LD No: 1013/13 Section: 3 Field: NATURAL

**Part I: Write "True" if the statement is correct and "False" if it is incorrect on the space provided only (1 x 5=5 points).**

1) The modulus of a complex number is always positive. TRUE
2) The modulus of product of two complex numbers is the product of their modulus. TRUE
3) Two sets $A$ and $B$ are said to be disjoint sets if $A \cap B = \{\phi\}$. TRUE
4) We can construct a set having 127 proper subsets. TRUE
5) The set of complex numbers doesn't satisfy the trichotomy property. FALSE

**Part II: Write the most simplified answer on the space provided only (1.5 x 4=6 points).**

6) The GCF and LCM of 50,60 and 70 are 10 and 2100 respectively.
7) If $Z = 2 - i$ is a complex number, then the standard form of its multiplicative inverse is given by $Z^{-1} =$ 1/2−i = 2/5 + i/5
8) If the sum and difference of the real and imaginary parts of a complex number $Z$ are 20 and $-4$ respectively, then $Z =$ x+7i=20, x−7i=−4. So, Z=12+7i
9) Suppose that the propositions $p, q, r$ and $s$ are assigned with truth values $T, F, F$ and $T$ respectively. Then the truth value of the compound proposition $(r \vee s) \Leftrightarrow (p \Rightarrow (\neg q \wedge \neg s))$ is F

**Part II: Choose the best appropriate answer from the given alternatives and write your answer on the space provided only (1.5 x 6=9 points).**

10) Let $W$ be a complex number such that $\left|\frac{3W}{-4+3i}\right| = (2-i)\overline{(2-i)}$. Then the modulus of $W$ is: C

A) $\frac{14}{3}$ B) $\frac{10}{3}$ C) $\frac{25}{3}$ D) $\frac{25}{9}$ E) None

11) Which one of the following sets has **no additive inverse**? B
A) Set of integer numbers. C) Set of real numbers. E) A and B
B) The set of whole numbers. D) The set of complex numbers.

12) Which logical connective is not commutative for two propositions $p$ and $q$? E
A) Implication B) Bi-implication C) Conjunction D) Disjunction E) None

13) Which mathematical operation is/are **not closed** under the set of integers? D
A) Addition B) Subtraction C) Multiplication D) Division E) B and D

14) For the complex umber $z = x + iy$, which one of the following statements represents the circle $(x-3)^2 + (y+1)^2 = 4$? C
A) $|z+3-i| = 4$ C) $|z-3+i| = 2$ E) None
B) $|z-3+i| = 4$ D) $|z+3-i| = 2$

15) Which one of the following statement is not true if the domain of the variable is the set of **real numbers**? A
A) $(\exists x)(x^2 = 2)$ C) $(\exists x)((x^2-1) = 5)$ E) C & D
B) $(\forall x)(x^2 \neq x)$ D) $(\forall x)((x^2+2) \geq 1)$



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**Part IV**: **Show all the necessary steps clearly and neatly on the attached papers one by one (10 points).**

16) Use formal proof of validity (rule of inference) to show the validity of the argument:
**"If the rain does not come, then the crops are ruined and the people will starve. The crops are not ruined or the people will not starve. Therefore, the rain comes."** (3 points).

P = If the rain does not come.
Z = then the crops are ruined & the people will starve.

then :- P ⇒ Z

¬Z = The crops are not ruined or the people will not starve.

Therefore:
Conclusion = ¬P

$P \Rightarrow Z, \neg Z \vdash \neg P$ OS

We start from ~~Simple statement~~ :-

¬Z = T. ---- ① Z = F

P ⇒ Z = T ---- ② P = F

¬P = T

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17) Use mathematical induction to show that:

a) $1^2+2^2+3^2+\ldots+n^2=\frac{n(n+1)(2n+1)}{6}$, $n=1,2,3,\ldots$ and using this result deduce that the sum

b) $1^2+2^2+3^2+\ldots+(30)^2=9455$

(3+1+4 points)

a) Step 1) $n=1$

$1^2+2^2+3^2+\ldots+n^2=\frac{n(n+1)(2n+1)}{6}$

$1^2=\frac{1(2)(3)}{6}$

$1^2=6/6$

$1=1$ ..... (1)

b) Step 2) $n=k$

$1^2+2^2+3^2+\ldots+k^2=\frac{k(k+1)(2k+1)}{6}$ ..... (1)

c) Step 3) $n=k+1$

$1^2+2^2+3^2+\ldots+n^2=\frac{n(n+1)(2n+1)}{6}$

$\underbrace{1^2+2^2+3^2+\ldots+k^2}+(k+1)^2=\frac{k+1(k+2)(2k+3)}{6}$

$=\underbrace{1^2+2^2+3^2+\ldots+k^2}=\frac{k(k+1)(2k+1)}{6}+(k+1)^2=\frac{(k+1)(k+2)(2k+3)}{6}$

$\frac{k(k+1)(2k+1)}{6}+(k+1)^2=\frac{(k+1)(k+2)(2k+3)}{6}$

$\frac{(k^2+k)(2k+1)}{6}+(k+1)^2=\frac{(k+1)(2k^2+7k+6)}{6}$

$\frac{2k^3+3k^2+k+6k^2+12k+6}{6}=\frac{2k^3+9k^2+13k+6}{6}$

$\frac{2k^3+9k^2+13k+6}{6}=\frac{2k^3+9k^2+13k+6}{6}$ ..... (1)

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18) Solve the equation $z^{3/2} - 8i = 0$, where $z$ is a complex number.

$z^{3/2} - 8i = 0$

$z^{3/2} = 8i = 0$

$z^3 - 8^2 = 0$

$z = x + yi$

$z^3 = (x+yi)^3 - 8^2 = 0$

$(x+yi)(x+yi)(x+yi) - 8^2 = 0$

$x^3 + xyi(yi) + x^2yi + y^3 = 64$

or

$x^2 + 2xyi + y^2(x+yi) = 64$

$x^2 + 2xyi - y^2(x+yi) = 64$

$x^3 + y^2 + 3x^2yi - 3xy^2 = 64$

$x^3 + 3x^2yi - 3xy^2 + y^2 = 64$

$Y = 0$, and

$x^3 + 3x^2yi - 3xy^2 = x^3$

$x^3 + 3x^2(0)i - 3x(0)^2 = x^3$

$x^3 = 64$, $x = 4$

$Y = 0$, $x = 4$

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