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CIRCULAR MOTION

Motion along a circular path : when a body is moving along a circular path with constant speed called uniform circular motion, when speed is not constant, motion is said to be non-uniform circular motion.

$$\vec{F}_c = \frac{mv^2}{r}(-\hat{r}) ; \hat{r} = \text{unit vector along radially outward}$$

A force required to keep body on circular path always acts in radially

inward direction called centripetal force whose magnitude is $\frac{mv^2}{r}$.

For non-uniform circular motion

$$\vec{F} = F_c(-\hat{r}) + F_t(\hat{n})$$

$\hat{n}$ = unit vector along direction of motion or velocity

$$F_c = \frac{mv^2}{r} \text{ and } F_t = m \frac{dv}{dt}$$

$\vec{a}$ = Resultant or net acceleration = $a_c(-\hat{r}) + a_t(\hat{n})$

$$= \frac{v^2}{r}(-\hat{r}) + \frac{dv}{dt}(\hat{n})$$

$$|\vec{a}| = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$$

$$\alpha = \tan^{-1}\left(\frac{dv/dt}{v^2/r}\right)$$

Angle of banking: Angle by which an outer edge of circular track is raised to provide the necessary centripetal force through the horizontal component of normal reaction.

$$\text{angle of banking } \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

Motion of vehicle on a horizontal circular track:

$$\frac{mv^2}{r} \text{ is being provided by force of static friction i.e., } F_s = \mu_s N \text{ and } N = mg \Rightarrow v^2 = \mu_s rg \text{ or } v = \sqrt{\mu_s rg}$$

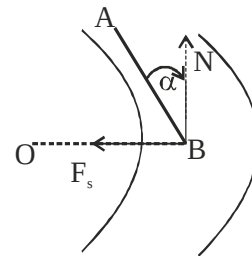
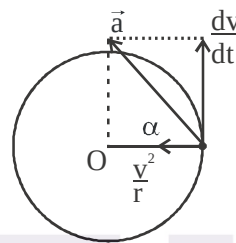
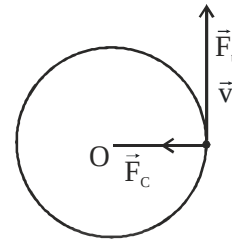
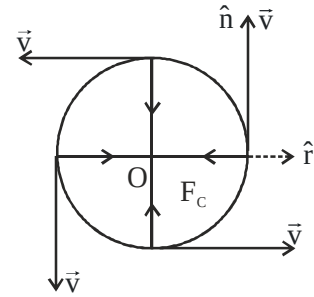
Condition for no skidding on circular track

$$F_s \geq \frac{mv^2}{r} \text{ or } \mu_s mg \geq \frac{mv^2}{r} \text{ or } v \leq \sqrt{\mu_s rg}$$

Angle of bending of a cyclist on a rough horizontal circular track to move on is given

$$\tan \theta = \frac{v^2}{rg} \Rightarrow \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

F_s provides necessary centripetal force $\frac{mv^2}{r}$ and $N = mg$. For safe turn there is a rotational equilibrium hence



no torque about A (Centre of gravity of cycle and cyclist).

Vertical Circular Motion

u is the velocity imparted at the bottom of the vertical circle. At P, equation of motion

$$T - mg \cos \theta = \frac{mv^2}{r} \quad \dots(i)$$

and from mechanical energy conservation principle,

$$\frac{1}{2} mu^2 = mgl(1 - \cos \theta) \Rightarrow v^2 = u^2 - 2gl(1 - \cos \theta) \quad \dots(ii)$$

from (i) and (ii) $T = mg \cos \theta + \frac{m}{l} [u^2 - 2gl(1 - \cos \theta)]$

$$= \frac{m}{l} [u^2 - 2gl + 3gl \cos \theta] \quad \dots(iii)$$

from (ii) and (iii) we have velocity and tension at any point on the vertical circular path

For just to complete the vertical circle

$$u = \sqrt{5gl} = \text{velocity}$$

At A, $v_A = \sqrt{5gl}$; $T_A = 6mg$ = tension in string when block is at A

At B, $v_B = \sqrt{3gl}$; $T_B = 3mg$

At C, $v_C = \sqrt{gl}$; $T_C = mg$

For no toppling of the automobile on horizontal circular track

$$F_s \cdot h \leq mga \quad ;$$

$$\frac{mv^2}{r} \cdot h \leq mga$$

h is the height of centre of gravity of automobile from surface of road.

$$v \leq \sqrt{\frac{arg}{h}}$$

While toppling wheels nearer to centre of track loose the contact.

