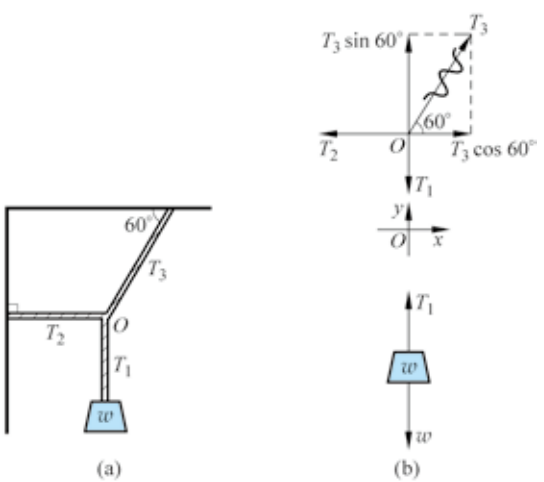


Fig. 1-6 (a)A block of weight w is suspended from a cord knotted at O to two other cord;(b) Free-body diagrams for the block and for the knot, showing the components of force acting on each.

Free-body diagrams for the block and the knot are shown in Fig. 1-6 (b), where T_1, T_2, and T_3 are the **magnitudes** of the forces shown; the directions of these forces are indicated by the vectors on the diagram. An x-y coordinate axis system is also shown, and the force of magnitude T_3 has been resolved into its x-and y-components.

Considering the block first, we note that there are no x-components of force. The

y-components of force exerted by the cord is in the positive y-components and is just T_1. The y-component of the weight is, however, in the negative y-direction and is $-w$. The equilibrium condition for the block, that the algebraic sum of y-components of force must be zero, is

$$T_1 + (-w) = 0$$

Hence,

$$T_1 = w \qquad (1)$$

The tension in the vertical cord equals the weight of the block, as we have already noted.

We now consider the knot at O. There are both x-and y-components of force acting on it, so there are two separate equilibrium conditions. The algebraic sum of the x-components must be zero, and the algebraic sum of y-components must separately be zero. (Note that x-and y-components are never added together in a single equation.) We find:

$$\sum F_x = 0: \qquad T_3 \cos 60° - T_2 = 0 \qquad (2)$$

$$\sum F_y = 0: \qquad T_3 \sin 60° - T_1 = 0 \qquad (3)$$

Because $T_1 = w$, the Eq. (3) can be rewritten as:

$$T_3 = \frac{T_1}{\sin 60°} = \frac{w}{\sin 60°} = 1.155w$$

This result can now be used in the Eq. (2):

$$T_2 = T_3 \cos 60° = (1.155w) \cos 60° = 0.577w$$

Thus all three tensions can be expressed as multiples of the weight of the block, which is assumed to be known. To summarize,

$$T_1 = w$$
$$T_2 = 0.577w$$
$$T_3 = 1.155w$$

Example 2. In order to handle a very hot object of mass $m = 5$kg, a worker hooks it with a light but stiff pole of length $L = 3$m, which he then carries horizontally with two hands (Fig. 1-7 (a)). Hand A is at one end of the pole, hand B is $l = 0.75$m farther down the pole, and the mass is at the other end. What forces must the hands of the worker exert on the pole?

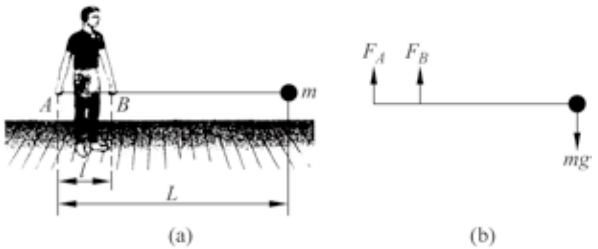


Fig. 1-7 (a)Example 2; (b) Extended force diagram.

Solution: We want to use the static equilibrium conditions to determine the forces the hands exert on the pole. To do so, we first note that the extended object on which the forces act is the pole along with the hot object at its end, and the external forces acting on it are gravity mg and contact forces (F_A and F_B) from each hand. In Fig. 1-7(b), a force diagram, we have used our intuition that the hands will exert upward forces, but as is usual with force diagrams, it is not necessary that the forces drawn correspond precisely in direction and magnitude to the forces to be determined, the static equilibrium conditions will themselves determine these forces.

Let us choose point A to be the origin. Then our static equilibrium conditions are

$$F_A + F_B - mg = 0$$

and

$$lF_B - Lmg = 0$$

Note that F_B and mg must exert torques in opposite directions, and that with the choice of A as origin, there is no torque due to F_A.

These equations can be solved for the unknown forces:

$$F_B = mg\left(\frac{L}{l}\right)$$

$$F_A = mg - mg\frac{L}{l} = mg\left(\frac{l-L}{l}\right)$$

Because $l < L$, F_A is negative. Hand A must exert a **downward** force. Inserting numbers, we have:

$$F_A = (5\text{kg})\ (9.8\text{m}\cdot\text{s}^{-2})\ \frac{0.75\text{m} - 3\text{m}}{0.75\text{m}} = -150\text{N}$$

$$F_B = (5\text{kg})\ (9.8\text{m}\cdot\text{s}^{-2})\ \frac{3\text{m}}{0.75\text{m}} = 200\text{N}$$

Example 3. A ladder whose length L is 12m and whose mass m is 45kg rests against a wall. Its upper end is a distance h of 9.3m above the ground, as in Fig. 1-8 (a). The center of gravity of the ladder is one-third of the way up the ladder. A firefighter whose mass M is 72kg climbs halfway up the ladder. Assume that the wall, but not the ground, is frictionless. What forces are exerted on the ladder by the wall and by the ground?

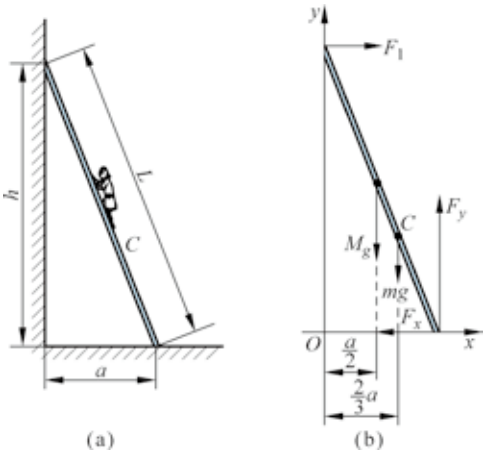


Fig. 1-8 (a)A firefighter climbs halfway up a ladder; (b) A free-body diagram.

Solution: Fig. 1-8 (b) shows a free-body diagram. The wall exerts a horizontal force F_1 on the ladder, it can exert no vertical force because the wall-ladder contact is assumed to be frictionless. The ground exerts a force $\boldsymbol{F}$ on the ladder with a horizontal component F_x and a vertical component F_y. We choose coordinate axes as shown, with the origin O at the point where the wall meets the ground. The distance a from the wall to the foot of the ladder is readily found from

$$a = \sqrt{L^2 - h^2} = \sqrt{(12\text{m})^2 - (9.3\text{m})^2} = 7.6\text{m}$$

From Eq. (1-4), the balance of forces equation, we have

$$F_1 - F_x = 0 \qquad (1')$$

and

$$F_y - Mg - mg = 0 \qquad (2')$$

Eq. (2') yields

$$F_y = g\,(M + m) = (9.8\text{m}\cdot\text{s}^{-2})\ (72\text{kg} + 45\text{kg}) = 1147 \approx 1150\text{N}$$

From Eq. (1-5), the balance of torques equation, we have, taking an axis through O, the point of contact of the wall with the ground.

$$F_1 h + Mg\frac{a}{2} + mg\frac{2}{3}a - F_y a = 0$$

$$F_1 = \frac{\left(F_y - \frac{1}{2}Mg - \frac{2}{3}Mg\right)a}{h}$$

$$= \frac{\left[1147\text{N} - \left(\frac{1}{2}\right)(72\text{kg})(9.8\text{m}\cdot\text{s}^{-1}) - \left(\frac{2}{3}\right)(45\text{kg})(9.8\text{m}\cdot\text{s}^{-2})\right](7.6\text{m})}{9.3\text{m}}$$

$$= 409\text{N} \approx 410\text{N}$$

From Eq. (1'), we have at once

$$F_x = F_1 = 409\text{N}$$

1.2 Straight Line Motion

Mechanics deals with the relations of force, matter, and motion. Motion may be defined as a continuous change of position. In actual motion of an extended body, different points of the body move along different paths, but we shall consider first a description in terms of a single point, the **particle.**

The simplest motion to describe is motion of a point (particle) along a straight line, which we shall take to coincide with a coordinate axis. In later chapters we shall see that more general motions in space can always be represented by means of their projections onto the three coordinate axes.

1.2.1 Constant Velocity

If a car is moving with constant velocity, then the distance traveled is directly proportional to the time. For constant velocity v,

$$x = vt$$

Where x is the distance covered in time t. In Fig. 1-9 x is plotted against t for an object moving with uniform (constant) velocity. If we divided both sides of the above equation by t we obtain

$$v = \frac{x}{t} \qquad (v \text{ is constant}) \qquad (1\text{-}6)$$

1.2.2 Instantaneous Velocity

We shall now proceed to the more general case where the velocity is permitted to change with time. The discussion will be restricted to the situation where the magnitude, but not the direction, of the velocity may change. This type of motion in a fixed direction or along a straight line, is called one-dimensional motion. We will, however, permit a reverse in direction. Then v will be negative which means x will be decreasing in value. Strictly speaking, we should use the word "speed" when we are only speaking of the magnitude of the velocity and not the direction. In our presentation of one-dimensional motion, we are keeping track of the direction by permitting the use of negative v.

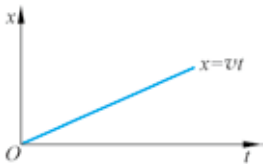


Fig. 1-9 Plot of distance x versus time t of an object moving with constant velocity.

Suppose a car is accelerating (speeding up or slowing down). The Eq. (1-6) will give the wrong answer for the "speedometer reading" unless a very small value of x is used. We will use

the symbol Δx to stand for a very small distance, and Δt for the time taken to travel the distance Δx. The instantaneous velocity is defined to be

$$v = \frac{\Delta x}{\Delta t}$$

More correctly

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{\mathrm{d}x}{\mathrm{d}t} \tag{1-7}$$

The above equation signifies that v is the limit of the ratio $\frac{\Delta x}{\Delta t}$ as Δx approaches zero. This is the mathematically rigorous definition of instantaneous velocity.

If the time interval Δt is not very small, Eq. (1-7) gives us the average velocity $\overline{v}$

$$\overline{v} = \frac{\Delta x}{\Delta t}$$

and

$$\Delta x = \overline{v}\,\Delta t \tag{1-8}$$

Example 1. Suppose the motion of a particle is described by the equation $x = 20\text{cm} + (4\text{cm}\cdot\text{s}^{-2})\,t^2$.

(a) Find the displacement (the same meaning as distance) of the particle in the time interval between $t_1 = 2\text{s}$ and $t_2 = 5\text{s}$.

(b) Find the average velocity in this time interval.

(c) Find the instantaneous velocity at time $t_1 = 2\text{s}$.

Solution:

(a) At time $t_1 = 2\text{s}$, the position is

$$x_1 = 20\text{cm} + (4\text{cm}\cdot\text{s}^{-2})\,(2\text{s})^2 = 36\text{cm}$$

at time $t_2 = 5\text{s}$,

$$x_2 = 20\text{cm} + (4\text{cm}\cdot\text{s}^{-2})\,(5\text{s})^2 = 120\text{cm}$$

The displacement is therefore

$$\Delta x = x_2 - x_1 = 120\text{cm} - 36\text{cm} = 84\text{cm}$$

(b) The average velocity in this time interval is

$$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{84\text{cm}}{3\text{s}} = 28\text{cm}\cdot\text{s}^{-1}$$

(c) The instantaneous velocity may be obtained by calculating the derivative of x with respect to t. Starting again with

$$x = 20\text{cm} + (4\text{cm}\cdot\text{s}^{-2})\,t^2$$

and using Eq. (1-7), we obtain

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left[\,20\text{cm} + (4\text{cm}\cdot\text{s}^{-2})t^2\,\right] = 2\,(4\text{cm}\cdot\text{s}^{-2})t$$

The instantaneous velocity at time $t_1 = 2\text{s}$ is

$$v = (2)\,(4\text{cm}\cdot\text{s}^{-2})\,(2\text{s}) = 16\text{cm}\cdot\text{s}^{-1}$$

1.2.3 Average and Instantaneous Acceleration

When the velocity of a moving body changes with time, the body is said to have an **acceleration**. Just as velocity is a quantitative description of rate of change of position with time, so acceleration is a quantitative description of rate of change of velocity with time.

Considering again the motion of a particle along the x-axis, as shown is Fig. 1-10 (a), we suppose that at time t_1 the particle is at point P and has velocity v_1, and that at a later time t_2 it si at point Q and has the velocity v_2.

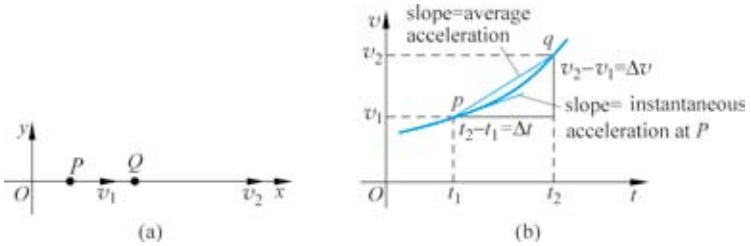


Fig. 1-10 (a) Particle moving on the x-axis; (b) Velocity-time graph of the motion. The average acceleration between t_1 and t_2 equals the slope of the chord pq. The instantaneous acceleration at P equals the slope of the tangent at P.

The **average acceleration** $\overline{a}$ of the particle as it moves from P to Q is defined as the ratio of the change in velocity to the elapsed time:

$$\overline{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

where t_1 and t_2 are the times corresponding to the velocities v_1 and v_2.

Fig. 1-10 (b) is a graph of instantaneous velocity v plotted as a function of time, points p and q corresponding to positions P and Q in Fig. 1-10 (a). The average acceleration is represented by the slope of the chord pq, computed using the appropriate scales and units of the graph.

The **instantaneous acceleration** of a body, that is, its acceleration at some one instant of time or at some one point of its path, is defined in the same way as instantaneous velocity. Let the second point Q in Fig. 1-10 (a) be taken closer and closer to the first point P, and let the average acceleration be computed over shorter and shorter intervals of time. The instantaneous acceleration at the first point is defined as the limiting value of the average acceleration when the second point is taken closer and closer to the first:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \qquad (1\text{-}9)$$

As point Q approaches point P in Fig. 1-10 (a), point q approaches point p in Fig. 1-10 (b), and the slope of the chord pq approaches the slope of the tangent to the velocity—time graph at point p. **The instantaneous acceleration at any point of the graph therefore equals the slope of the tangent to the graph at that point.**

The acceleration $a = dv/dt$ can be expressed in various ways. Since $v = dx/dt$, it follows that

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

The acceleration is therefore the second derivative of the coordinate with respect to time.

Instantaneous acceleration plays an important part in the laws of mechanics; average acceleration is less frequently used. From now on when the term "acceleration" is used we shall understand it to mean instantaneous acceleration.

Example 1. Suppose the velocity of the particle in Fig. 1-10 is given by the equation

$$v = 10\text{cm}\cdot\text{s}^{-1} + (2\text{cm}\cdot\text{s}^{-3})\, t^2$$

(a) Find the change in velocity of the particle in the time interval between $t_1 = 2$s and $t_2 = 5$s.

(b) Find the average acceleration in this time interval.

(c) Find the instantaneous acceleration at time $t_1 = 2$s.

Solution:

(a) At time $t_1 = 2$s

$$v_1 = 10\text{cm}\cdot\text{s}^{-1} + (2\text{cm}\cdot\text{s}^{-3})\ (2\text{s})^2 = 18\text{cm}\cdot\text{s}^{-1}$$

At time $t_2 = 5\text{s}$

$$v_2 = 10\text{cm}\cdot\text{s}^{-1} + (2\text{cm}\cdot\text{s}^{-3})\ (5\text{s})^2 = 60\text{cm}\cdot\text{s}^{-1}$$

The change in velocity is therefore

$$\Delta v = v_2 - v_1 = 60\text{cm}\cdot\text{s}^{-1} - 18\text{cm}\cdot\text{s}^{-1} = 42\text{cm}\cdot\text{s}^{-1}$$

(b) The average acceleration is

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{42\text{cm}\cdot\text{s}^{-1}}{3\text{s}} = 14\text{cm}\cdot\text{s}^{-2}$$

This corresponds to the slope of the chord *pq* in Fig. 1-10 (b).

(c) The instantaneous acceleration can be obtained directly by taking the time derivative of the instantaneous velocity. Starting with

$$v = 10\text{cm}\cdot\text{s}^{-1} + (2\text{cm}\cdot\text{s}^{-3})\ t^2$$

and using Eq. (1-9), we find

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left[\ 10\text{cm}\cdot\text{s}^{-1} + (2\text{cm}\cdot\text{s}^{-3})t^2\right] = 2\ (2\text{cm}\cdot\text{s}^{-3})t$$

The instantaneous acceleration when $t_1 = 2\text{s}$ is

$$a = (2)\ (2\text{cm}\cdot\text{s}^{-3})\ (2\text{s}) = 8\text{cm}\cdot\text{s}^{-2}$$

This corresponds to the slope of the tangent at point *p* in Fig. 1-10 (b).

1.2.4 Motion with Constant Acceleration

The simplest kind of accelerated motion is straight-line motion in which the acceleration is constant, that is, in which the velocity changes at the same rate throughout the motion. The velocity-time graph is then a straight line, as in Fig. 1-11, the velocity increasing by equal amounts in equal intervals of time. The slope of a chord between any two points on the line is the same as the slope of a tangent at any point, and the average and instantaneous accelerations are equal. Hence the average acceleration $\bar{a}$ can be replaced by the constant acceleration a, and we have

$$a = \frac{v_2 - v_1}{t_2 - t_1}$$

Now let $t_1 = 0$ and let t_2 be any arbitrary later time t. Let v_0 represent the velocity when $t = 0$, (called the **initial** velocity), and let v be the velocity at the later time t. Then the preceding equation becomes

$$a = \frac{v - v_0}{t - 0}$$

or

$$v = v_0 + at \qquad (1\text{-}10)$$

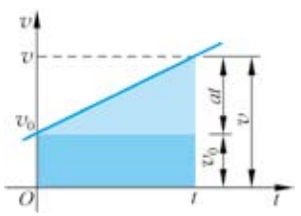


Fig. 1-11 Velocity-time graph for rectilinear motion with constant acceleration.

This equation can be interpreted as follows: The acceleration a is the constant rate of change of velocity, or the change of velocity per unit time. The term at is the product of the change in velocity per unit time, a , and the duration of the time interval, t ; therefore it equals the total change in velocity. The velocity v at the time t then equals the velocity v_0 at the time $t = 0$, plus the change in velocity at. Graphically, the ordinate v at time t , in Fig. 1-11 , can be considered as the sum of two segments: one of length v_0 equals to the initial velocity , the other of length at equals to the change in velocity in time t .

From Eq. (1-10) , the average velocity $\bar{v}$ during the time interval t is given by

$$\bar{v} = \frac{1}{2}(v_0 + v) = \frac{1}{2}(v_0 + v + at) = v_0 + \frac{1}{2}at$$

So , according to Eq. (1-8) , the distance Δx traveled during the time interval t is

$$\Delta x = \bar{v}\, t = v_0 t + \frac{1}{2}at^2 \qquad (1\text{-}11)$$

The distanc.e Δx equals the shaded area under the velocity-time graph (Fig. 1-11) .

Combing Eqs. (1-10) and (1-11) together, and canceling a factor of t , we get

$$v^2 - v_0^2 = 2a\Delta x \qquad (1\text{-}12)$$

Eqs. (1-10) , (1-11) and (1-12) are the **equations of motion with constant acceleration.**

Example 1. A body moves along the x-axis with constant acceleration $a = 4\text{m}\cdot\text{s}^{-2}$. At time $t = 0$ it is at $x_0 = 5\text{m}$ and has velocity $v_0 = 3\text{m}\cdot\text{s}^{-1}$

(a) Find the position and velocity at time $t = 2\text{s}$.

(b) Where is the body when its velocity is $5\text{m}\cdot\text{s}^{-1}$?

Solution:

(a) From Eq. (1-11) ,

$$\Delta x = x - x_0 = v_0 t + \frac{1}{2}at^2 = (3\text{m}\cdot\text{s}^{-1})(2\text{s}) + \frac{1}{2}(4\text{m}\cdot\text{s}^{-2})(2\text{s})^2 = 14\text{m}$$

So the position x at time $t = 2\text{s}$ is

$$x = x_0 + \Delta x = 5\text{m} + 14\text{m} = 19\text{m}$$

From Eq. (1-10) ,

$$v = v_0 + at = 3\text{m}\cdot\text{s}^{-1} + (4\text{m}\cdot\text{s}^{-2})(2\text{s}) = 11\text{m}\cdot\text{s}^{-1}$$

(b) From Eq. (1-12)

$$v^2 - v_0^2 = 2a\Delta x = 2a(x - x_0)$$

so

$$x = \frac{v^2 - v_0^2}{2a} + x_0 = \frac{(5\text{m}\cdot\text{s}^{-1})^2 - (3\text{m}\cdot\text{s}^{-1})^2}{2(4\text{m}\cdot\text{s}^{-2})} + 5\text{m} = 7\text{m}$$

Alternatively, we may use Eq. (1-10) to find first the **time** when $v = 5\text{m}\cdot\text{s}^{-1}$

$$5\text{m}\cdot\text{s}^{-1} = 3\text{m}\cdot\text{s}^{-1} + (4\text{m}\cdot\text{s}^{-1})\, t$$

$$t = \frac{1}{2}s$$

Then from Eq. (1-11) ,

$$x = x_0 + \Delta x = 5\text{m} + (3\text{m}\cdot\text{s}^{-1})\left(\frac{1}{2}s\right) + \frac{1}{2}(4\text{m}\cdot\text{s}^{-2})\left(\frac{1}{2}s\right)^2 = 7\text{m}$$

Example 2 . An elevator and its load have a total mass of 800kg. Find the tension T in the supporting cable when the elevator, originally moving downward at $10\text{m}\cdot\text{s}^{-1}$, is brought to rest with constant acceleration in a distance of 25m. (See Fig. 1-12)

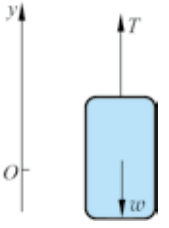


Fig. 1-12 The resultant vertical force is T-w

Solution: The weight of elevator is

$$w = mg = (800\text{kg})(9.8\text{m}\cdot\text{s}^{-2}) = 7840\text{N}$$

From the equations of motion with constant acceleration,

$$v^2 + v_0^2 = 2ay$$

$$a = \frac{v^2 - v_0^2}{2y}$$

The initial velocity v_0 is $-10\text{m}\cdot\text{s}^{-1}$; the final velocity v is zero . If we take the origin at the point where the acceleration begins , then $y = -25\text{m}$. Hence,

$$a = \frac{0 - (-10\text{m}\cdot\text{s}^{-1})^2}{2(-25\text{m})} = 2\text{m}\cdot\text{s}^{-2}$$

The acceleration is therefore positive (upward). From the free-body diagram (Fig. 1-12) the resultant force is

$$\sum F = T - w = T - 7840\text{N}$$

Since $\sum F = ma$,

$$T - 7840\text{N} = (800\text{kg})\ (2\text{m}\cdot\text{s}^{-2}) = 1600\text{N}$$

$$T = 9440\text{N}$$

The tension must be **greater** than the weight by 1600N to cause the upward acceleration while the elevator is stopping.

1.2.5 Freely Falling Bodies

The most common example of motion with (nearly) constant acceleration is that of a body falling toward the earth—the freely falling body. The acceleration of a freely falling body is called **acceleration due to gravity** , or the **acceleration of gravity** , and is denoted by the letter ***g*** . At or near the earth's surface its magnitude is approximately $9.8\text{m}\cdot\text{s}^{-2}$.

If we use coordinate y , rather than x , to denote the position of a freely falling body , then the equations of motion of a freely falling body will be

$$v = v_0 + gt \qquad (1\text{-}13)$$

$$y = y_0 + v_0 t + \frac{1}{2} gt^2 \qquad (1\text{-}14)$$

$$v^2 - v_0^2 = 2g\ (y - y_0) \qquad (1\text{-}15)$$

Example 1 . A pitcher throws a baseball straight up , with an initial speed of $25\text{m}\cdot\text{s}^{-1}$.

(a) How long does it take to reach its highest point ?

(b) How high does the ball rise above its release point ?

(c) How long will it take for the ball to reach a point 25m above its release point ?

Solution:

(a) By **highest point** we mean the point at which its velocity v is zero. Eq. (1-13) is the one we want . (The value of g is negative value because g and v_0 are opposite directions.)

$$t = \frac{v - v_0}{\text{g}} = \frac{0 - 25\text{m}\cdot\text{s}^{-1}}{-\ 9.8\text{m}\cdot\text{s}^{-2}} = 2.6\text{s}$$

(b) We take the release point of the ball to be the origin of the y-axis . Putting $y_0 = 0$ in Eq. (1-15) and solving for y , we obtain

$$y = \frac{v^2 - v_0^2}{2\text{g}} = \frac{0 - (25\text{m}\cdot\text{s}^{-1})^2}{-\ 2\ (9.8\text{m}\cdot\text{s}^{-2})} = 32\text{m}$$

If we wanted to take advantage of the fact that we also know the time of flight , having found it in (a) above , we can also calculate the height of rise from Eq. (1-14) . Check it out.

(c) We use Eq. (1-14) . With $y_0 = 0$, substitution yields

$$y = v_0 t + \frac{1}{2} gt^2$$

$$25\text{m} = (25\text{m}\cdot\text{s}^{-1})\ t - \left(\frac{1}{2}\right)(98\text{m}\cdot\text{s}^{-2})\ t^2$$

We can rewrite this as

$$4.9t^2 - 25t + 25 = 0$$

Solving this quadratic equation for t yields

$$t = 1.4\text{s} \quad \text{and} \quad t = 3.7\text{s}$$

There are two such times . This is not really surprising because the ball passes twice through $y = 25\text{m}$, once on the way up and once on the way down .

1.3 Motion in a Plane

1.3.1 Projectile Motion

Consider the motion of a golf ball hit off a tee . The golf ball is an example of a **projectile** that moves under the effect of gravity . In the absence of air resistance, what is the trajectory of such a projectile as it moves under the constant acceleration of gravity ? To answer this question , we can calculate the trajectory by breaking up the motion into horizontal and vertical components. The coordinate system is best taken with the origin at the golf tee , with the y-direction vertically up and the x-direction horizontal (Fig. 1-13) . The initial position of the ball is $x_0 = y_0 = 0$.

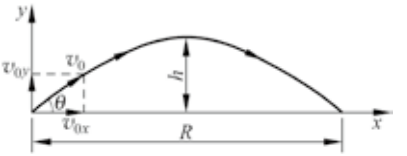


Fig. 1-13 A golf ball hit off a tee leaves the tee with an initial velocity v_0 and an elevation angle θ.

We write the initial velocity at $t = 0$ as v_0 . The angle that the golf ball's trajectory initially makes with the horizontal axis is the elevation angle θ . The components of v_0 are thus

$$v_{0x} = v_0 \cos\theta \qquad \text{and} \qquad v_{0y} = v_0 \sin\theta$$

The components of the acceleration are the constants

$$a_x = 0 \qquad \text{and} \qquad a_y = -g$$

So

$$\begin{cases} x = (v_0 \cos\theta)t \\ y = (v_0 \sin\theta)t - \dfrac{1}{2} g t^2 \end{cases} \tag{1-16}$$

And

$$\begin{cases} v_x = v_0 \cos\theta \\ v_y = v_0 \sin\theta - gt \end{cases} \tag{1-17}$$

Example 1 . Let us consider the case of a cannon fired at an angle of $\theta = 45°$ from the horizontal as shown Fig. 1-13 . Assume the projectile velocity is $v_0 = 1000\text{m} \cdot \text{s}^{-1}$ as it leaves the cannon and ignore air resistance. Determine:

(a) how long the projectile is in flight ;

(b) how high it goes ; and

(c) how far it goes (what the range is of this cannon) .

Solution:

(a) For $\theta = 45°$, $v_{0x} = v_{0y} = v_0 \sin 45° = \dfrac{1000\text{m} \cdot \text{s}^{-1}}{1.414} = 707\text{m} \cdot \text{s}^{-1}$

According to Eq. (1-17)

$$v_y = v_0 \sin\theta - gt = v_{0y} - gt$$

Nothing that when the projectile reaches its maximum height h , $v_y = 0$, therefore

$$t_1 = \frac{v_{0y} - 0}{g} = \frac{707\text{m} \cdot \text{s}^{-1} - 0}{9.8\text{m} \cdot \text{s}^{-2}} = 72\text{s}$$

The total time of travel will be twice this or 144s. This is the answer.

(b) The time required to reach maximum height h is $t_1 = 72\text{s}$. According to Eq. (1-16) ,

$$y = (v_0 \sin\theta)\, t - \frac{1}{2} g t^2 = v_{0y} t - \frac{1}{2} g t^2$$

we have

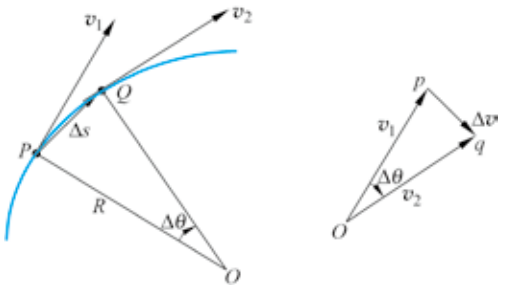
$$h = v_{0y}\, t_1 - \frac{1}{2} g t_1^{\,2} = (707\text{m}\cdot\text{s}^{-1})\,(72\text{s}) - \left(\frac{1}{2}\right)(9.8\text{m}\cdot\text{s}^{-2})\,(72\text{s})^2 = 2.55\times 10^4\text{m}$$

(c) From $x = (v_0 \cos\theta)\, t = v_{0x} t$,we can get the range R of the cannon if we use the total time of travel . So the range

$$R = v_{0x} t = 2 v_{0x}\, t_1 = (2)(707\text{m}\cdot\text{s}^{-1})\,(72\text{s}) = 1.02\times 10^5\text{m}$$

1.3.2 Uniform Circular Motion

In projectile motion the acceleration ***g*** is perpendicular to the velocity at the time the projectile reaches its maximum height. The projectile is moving in a curved path. We now study the special case where the acceleration is **always** perpendicular to the velocity. If a particle is moving in a circular path with constant speed, such motion is called **uniform circular motion**. In this case, its acceleration is always perpendicular to its velocity and thus points toward the center of the circle.



(a) (b)

Fig. 1-14 Construction for finding change in velocity,Δv, of a particle moving in a circle.

Fig. 1-14 (a) represents a particle moving in a circular path of radius R with center at O. Vectors v_1 and v_2 represent its velocities at points P and Q. The vector change in velocity, Δv, is shown in Fig. 1-14 (b) . The particle moves from P to Q in a time Δt .

The triangles OPQ and Opq in Fig. 1-14 are **similar** , since both are isosceles triangles and the angles labeled $\Delta\theta$ are the same . Hence ,

$$\frac{\Delta v}{v_1} = \frac{\Delta s}{R} \qquad \text{or} \qquad \Delta v = \frac{v_1}{R}\Delta s$$

The magnitude of the average acceleration $\bar{a}$ during Δt is $\Delta v/\Delta t$; from the above equation , this is equal to

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_1 \Delta s}{R \Delta t}$$

The **instantaneous** acceleration a at point P is the limiting value of this expression , as point Q is taken closer to point P , and as $\Delta t \to 0$:

$$a = \lim_{\Delta t\to 0} \frac{v_1}{R}\frac{\Delta s}{\Delta t} = \frac{v_1}{R}\lim_{\Delta t\to 0}\frac{\Delta s}{\Delta t}$$

But the limiting value of $\Delta s/\Delta t$ is the speed v_1 at point P , and since P can be any point of the path , we can drop the subscript from v_1 and let v represent the speed at any point . Then

$$a = \frac{v^2}{\text{R}} \tag{1-18}$$

The magnitude of the instantaneous acceleration is therefore equal to the square of the speed divided by the radius . The direction is perpendicular to v and inward along the radius, toward the center of the circle. Because of this it is called a **centripetal**, or a **radial** acceleration.

Fig. 1-15 shows the directions of the velocity and acceleration vectors at a number of points, for a particle moving in a circle with constant speed.

Newton's second law governs circular motion as well as all other motion of a particle. The acceleration toward the center of the circle, for a particle in uniform circular motion , must be caused by a **force** also directed toward the center. Since the magnitude of the centripetal acceleration equals v^2/R, and its direction is toward the center, the magnitude of the radial force on a particle of mass m is

$$F = ma = m\frac{v^2}{R} \qquad (1\text{-}19)$$

A familiar example of such a force occurs when one ties an object to a string and whirls in a circle . The string must constantly pull in toward the center ; if it breaks , then the inward force no longer acts , and the objects flies off along a tangent to the circle .

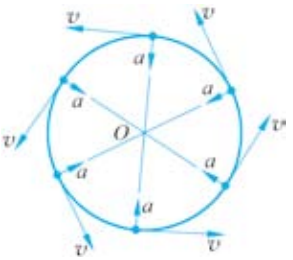


Fig. 1-15 Velocity and acceleration vectors of a particle in uniform circular motion.

More generally, there may be several forces acting on a body in uniform circular motion . In this case the vector sum of all forces on the body must have the magnitude given by Eq. (1-19) and must be directed toward the center of the circle. For such problems the problem—solving techniques developed in Section 1.1 are applicable, as the following examples will show.

The force in Eq. (1-19) is sometimes called **centripetal force.** The usage is unfortunate because it implies that this force is somehow different from ordinary forces , or that the fact of circular motion somehow generates an additional force ; neither is correct. **Centripetal** refers to the **effect** of the force, namely to the fact that it causes circular motion in which the direction of the velocity changes but not its magnitude . In the equation $\sum F = ma$, the sum of forces must include only the real physical forces , pushes or pulls exerted by strings , rods , or other agencies ; the quantity mv^2/R does not appear in $\sum F$ but belongs on the $m\boldsymbol{a}$ side of the equation .

Example 1 . An air-hockey puck ($m = 0.1$kg) moves without friction in a circle of radius 0.5m on a horizontal table. The puck is attached to the center of rotation by a string of negligible mass. The speed of the puck is $6\text{m}\cdot\text{s}^{-1}$. Determine

(a) the centripetal acceleration of the puck , and

(b) the string tension .

Solution:

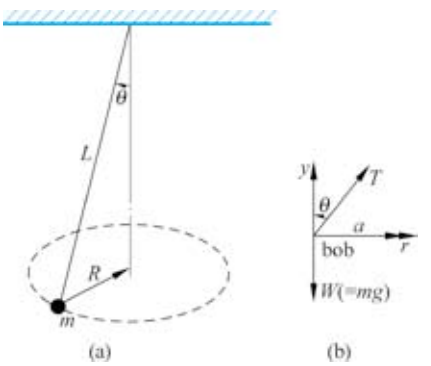


Fig. 1-16 (a) A conical pendulum, its cord making an angle θ with the vertical; (b) A free-body diagram for the pendulum bob. The resultant force (and thus the acceleration) points radially inward.

(a) From $a = \dfrac{v^2}{R}$

we get

$$a = \frac{v^2}{R} = \frac{(6\text{m}\cdot\text{s}^{-1})^2}{(0.5\text{m})} = 72\text{m}\cdot\text{s}^{-2}$$

(b) According to Newton's second law , the string tension should be

$$F = ma = (0.1\text{kg})(72\text{m}\cdot\text{s}^{-1}) = 72\text{N}$$

Example 2 . Fig. 1-16 (a) shows a **conical pendulum.** Its bob, whose mass m is 1.5kg , whirling around in a horizontal circle at constant speed v at the end of a cord those length L, measured to the center of the bob, is 1.7m. The cord makes an angle θ of 37° with the vertical. As the bob swings around, the cord sweeps out the surface of a cone. Find the

period of the pendulum, that is, the time τ for the bob to make one complete revolution.

Solution: Fig. 1-16 (b) shows the free-body diagram for the bob. The forces that act on it are the tension $\boldsymbol{T}$ in the cord and the weight $\boldsymbol{W}$ ($= mg$) of the bob, acting straight down. Applying Newton's second law in the vertical direction yields (bearing in mind that the acceleration a_y in this direction is zero)

$$T\cos\theta - mg = ma_y = 0 \qquad \text{or} \qquad T\cos\theta = mg \tag{1''}$$

In the radial direction, we have

$$T\sin\theta = \frac{mv^2}{R} \tag{2''}$$

where R is the radius of the circular path. Dividing Eq. (2″) by Eq. (1″) and solving for v, we obtain

$$v = \sqrt{\frac{gR\sin\theta}{\cos\theta}}$$

We can substitute $2\pi R/\tau$ for the speed v of the bob. Doing so and solving for period τ , we obtain

$$\tau = 2\pi\sqrt{\frac{R\cos\theta}{g\sin\theta}} \tag{3''}$$

However, from Fig. 1-16 (a) we see that $R = L\sin\theta$. Making this substitution in Eq. (3'') yields

$$\tau = 2\pi\sqrt{\frac{L\cos\theta}{g}} = 2\pi\sqrt{\frac{(1.7\text{m})(\cos 37°)}{9.8\text{m}\cdot\text{s}^{-2}}} = 2.3\text{s}$$

The period τ does not depend on the mass of the bob, only on $L\cos\theta$, the vertical distance of the bob from the level of its point of support.

Example 3. An earth satellite revolves in a circular orbit at a height of 300km above the earth's surface.

(a) What is the speed of the satellite, assuming the earth's radius to be 6380km, and its mass is $m_E = 5.98 \times 10^{24}\text{kg}$.

(b) What is the period τ ?

(c) What is the centripetal acceleration of the satellite ?

Solution: (a) From the problem, we know that the earth's radius $R = 6380\text{km} = 6.38 \times 10^6\text{m}$ and the radius of the orbit of the satellite

$$r = 6380\text{km} + 300\text{km} = 6680\text{km} = 6.68\times 10^6\text{m}$$

Because the gravitational force is the centripetal force, so we have

$$G\frac{m_E m}{r^2} = m\frac{v^2}{r}$$

$$v = \sqrt{\frac{Gm_E}{r}} = \sqrt{\frac{(6.67\times 10^{-11}\text{N}\cdot\text{m}^2\cdot\text{kg}^{-2})(5.98\times 10^{24}\text{kg})}{6.68\times 10^6\text{m}}} = 7.73\times 10^3\text{m}\cdot\text{s}^{-1}$$

(b) The period τ is

$$\tau = \frac{2\pi r}{v} = \frac{(2)(3.14)(6.68\times 10^6\text{m})}{7.73\times 10^3\text{m}\cdot\text{s}^{-1}} = 5.43\times 10^3\text{s}$$

(c) The centripetal acceleration of the satellite is

$$a = \frac{v^2}{r} = \frac{(7.73\times 10^3\text{m}\cdot\text{s}^{-1})^2}{6.68\times 10^6\text{m}} = 8.95\text{m}\cdot\text{s}^{-2}$$

1.4 Work and Energy

1.4.1 Work done by Constant Force

When a body moves a distance (or displacement) d along a straight line, while acted on by a constant force of magnitude F in the same direction as the motion, the work W done by the force is defined as

$$W = Fd$$

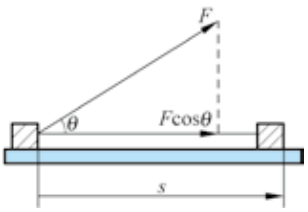


Fig. 1-17 The work done by the force $\boldsymbol{F}$ in a displacement $\boldsymbol{s}$ is ($F\cos\theta$) s.

More generally, the force need not have the same direction as the displacement. In Fig. 1-17, the force $\boldsymbol{F}$, assumed constant, makes an angle θ with the displacement. The work W of this force (or the work done by the force) , when its point of application undergoes a displacement $\boldsymbol{s}$, is defined as the product of the magnitude of the displacement and the **component** of the force in the direction of the displacement.

The component of $\boldsymbol{F}$ in the direction of $\boldsymbol{s}$ is $F\cos\theta$, then

$$W = (F\cos\theta)\, s = \boldsymbol{F}\cdot\boldsymbol{s} \tag{1-20}$$

Although W is calculated from two vector quantities, work itself is a **scalar** quantity. Work is an **algebraic** quantity, it can be positive or negative. When the component of the force is in the **same direction** as the displacement, the work W is **positive**. When it is **opposite** to the displacement, the work is **negative**. If the force is at **right angle** to the displacement, it has no component in the direction of the displacement and the work is zero.

Thus, when a body is lifted, the work done by the lifting force is positive; when a spring is stretched, the work done by the stretching force is positive; when a gas is compressed in a cylinder, the work done by the compressing force is positive. On the other hand, the work done by the gravitational force on a body being lifted is negative, since the (downward) gravitational force is opposite to the (upward) displacement. When a body slides along a surface, the work done by the frictional force acting on the body is negative and that of the normal force acting on the body is zero. When a body moves in a circle, the work done by the centripetal force on the body is also zero.

In SI system the unit of force is the Newton and the unit of distance is the meter, thus in this system the unit of work is one **Newton meter** ($1\mathrm{N}\cdot\mathrm{m}$) . This combination of units appears so frequently that it is given a special name, the **joule** (abbreviated J)

$$1\text{ joule} = (1\text{ Newton})\,(1\text{ meter})\quad\text{or}\quad 1\mathrm{J} = 1\mathrm{N}\cdot\mathrm{m}$$

When several external forces act on a body, it is useful to consider the works of the separate forces. Each of these may be computed from the definition of work in Eq. (1-20). Then, since work is a scalar quantity, the **total** work is the **algebraic sum** of the individual works.

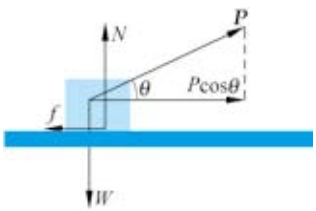


Fig. 1-18 An object on a rough horizontal surface moving to the right under the action of a force P in inclined at an angle θ.

Example 1. Fig. 1-18 shows a box being dragged along a horizontal surface by a constant force $\boldsymbol{P}$ making a constant angle θ withthe direction of motion. The other forces on the box are its weight $\boldsymbol{W}$, the upward normal force $\boldsymbol{N}$ exerted by the surface,

and the friction force ***f***. What is the work of each force when the box moves a distance s along the surface to the right ? (Suppose that $P = 50\text{N}$, $f = 15\text{N}$, $\theta = 37°$, and $s = 20\text{m}$)

Solution: The component of ***P*** in the direction of motion is $P\cos\theta$. The work of the force ***P*** is therefore

$$W_P = (P\cos\theta)\, s = (50\text{N})\,(0.80)\,(20\text{m}) = 800\text{J}$$

The forces ***W*** and ***N*** are both at right angles to the displacement. Hence,

$$W_W = 0, \qquad W_N = 0$$

The friction force ***f*** is opposite to be the displacement, so the work of the friction force is

$$W_f = -fs = (-15)\,(20\text{m}) = -300\text{J}$$

The total work W of all forces on the body is the algebraic sum of the individual works:

$$W = W_P + W_W + W_N + W_f = (P\cos\theta)\, s + 0 + 0 - fs = (P\cos\theta - f)s$$

$$= [(50\text{N})(0.80) - 15\text{N}](20\text{m}) = 500\text{J}$$

But $(P\cos\theta - f)$ is the **resultant** force (the net force) on the body. Hence **the total work of all forces is equal to the work of the resultant force (the net force).**

1.4.2 Work done by Varying Force

When external forces act on a particle, the particle does not always move in a straight line. Therefore, we must extend our definition of work to include motion on a curved path. Consider a particle moving along the curved path shown in Fig. 1-19 and acted by varying force. We can take a path segment, Δs_i , small enough so that the work done on the particle for that segment is

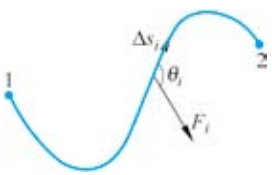


Fig. 1-19 Work done by varying force.

$$\Delta W_i = \boldsymbol{F_i} \cdot \Delta s_i$$

The smaller Δs_i , the more accurate will be the approximation of ΔW_i . We divide the path from the initial position of the particle, labeled 1, to its final position, labeled 2, into a large number of segments. Each segment contributes a term like ΔW_i to the work. Using a limiting procedure, we obtain the general definition of work.

$$W = \int_1^2 \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{s} = \int_1^2 F\cos\theta\, \mathrm{d}s \qquad (1\text{-}21)$$

Example 1. Compute the work required to stretch a spring a distance X from its unstretched length, if the force required increases in direct proportion to the amount of elongation.

Solution: To keep a spring stretched at an elongation x beyond its unstretched length, a force ***F*** must be exerted at one end and a force equal in magnitude but opposite in direction at the other end, as shown in Fig. 1-20. If the elongation is not too great, F is directly proportional to x :

$$F = kx$$

where k is a constant called the **force constant** or the **stiffness** of the spring. This direct proportion between force and elongation, for elongations that are not too great, was discovered by Robert Hooke (1635—1703) in 1678 and is known as **Hook's law.**

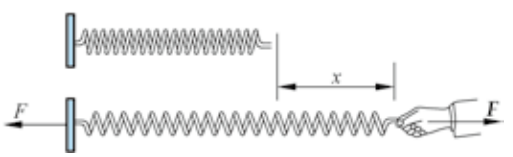


Fig. 1-20 The force to stretch a spring is proportional to its elongation; $F = kx$.

Suppose that forces equal in magnitude and