

Grade 12 students

G12 +

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CHOOSE THE BEST ANSWER FROM THE GIVEN ALTERNATIVES

ENTERANCE EXAM RELATED TO UNIT ONE AND UNIT TWO OF GRADE 11

1. Which of the following is the inverse of $f(x) = 8x^3 + 2$? **Ans. B**

   A. $f^{-1}(x) = \frac{1}{8x^3} + 2$ B. $f^{-1}(x) = \frac{1}{2}\sqrt[3]{x-2}$

   C. $f^{-1}(x) = 8x^{-3} - 2$ D. $f^{-1}(x) = \frac{1}{8}\sqrt[3]{x-2}$

Solution:

$$\text{Let } f(x) = y, \text{ then } y = 8x^3 + 2$$
$$\Rightarrow x = 8y^3 + 2$$
$$\Rightarrow x - 2 = 8y^3$$
$$\Rightarrow y^3 = \frac{x-2}{8}$$
$$\Rightarrow y = \sqrt[3]{\frac{x-2}{8}}$$
$$\Rightarrow y = \frac{1}{2}\sqrt[3]{x-2}$$

$$\text{Therefore, } f^{-1}(x) = \frac{1}{2}\sqrt[3]{x-2}$$

2. Let $R = \{(x, y): y \geq x^2 + 1 \text{ and } y \leq 5$ be a relation.then which one of the following defines the inverse of R.

   A. $\{(x, y): x \leq y^2 + 1 \text{ and } x \geq 5\}$

   B. $\{(x, y): x \leq y^2 - 1 \text{ and } x \leq 5\}$

   C. $\{(x, y): x \geq y^2 + 1 \text{ and } x \leq 5\}$

   D. $\{(x, y): x \geq y^2 + 1 \text{ and } x \geq 5\}$

Solution:

$R^{-1} = \{(x, y): x \geq y^2 + 1 \text{ and } x \leq 5\}$ **Ans. C**

3. The inverse of the function $y = 3x - 5$ *is equal to*

   A. $y = \frac{x+3}{5}$ B. $y = \frac{x+5}{3}$

   C. $y = -5x + 3$ D. $y = \frac{x-5}{3}$

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Solution:

$$x = 3y - 5$$
$$\Rightarrow x + 5 = 3y$$
$$\Rightarrow y = \frac{x+5}{3}$$

Ans. B

4. Let $f(x) = 3 - 2x$, what is the inverse of f ?

A. $f^{-1}(x) = \frac{x}{2} - \frac{3}{3}$ B. $f^{-1}(x) = \frac{3}{2} - \frac{x}{2}$

C. $f^{-1}(x) = 3 + 2x$ D. $f^{-1}(x) = 2x - 3$

Solution:

$$y = 3 - 2x$$
$$\Rightarrow x = 3 - 2y$$
$$\Rightarrow x - 3 = -2y$$
$$\Rightarrow y = \frac{3-x}{2}$$

Ans. B

5. If $f(x) = ax - b$ and $f^{-1}(x+1) = \frac{1}{2}x + 2 \; for \; each \; x \in \mathbb{R}$, then what must be the value of $a \; and \; b$?

A. $a = 2, b = 3$ $B. a = 2, b = 2$

$C. a = 1, b = 1$ $D. a = \frac{1}{2}, b = -2$

Solution:

Since $f^{-1}(x)$ is the inverse of $f(x)$, then $(fof^{-1})(x+1) = x + 1$

$$f\left(\frac{1}{2}x + 2\right) = x + 1$$
$$\Rightarrow \frac{ax}{2} + 2a - b = x + 1$$
$$\Rightarrow \frac{a}{2} = 1 \text{ and } 2a - b = 1$$
$$\Rightarrow a = 2 \text{ and } 4 - b = 1$$
$$\Rightarrow b = 4 - 1 = 3$$

Ans. A

6. Which one of the following is an example of power function?

A. $4x^2 + x$ B. x^x C. $\frac{\sqrt{2}}{\pi}x^{-\frac{2}{5}} - 2^x$ $D. \frac{3}{2x^{\frac{-1}{3}}}$

Ans. D

Solution:

- A function which is written in the form of $f(x) = ax^r, r \in \mathbb{R}$ and a is non zero.
- The other functions are not written in the form of $f(x) = ax^r$, except $\frac{3}{2x^{\frac{-1}{3}}}$
- $4x^2 + x$ is polynomial not power, x^x is exponential not power

7. What is the respective domain and range of the function $f(x) = 3x^{-\frac{6}{5}}$?

A. $\mathbb{R}$ and $[0,\infty)$ B. $(0,\infty)$ *and* $(0,\infty)$

C. $\mathbb{R}/\{0\}$ *and* $(0,\infty)$ D. $[0,\infty)$*and*$[0,\infty)$

Solution:

- Domain of $f(x) = \mathbb{R}/\{0\}$ and its range is $(0,\infty)$ **Ans. C**
- Let $y = 3x^{-\frac{6}{5}} \Rightarrow x = 3y^{-\frac{6}{5}} \Rightarrow 3y = x^{-\frac{5}{6}} \Rightarrow y = \frac{x^{\frac{-5}{6}}}{3} = \frac{1}{3x^{\frac{5}{6}}}$
- Domain of y *is* $(0,\infty) = Range\ of\ f(x)$

8. Which one of the following is necessary true?

A. All power function is polynomial.

B. All polynomial is power function.

C. If some function is power function, then it is polynomial

D. If some function not is polynomial, then it is power function.

Solution: **Ans. C and D**

9. If $x, y \in \mathbb{R}$, then which one of the following is NOT always true? **Ans. C and B**

A. $|x-y| = |y-x|$ C. $|x+y| = |x| + |y|$

B. $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$ D. $|xy| = |y||x|$

Solution:

For any real numbers $|x-y| = |y-x|$, $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$, if $y \neq 0$ and $|xy| = |x||y|$, $|x+y| = |x| + |y|$ some time but not equal always.

10. What is the solution set of the equation $\frac{3}{2} - 2\left|1 - \frac{2x}{5}\right| = -1$? **Ans B**

A. $\{\frac{-5}{2}, \frac{15}{2}\}$ B. $\{\frac{-5}{8}, \frac{15}{8}\}$ C.$\{\}$ D. $\{-2, 3\}$

Solution:

$$\frac{3}{2} - 2 + \frac{4x}{5} = -1 \Rightarrow \frac{4x}{5} = -1 + \frac{1}{2} \Rightarrow \frac{4x}{5} = -\frac{1}{2} \Rightarrow x = -\frac{5}{8}$$

$$\frac{3}{2} - 2 + \frac{4x}{5} = 1 \Rightarrow \frac{4x}{5} = 1 + \frac{1}{2} \Rightarrow x = \frac{2+1}{2} \times \frac{5}{4} \Rightarrow x = \frac{15}{8}$$

11. Which one of the following is the solution of the equation $|9x - (1 - x)| + 4x = 1$?

A. $\{0, \frac{1}{7}\}$ B. $\{\frac{1}{7}, \frac{1}{6}\}$ C.$\{\frac{-1}{7}, 0\}\}$ D. $\{0, \frac{1}{6}\}$ **ANS, A**

Solution:

- $|9x - (1 - x)| + 4x = 1 \Rightarrow 9x - 1 + x + 4x = 1 \Rightarrow 14x = 2 \Rightarrow x = \frac{2}{14} \Rightarrow x = \frac{1}{7}$
- $|9x - (1 - x)| + 4x = 1 \Rightarrow 9x - 1 + x + 4x = -1 \Rightarrow 14x = 0 \Rightarrow x = 0$

12. If f is greatest integer function and g is absolute value function, then what is the value of $fog\left(\frac{3}{2}\right) + gof\left(\frac{-4}{3}\right)$? $A.1$ $B.-1$ $C.3$ $D.2$

Solution **Ans C**

$$(fog)\left(\frac{3}{2}\right) + \text{gof}\left(\frac{-4}{3}\right) = \text{f}\left(g\left(\frac{3}{2}\right)\right) + \text{g}\left(f\left(\frac{-4}{3}\right)\right)$$

$$\Rightarrow \text{f}\left(\frac{3}{2}\right) + \text{g}(-2) = 1 + 2 = 3$$

13. The solution set of the equation $|2x - 5| - 3|x + 1| = 0$ *is equal to* ___

A. $\{-4, \frac{2}{5}\}$ B. $\left\{-8, \frac{-8}{5}\right\}$ C. $\left\{-8, \frac{2}{5}\right\}$ D. $\left\{-8, \frac{8}{5}\right\}$ **Ans. C**

Solution:

- $|2x - 5| - 3|x + 1| = 0 \Rightarrow 2x - 5 - 3x - 3 = 0 \Rightarrow -x = 8 \Rightarrow x = -8$
- $|2x - 5| - 3|x + 1| = 0 \Rightarrow 2x - 5 + 3x + 3 = 0 \Rightarrow 5x = 2 \Rightarrow x = \frac{2}{5}$

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14. All solution set of the equation $|x-3|-2|x+5|=10$ *is equal to* ___.

A. $\{-23\}$ B.$\{-3,\frac{-17}{3}\}$ C. $\emptyset$ D. $\{-3,\frac{-17}{3},-23\}$

Solution: **Ans. C**

$|x-3|-2|x+5|=10 \Rightarrow x-3-2x-10=10 \Rightarrow -x=23 \Rightarrow x=-23$

15. If $x<0$, *what is the simplest form of* $\frac{2x-|5x|}{x}$? **Ans.B**

A. -3 B. 7 C. 3 D.-7

Solution:

➢ $\frac{2x-|5x|}{x}=\frac{2x+5x}{x}=\frac{7x}{x}=7$

16. Which one of the following is true about the graph of $f(x)=|x+2|-1$?

A. The graph is obtained by shifting the graph of $(x)=|x|$, 2 unit to the right and 1 unit down.

B. The graph is obtained by shifting the graph of $(x)=|x|$, 2 unit to the left and 1 unit up.

C. Range of f is $y\leq -1$

D. The graph is obtained by shifting the graph of $(x)=|x|$, 2 unit to the left and 1 unit down.

Solution: **Ans. D**

➢ The graph of $f(x)=|x+2|-1$ is shifting the graph of

➢ $F(x)=|x|$ 2 unit to the left and 1 unit down.

➢ The range of $f(x)$ *is* $y\geq -1$

17. What is the range of the function $f(x)=sgnx+3$? **Ans. D**

A. $2\leq y\leq 4$ B.$-1\leq y\leq 1$ C. $\{-1,0,1\}$ D. $\{2,3,4\}$

Solution: $(sgnx)+3 \Rightarrow (-1,0,1)+3=\{2,3,4\}$

➢ Therefore, the range of f(x)=$\{2,3,4\}$

18. Which one of the following is NOT true about the graph of $f(x)=|x-3|+2$?

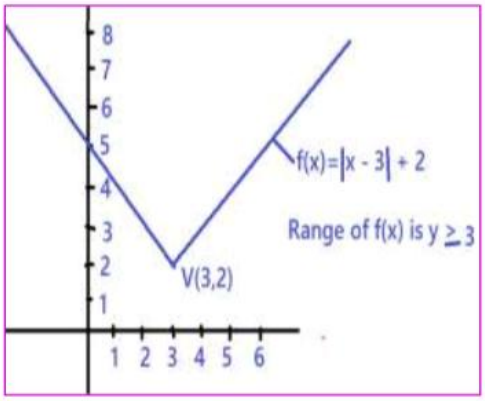
A. Its vertex is (3,2)

B. Its range is $[2,\infty)$

C. It intersect x-axis at $x=3$

D. The graph of $f(x)$ is the graph of $y=|x|$ translated up 2 unit and right 3 unit.

Solution: **Ans. C**



- The vertex of the graph of the function f(x) is at (3,2).
- The graph of f(x) is obtained by shifting the graph of f(x)=|x| 2 unit up ward and 3 unit to the right.
- The range of the function is $y \geq 2$.
- The graph of the function never cross the x-axis.

19. What are the domain and range of the function $f(x) = 2x^{\frac{3}{4}}$? **Ans. B**

A. $[0,\infty) and\ (0,\infty)$ B. $[0,\infty) and\ [0,\infty)$

C. $\mathbb{R}$ and $[0,\infty)$ D. $[0,\infty)\ and\ \mathbb{R}$

Solution: The domain of the function is $[0,\infty) and$ The range of the function is $[0,\infty)$

20. What is the range of the function $f(x) = x - [x]$? **Ans. C**

A. $\mathbb{R}$ B. $\mathbb{Z}$ C. $[0,1)$ D. $[0,1]$

Solution: For any real number x the result is always including zero and between 1.

21. If $f(x) = 1 + \log^{(3x+1)}$ and $g(x) = 10x + 3$, then what is the value of $fog(3)$?

A. 33 B. 100 C. 3 D. 2 **Ans. C**

Solution:

$$fog(3) = f(g(3)) = f(33) = 1 + \log(3 \times 33 + 1) = 1 + \log 100 = 3$$

22. Suppose $f(x) = \frac{x-1}{x+3}$ and $g(x) = \frac{3x+1}{6-2x}$ then what is the value of $f^{-1}(g\left(\frac{1}{2}\right))$?

A. 5 B. 4 C. $\frac{2}{3}$ D. $\frac{1}{2}$ **Ans.A**

Solution: $y = \frac{x-1}{x+3} \Rightarrow x = \frac{y-1}{y+3} \Rightarrow xy + 3x = y - 1$

$$\Rightarrow y(x-1) = -3x - 1 \Rightarrow y = \frac{-3x-1}{x-1}$$

$$f^{-1}(x) = \frac{3x+1}{1-x} \Rightarrow f^{-1}\left(g\left(\frac{1}{2}\right)\right) \Rightarrow f^{-1}\left(\frac{1}{2}\right) = 5$$

23. The inverse of the function $\frac{1-2x}{1+x}$ *is* **Ans.A**

A. $\frac{1-x}{x+2}$ B. $\frac{x+1}{x-2}$ C. $\frac{2x-1}{x+1}$ D. $\frac{x+1}{1-2x}$

Solution:

$$\text{Let } y = \frac{1-2x}{1+x} \Rightarrow x = \frac{1-2y}{1+y} \Rightarrow x + xy = 1 - 2y$$

$$\Rightarrow y(x+2) = 1 - x \Rightarrow y = \frac{1-x}{x+2}$$

24. Which one of the following is the inverse of $f(x) = \frac{1+e^x}{1-e^x}$? **Ans. A**

A. $\ln\left(\frac{1-x}{x+1}\right)$ B. $\frac{1+lnx}{1-lnx}$ C. $2 + e^x$ D. x

Solution: Let $y = \frac{1+e^x}{1-e^x} \Rightarrow x = \frac{1+e^y}{1-e^y} \Rightarrow x - xe^y = 1 + e^y$

$$\Rightarrow e^y + xe^y = 1 - x \Rightarrow e^y(1+x) = 1 - x$$

$$\Rightarrow e^y = \frac{1-x}{1+x} \Rightarrow y = \ln\left(\frac{1-x}{1+x}\right)$$

25. Let $f(x) = x - x^2$ and $g(x) = \frac{1}{x}$. Then What is $g\left(f\left(\frac{1}{x}\right)\right)$ eqaul to:

A. $\frac{x^2}{x-1}$ B. $x - x^2$ C. $\frac{1}{x^2-x}$ D. $\frac{x-1}{x^2}$ **Ans. A**

Solution: $g\left(f\left(\frac{1}{x}\right)\right) = g\left(\frac{1}{x} - \left(\frac{1}{x}\right)^2\right) = g\left(\frac{x-1}{x^2}\right) \Rightarrow \frac{1}{\frac{x-1}{x^2}} = \frac{x^2}{x-1}$

26. *Let* $f(x) = \frac{3x+1}{x-2}$. *Then what is the range of* $f(x)$? **Ans. D**

A. $\mathbb{R}/\{2\}$ B. $\mathbb{R}$ C. $\mathbb{R}/\{1,3\}$ D. $\mathbb{R}/\{3\}$

Solution: $Let\ y = \frac{3x+1}{x-2} \Rightarrow x = \frac{3y+1}{y-2} \Rightarrow xy - 2x = 3y + 1$

$$\Rightarrow xy - 3y = 2x + 1 \Rightarrow y(x-3) = 2x + 1 \Rightarrow y = \frac{2x+1}{x-3}$$

Therefore, the range of f(x) = $\mathbb{R}/\{3\} = Domain\ of\ f^{-1}(x)$

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27. What is the Partial fraction decomposition of $\frac{x^2+x+1}{(x+2)(x^2+1)}$? **Ans. B**

A. $\frac{5}{3(x+2)}+\frac{2x+1}{3(x^2+1)}$ B. $\frac{3}{5(x+2)}+\frac{2x+1}{5(x^2+1)}$

C. $\frac{5}{2(x+2)}+\frac{5x+1}{2(x^2+1)}$ D. $\frac{3}{5(x+2)}+\frac{5x+1}{2(x^2+1)}$

Solution: $\frac{x^2+x+1}{(x+2)(x^2+1)}=\frac{A}{x+2}+\frac{Bx+C}{x^2+1}\Rightarrow\frac{A(x^2+1)+(Bx+C)(x+2)}{(x+2)(x^2+1)}$

$$\Rightarrow\frac{Ax^2+A+Bx^2+2Bx+Cx+2c}{(x+2)(x^2+1)}\Rightarrow\begin{cases}A+B=1\\2B+C=1\\A+2C=1\end{cases}$$

$A+B=1 \Rightarrow A=1-B$

$2B+C=1$

$A+2C=1\Rightarrow 1-B+2C=1\Rightarrow -B+2C=0$

$$\begin{cases}2B+C=1\\-B+2C=0\end{cases}\Rightarrow\begin{cases}2B+C=1\\-2B+4C=0\end{cases}\Rightarrow 5C=1\Rightarrow C=\frac{1}{5}$$

$$-B+2C=0\Rightarrow B=\frac{2}{5}$$

$$A=1-\frac{2}{5}=\frac{3}{5}\Rightarrow A=\frac{3}{5}$$

Therefore, $\frac{x^2+x+1}{(x+2)(x^2+1)}=\frac{3}{5(x+2)}+\frac{2x+1}{5(x^2+1)}$

28. Which one of the following is true about signum, absolute value and greatest integer functions?

A. $\text{sgn}(x)=\pm|x|$, for all $x\in\mathbb{R}$. B. $|x|=x\,\text{sgn}(x)$, for all $x\in\mathbb{R}$

C. $\text{sgn}(x)\le|x|$, for all $x\le 0$ D. $\text{sgn}(x)\le|x|$, for all $x\ge 0$

Ans. B

29. If $f(x)=\sqrt{x^3}$ and $(fog)(x)=\sqrt[4]{x}$, then what is the value of $g(8)$?

A. $\sqrt[3]{2}$ B. 2 C. $\sqrt{2}$ D. $2\sqrt{2}$ **Ans.C**

Solution: $(fog)(x)=\sqrt[4]{x}\Rightarrow f(g(x))=\sqrt[4]{x}$

$$\Rightarrow \sqrt{(g(x))^3} = \sqrt[4]{x} \Rightarrow (g(x))^3 = \sqrt[4]{x^2}$$

$$\Rightarrow (g(x))^3 = \sqrt{x} \Rightarrow g(x) = \sqrt[3]{\sqrt{x}}$$

$$\Rightarrow g(8) = (8)^{\frac{1}{2}\times\frac{1}{3}} = 2^{\frac{3}{6}} = 2^{\frac{1}{2}} = \sqrt{2}$$

30. Which one of the following is the inverse of $f(x) = 8x^3 + 2$? **Ans. B**

A. $f^{-1}(x) = \frac{1}{8x^2+2}$ B. $f^{-1}(x) = \frac{1}{2}\sqrt[3]{x-2}$

C. $f^{-1}(x) = 8x^{-3} + 2$ D. $f^{-1}(x) = \frac{1}{8}\sqrt[3]{x-2}$

Solution: Let $y = 8x^3 + 2 \Rightarrow x = 8y^3 + 2$

$$\Rightarrow 8y^3 = x - 2 \Rightarrow y^3 = \frac{x-2}{8}$$

$$\Rightarrow y = \sqrt[3]{\frac{x-2}{8}} \Rightarrow y = \frac{\sqrt[3]{x-2}}{2}$$

31. Which one of the following is function is a one to one correspondence?

A. $f: [0, \infty) \to \mathbb{R}$ defined by $f(x) = |x|$

B. $f: \mathbb{R} \to [0, \infty)$ defined by $f(x) = x^2$

C. $f: \mathbb{R} \to [0, \infty)$ defined by $f(x) = 3^x$

D. $f: (0, \infty) \to \mathbb{R}$ defined by $f(x) = \log_2 x$

Solution: **Ans. A**

- One to one correspondence function is a function of the form whose domain and range are equivalent.

32. Suppose $f(x) = \frac{q(x)}{x(x^2-1)}$ where $q(x)$ is a quadratic function.

Which one of the following is necessarily true bout the graph of f?

A. $x = 0, x = 1$ and $x = -1$ are vertical asymptotes of the graph of f,

B. The graph of f does not intersect with its horizontal asymptote.

C. The vertical asymptotes of the graph of f is only $x = -1$

D. The vertical asymptotes of the graph of f are $x = 1$ and $x = -1$ if $q(x) = 2x^2$

Solution: **Ans. D**

The vertical asymptotes of the graph of f are x = 1 and x = -1 if $q(x) = 2x^2$

33. Which one of the following is NOT true about the graph of $f(x) = 5 + \frac{|x|}{x+1}$?

A. $y = 5 + x$ is an oblique asymptote
B. $y = 6$ is horizontal asymptote of f
C. $y = 4$ is horizontal asymptote
D. $x = -1$ is its vertical asymptote of f

Solution: **Ans. A**

- The graph of f(x) has no oblique asymptote
- If $x > 0$, then horizontal asymptote is $y = 6$.
- If $x < 0$, then the horizontal asymptote is $y = 4$
- $x = -1$ is the only vertical asymptote of f(x).

34. Which one of the following is true about the horizontal asymptote(s) of the graph of $y = \frac{|x|+2}{x}$?

A. $y = 2$ is the only horizontal asymptote of the graph.
B. $y = 1$ and $y = -1$ are the horizontal asymptotes of the graph.
C. $y = 2$ and $y = -2$ are the horizontal asymptotes of the graph.
D. $y = 1$ is the only horizontal asymptote of the graph.

Solution: **Ans. B**

The possible horizontal asymptote of the graph of $y = \frac{|x|+2}{x}$ *are*:

- If x > 0, then the horizontal asymptote is y = 1
- If x < 0, then the horizontal asymptote is y = -1

35. Which one of the following is the simplified form of $\frac{4x^4-64}{2x-4}$ for $x \neq 2$? **Ans. D**

A. $-4(x-2)(x^2+4)$ B. $-4(x-2)(x^2+4)$
C. $4(x-2)(x^2-4)$ D. $2(x+2)(x^2+4)$

Solution:

$$\frac{4x^4-64}{2x-4} = \frac{4(x^2-4)(x^2+4)}{2(x-2)} = \frac{4(x-2)(x+2)(x^2+4)}{2(x-2)} = 2(x+2)(x^2+4)$$

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36. Let $R = \{(x, y): y \geq x^2 + 1 \text{ and } y \leq 5\}$ be a relation **Ans. A**

Then which one of the following defines the inverse of R?

A. $\{(x, y): x \geq y^2 + 1, x \leq 5\}$ B. $\{(x, y): x \geq y^2 + 1, x \geq 5\}$

C. $\{(x, y): x \leq y^2 + 1, x \geq 5\}$ D. $\{(x, y): x \geq y^2 - 1, x \leq 5\}$

Solution: $x \geq y^2 + 1 \text{ and } x \leq 5$

37. Let $f(x) = \sqrt{x+2}$ and $g(x) = x^2 - 1$. What are the domain and range of the composite of f with g fog respectively? **Ans. A**

A. $\mathbb{R}$ and $[1, \infty)$ B. $\mathbb{R}$ and $[0, \infty)$

C. $[0, \infty)$and $[1, \infty)$ D. $[0, \infty)$and $[0, \infty)$

Solution: $(fog)(x) = f(g(x)) = f(x^2 - 1) = \sqrt{x^2 + 1}$

- Domain of $(fog)(x) = \mathbb{R}$.

To obtaine the range of $fog(x)$ first we find its inverse.

$$\text{Let } y = \sqrt{x^2+1} \Rightarrow x = \sqrt{y^2+1} \Rightarrow y^2 + 1 = x^2$$

$$\Rightarrow y^2 = x^2 - 1 \Rightarrow y = \sqrt{x^2 - 1}$$

- Therefore, range of $fog(x)$ is $[1, \infty)$

38. Given a rational function $f(x) = \frac{2x+4}{x-1}$,

Which of the following is NOT true about its graph? The grph has:

A. A horizontal asymptote, y=2 B. A vertical asymptote x= - 1

C. y- intercept (0,-4) D. x- intercept at (-2,0)

Solution: **Ans. B**

- The horizontal asymptote is of f(x) is y = 2.
- The vertical asymptote of f(x) is x = 1
- The x- intercept is (-2, 0) and y-intercept is (0,-4)

39. *Let $f(x) = 4 - 2x$. what is the inverse of f?* **Ans.C**

A. $f^{-1}(x) = 2x - 4$ B. $f^{-1}(x) = 3 + 2x$

C. $f^{-1}(x) = \frac{4-x}{2}$ D. $f^{-1}(x) = \frac{x-4}{2}$

Solution:

Let $y = 4 - 2x \Rightarrow x = 4 - 2y \Rightarrow 2y = 4 - x \Rightarrow \mathbf{y = \frac{4-x}{2}}$

49. What are the domain and range of the function $f(x) = 2x^{\frac{3}{4}}$ *respectively*?

Ans. D

A. $\mathbb{R}$ and $[0, \infty)$
B. $[0, \infty)$ and $(0, \infty)$
C. $[0, \infty)$ and $\mathbb{R}$
D. $[0, \infty)$ and $[0, \infty)$

Solution:

The domain of f(x) is $[0, \infty)$ *and range of* $f(x)$ *is* $[0, \infty)$

50. Which of the following is the universal set of the expression $\frac{1}{x^2} + \frac{x+1}{x^2+x}$? Ans.D

A. $\mathbb{R}/\{0, 1\}$
B. $\mathbb{R}/\{0\}$
C. $\mathbb{R}/\{0, 1, -1\}$
D. $\mathbb{R}/\{0, -1\}$

Solution:

➢ $\mathbf{x^2 + x = 0 \Rightarrow x(x + 1) = 0 \Rightarrow x = 0 \text{ or } x = -1}$

51. The inverse of the function $y = 3x - 5$ *is equal to*: Ans. B

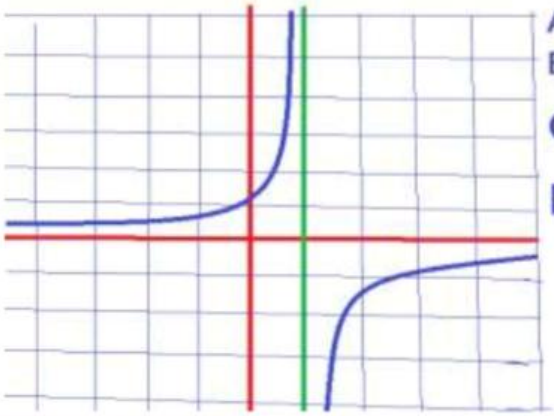
A. $\mathbf{y = \frac{x+5}{3}}$
B. $\mathbf{y = \frac{x+3}{5}}$
C. $\frac{x-5}{3}$
D. $-5x + 3$

Solution:

➢ $y = 3x - 5 \Rightarrow x = 3y - 5 \Rightarrow 3y = x + 5 \Rightarrow \mathbf{y = \frac{x+5}{3}}$

52. The graph of the rational function is given as in the figure below.

Which of the following is NOT true about the graph? Ans. C and D



A. x=1 is its vertical asymptote
B. y=0 is its horizontal asymptote.
C. It is increasing on $(0, \infty)$
D. It is decreasing on $(-\infty, 1)$

- The horizontal asymptote is y = 0
- The vertical asymptote is x = 1
- The graph is increasing on both $(-\infty, 1) and\ (1, \infty)$

53. Which of the following is true about a function f defined by $f(x) = \sqrt{\frac{1}{5x-7}}$?

A. Range of f is $\{f(x) \in \mathbb{R}: f(x) \geq 1\}$
B. Domain of f is $\{x \in \mathbb{R}: x \geq 7\}$
C. Domain of f is $\left\{x \in \mathbb{R}: x \geq \frac{7}{5}\right\}$
D. Range of f is $\{f(x) \in \mathbb{R}: f(x) > 0\}$

Solution: **Ans. D**

- Domain of $f(x)$ is $\frac{1}{5x-7} > 0 \Rightarrow x > \frac{7}{5}$
- Range of $f(x)$ = Domain of $f^{-1}(x) = \frac{7x^2+1}{5x^2}$ is $(0, \infty)$

54. Which one of the following is true about the zeros of a function?

A. If h is a function defined by $h(x) = \frac{x^3-27}{x-3}$, then 3 is the zero of h.
B. The zero of a function f is the functional value of the function at zero.
C. The functions g and h given by $g(x)=\sqrt{1-x^2}+1-x^2$ and $h(x) = \frac{1}{\sqrt{1-x^2}}$ have identical zeros.
D. The zeros of a given function f and cf for $c \neq 0$ are identical, if it exist(s)

Solution: **Ans. D**

- $h(x) = \frac{x^3-27}{x-3} \Rightarrow \frac{(x-3)(x^2+3x+9)}{x-3} = x^2 + 3x + 9$ $h(x)$ has no solution
- $h(x)$ has no zero and $g(x)$ has a zeros.
- $f(x)$ and $cf(x)$ has the same solution.

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55. Which of the following is the solution set of equation $\frac{3}{x+1}-\frac{1}{x-1}=\frac{2}{x}$?

A. $\left\{-\frac{1}{2}\right\}$ B. $\left\{\frac{1}{2}\right\}$ C. $\{-1\}$ D. $\{1\}$ **Ans. A**

Solution: *The domain of the equation is* $\mathbb{R}/\{-1,0,1\}$

$\frac{3}{x+1}-\frac{1}{x-1}=\frac{2}{x}\Rightarrow\frac{3}{x+1}-\frac{1}{x-1}-\frac{2}{x}=0\Rightarrow\frac{3x^2-3x-x^2-x-2x^2-2}{(x-1)(x+1)x}=0$

$-4x-2=0\Rightarrow x=-\frac{1}{2}\Rightarrow$ S.S=$\left\{-\frac{1}{2}\right\}$

56. Which one of the following pairs of functions, f and g are inverses of each other?

A. $f(x)=\frac{2x-1}{x+3}, x\neq -3$ *and* $g(x)=\frac{3x+1}{2-x}, x\neq 2$

B. $f(x)=5^x$ *and* $g(x)=\log_x 5$

C. $f(x)=(x-13)^2$ *and* $g(x)=x+\sqrt{13}$

D. $f(x)=\sqrt[5]{x+11}$ *and* $g(x)=x^5+11$

Solution: **Ans. A**

➢ Let $y=\frac{2x-1}{x+3}\Rightarrow x=\frac{2y-1}{y+3}\Rightarrow xy+3x=2y-1$

$\Rightarrow xy-2y=-1-3x\Rightarrow y(x-2)=-1-3x$

$\Rightarrow y=\frac{-1-3x}{x-2}$

$\Rightarrow y=\frac{3x+1}{2-x}.$

Therefore, $f^{-1}(x)=\frac{3x+1}{2-x}=g(x)$

➢ *Let* $y=5^x\Rightarrow x=5^y\Rightarrow y=\log_5 x$

Therefore, $f^{-1}(x)=\log_5 x\neq g(x)$

➢ *Let* $y=(x-13)^2\Rightarrow x=(y-13)^2\Rightarrow y-13=\sqrt{x}$

$\Rightarrow y=\sqrt{x}+13.$

Therefore, $f^{-1}(x)=\sqrt{x}+13\neq g(x)$

➢ Let $y=\sqrt[5]{x+11}\Rightarrow x=\sqrt[5]{y+11}\Rightarrow y+11=x^5\Rightarrow y=x^5-11$

Therefore, $f^{-1}(x)=x^5-11\neq g(x)$

57. Which one of the following expressions is equal to $\frac{x-3}{x+1} \div \frac{x+1}{x-1}$?

A. $\frac{x^2-4x+3}{x^2+2x+1}$ **B.** $\frac{x-1}{x-3}$ **C.** $\frac{x-3}{x-1}$ **D.** $\frac{x^2+2x+1}{x^2-4x+3}$ **Ans. A**

Solution:

$$\frac{x-3}{x+1} \div \frac{x+1}{x-1} \Rightarrow \frac{x-3}{x+1} \times \frac{x-1}{x+1} = \frac{x^2-x-3x+3}{x^2+x+x+1}$$

$$\Rightarrow \frac{x^2-4x+3}{x^2+2x+1}$$

59. Consider the functions f,g,h and m defined by $f(x) = \sqrt{x-1}$, $g(x) = \log_5 x$, $h(x) = 2^{x+1}$, and $m(x) = (x+1)^2$.

Which one of the following is true?

A. The range of the function fog *is* $\{y \in \mathbb{R}: y = fog(x) > 0\}$.

B. The domain of the function fog is $\{x \in \mathbb{R}: x > \sqrt{7}\}$.

C. The range of hom *is* $\{y \in \mathbb{R}: y = (\text{hom})(x) > 0\}$.

D. The domain of the function hom *is* $\{x \in \mathbb{R}: x > 0\}$

Solution: **Ans. A**

$fog(x) = f(g(x)) = f(\log_5 x) = \sqrt{\log_5 x - 1}$

$\Rightarrow let\ y = \sqrt{\log_5 x - 1} \Rightarrow x = \sqrt{\log_5 y - 1}$

$\Rightarrow \log_5 y - 1 = x^2 \Rightarrow y - 1 = 5^{x^2} \Rightarrow y = 5^{x^2} + 1$

$\Rightarrow$ $Therefore, range\ fog(x) is\ (0, \infty)$

➢ The domain of fog(x) is $[5, \infty)$

$$\text{hom}(x) = h(m(x)) = h((x+1)^2) = 2^{(x+1)^2+1}$$

$$y = 2^{(x+1)^2+1} \Rightarrow x = 2^{(y+1)^2+1}$$

$$\Rightarrow (y+1)^2 + 1 = \log_2 x \Rightarrow (y+1)^2 = \log_2 x - 1$$

$$\Rightarrow y + 1 = \sqrt{\log_2 x - 1} \Rightarrow y = \sqrt{(\log_2 x) - 1} - 1$$

Therefore, Domain of $\text{hom}^{-1}(x)$ is $[2, \infty)$ = Rnge of hom(x)

60. What is the value of $|x| + 2x$ if $x \leq 0$? **Ans. D**

A. $-3x$ B. $3x$ C. $-x$ D. x

Solution:

➢ $|x| + 2x$, if $x < 0$, then $-x + 2x = x$

61. Which one of the following functions has NO vertical asymptote? **Ans. C**

A. $f(x) = \ln(x+1)$ B. $f(x) = \frac{x^2+1}{x^3+8}$

C. $f(x) = \frac{x^2-9}{x-3}$ D. $f(x) = \frac{x-1}{x^3-x}$

Solution:

➢ $f(x) = \ln x + 1$ has a vertical asymptote $x = -1$.

➢ The graph of $f(x) = \frac{x^2+1}{x^3+8} = \frac{x^2+1}{(x+2)(x^2-2x+4)}$ has a vertical asymptote $x = -2$

➢ The graph of $f(x) = \frac{x^2-9}{x-3} = \frac{(x+3)(x-3)}{x-3} = x + 3$ has no vertical asymptote.

➢ The graph of $\frac{x-1}{x^3-x} = \frac{x-1}{x(x-1)(x+1)} = \frac{1}{x(x+1)}$ has vertical asymptotes $x = 0$ and $x = -1$

62. If $f(x) = \frac{1}{e^x+1}$, then which one of the following is equal to $f^{-1}(x)$ for $0 < x < 1$

A. $\ln(1-x) - \ln x$ B. $e^x + 1$ C. $\ln(1 - \frac{1}{x})$ D. $\frac{1}{e^x+1}$

Solution: **Ans. A**

$$Let\ y = \frac{1}{e^x+1} \Rightarrow x = \frac{1}{e^y+1} \Rightarrow xe^y + x = 1 \Rightarrow xe^y = 1 - x$$

$$e^y = \frac{1-x}{x} \Rightarrow y = \ln\left(\frac{1-x}{x}\right) = \ln(1-x) - \ln x$$

63. Given $f(x) = \ln(x-1)$ and $g(x) = \sqrt{1-2x}$, which one of the following is the domain of fog? **Ans. C**

A. $\{x \in \mathbb{R}: x > 1\}$ B. $\left\{x \in \mathbb{R}: x \leq \frac{1}{2}\right\}$

C. $\{x \in \mathbb{R}: x < 0\}$ D. $\left\{x \in \mathbb{R}: x > \frac{1}{2}\right\}$

Solution:

$$fog(x) = f(g(x)) = f(\sqrt{1-2x}) = \ln\left((\sqrt{1-2x}) - 1\right)$$

$$\sqrt{1-2x} - 1 \geq 0 \Rightarrow \sqrt{1-2x} \geq 1 \Rightarrow 1 - 2x \geq 1$$

$$\Rightarrow -2x \geq 0 \Rightarrow x < 0$$

$$Therefore, Domain\ of\ fog(x)\ is\ (-\infty, 0)$$

64. The inverse of the function defined by $g(x) = \frac{2x}{x+3}$ *is equal to*:

A. $g^{-1}(x) = \frac{2x}{x-3}$ B. $g^{-1}(x) = \frac{-3x}{x-2}$

C. $g^{-1}(x) = -\frac{x+3}{2x}$ D. $g^{-1}(x) = -\frac{x+2}{3x}$

Solution: **Ans. B**

- $Let\ y = \frac{2x}{x+3} \Rightarrow x = \frac{2y}{y+3} \Rightarrow xy + 3x = 2y$
- $\Rightarrow 2y - xy = 3x \Rightarrow y = \frac{3x}{2-x}$

65. Which one of the following is NOT a rational Expression? **Ans. C**

A. $\frac{3x+4}{\sqrt{x^2+3}}$ B. $\frac{3x+6}{(x^2+2x+3)^{\frac{1}{2}}}$ C. $\frac{\sqrt{2}x}{\sqrt{(x^2+2)^2}}$ D. $\frac{\sqrt{3x+4}}{x+4}$

- A, B and D are not a rational expression. For A and B the denominators are not polynomial.
- In D the numerator is not a polynomial expression.

66. What is the universal set of a rational expression $\frac{2x^2-8x}{x^2-4}$?

A. $\mathbb{R}/\{2\}$ B. $\mathbb{R}/\{-2, 2\}$ C. $\mathbb{R}$ D. $\mathbb{R}/\{-2\}$

Solution:

- $\frac{2x^2-8x}{x^2-4} = \frac{2x(x-4)}{(x-2)(x+2)} \Rightarrow x - 2 = 0\ or\ x + 2 = 0$
- $\Rightarrow x = 2\ or\ x = -2$
- Therefore, the domain of $\frac{2x^2-8x}{x^2-4}$ *is* $\mathbb{R}/\{-2, 2\}$

67. If $\frac{p(x)}{q(x)}=\frac{2x+4}{x^2-9}$ and $\frac{r(x)}{s(x)}=\frac{x^2+5x+6}{x^2-4}$, then $\frac{p(x)}{q(x)}\div\frac{r(x)}{s(x)}=$ ___

Ans. B

A. $\frac{x^2-3x+8}{x^2-9}$ B. $\frac{2x^2-8}{x^3+3x^2-9x+27}$ C. $\frac{2x^2-3x+8}{9x^2+6}$ D. $\frac{2x}{x+3}$

Solution:

$$\frac{p(x)}{q(x)}\div\frac{r(x)}{s(x)}=\frac{2x+4}{x^2-9}\div\frac{x^2+5x+6}{x^2-4}=\frac{2(x+2)}{(x-3)(x+3)}\times\frac{(x+2)(x-2)}{(x+2)(x+3)}=\frac{2x^2-8}{x^3+3x^2-9x-27}$$

68. The simplified form of $\frac{x^3-64}{x^2-4x}$ is equal to _____

Ans. B

A. $\frac{x}{x^2+4x+16}$ B. $\frac{x^2+4x+16}{x}$ C. $\frac{x^2+4x+8}{x}$ D. $\frac{x^2+2x+16}{x}$

Solution:

$$\frac{x^3-64}{x^2-4x}=\frac{(x-4)(x^2+4x+16)}{x(x-4)}=\frac{x^2+4x+16}{x}$$

69. If $\frac{7x+6}{x^2+6x+8}=\frac{A}{x+2}+\frac{B}{x+4}$, then what is the value of A and B?

A. 7 and 8 B. -4 and 11 C. 2 and 9 D. -2 and 11

Solution:

Ans. B

$$\frac{7x+6}{x^2+6x+8}=\frac{A}{x+2}+\frac{B}{x+4}$$

$$\Rightarrow\frac{7x+6}{(x+2)(x+4)}(x+2)=\frac{A}{x+2}(x+2)+\frac{B}{x+4}(x+2)$$

$$\Rightarrow\frac{7x+6}{x+4}=A+\frac{B}{x+4}(x+2)\text{ let solve for }x=-2$$

$$\Rightarrow\frac{7(-2)+6}{-2+4}=A+\frac{B}{-2+4}(-2+2)$$

$$\Rightarrow\frac{-14+6}{2}=A+0\Rightarrow A=-4$$

In the same way $\frac{7x+6}{x+2}=B\Rightarrow B=\frac{7(-4)+6}{-4+2}\Rightarrow B=\frac{-28+6}{-2}=\frac{-22}{-2}=11$

70. If $\frac{x^2+4x+22}{(x-1)(x+2)^2}=\frac{A}{x-1}+\frac{B}{x+2}+\frac{C}{(x+2)^2}$, then what is the value of A, B and C respectively?

Ans. B

A. $2-,3,-6$ B. $3,-2,-6$ C. $-6,3,-2$ D. $3,-6,2$

Solution: $\frac{x^2+4x+22}{(x-1)(x+2)^2} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2} = \frac{A(x+2)^2+B(x-1)(x+2)+C(x-1)}{(x-1)(x+2)^2}$

$$\Rightarrow \frac{Ax^2 + 4Ax + 4A + Bx^2 + Bx - 2B + Cx - c}{(x-1)(x+2)^2}$$

$$A + B = 1 \qquad \Rightarrow A = 1 - B \ldots\ldots (1$$

$$4A + B + C = 4 \qquad \Rightarrow -3B + C = 0 \ldots\ldots (2)$$

$$4A - 2B - C = 22 \qquad \Rightarrow -6B - C = 18 \ldots\ldots (3)$$

From equation (2) and (3) we have $\begin{cases} -3B + C = 0 \\ -6B - C = 18 \end{cases}$

Add the two equation and solve for B

$$-9B = 18 \Rightarrow B = -2, A = 3 \text{ and } C = -6$$

71. If $\frac{x^2+5x+6}{(x+1)(x-2)(x^2+2)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{Cx+D}{x^2+2}$, then what is the value of A, B, C and D respectively? **Ans. C**

A. $\frac{22}{27}, \frac{-31}{27}, \frac{-14}{27}, \frac{-8}{27}$

B. $\frac{-14}{27}, \frac{22}{27}, \frac{-8}{27}, \frac{-31}{27}$

C. $\frac{-2}{9}, \frac{10}{9}, \frac{-8}{9}, \frac{-13}{9}$

D. $\frac{27}{22}, \frac{31}{22}, \frac{14}{27}, \frac{27}{8}$

Solution

- By cover up method It is possible to find A and B
- $A = \frac{(-1)^2+5(-1)+6}{(-1-2)((-1)^2+2)} = \frac{1-5+6}{(-3)(3)} = \frac{2}{-9} = -\frac{2}{9}$
- $B = \frac{(2)^2+5(2)+6}{(2+1)((2)^2+2)} = \frac{4+10+6}{(3)(6)} = \frac{20}{18} = \frac{10}{9}$
- To find C and D
- Multiply both sides by LCM
- $x^2 + 5x + 6 = A(x-2)(x^2+2) + B(x+1)(x^2+2) + (Cx+D)(x+1)(x-2)$
- $x^2 + 5x + 6 = A(x^3 + 2x - 2x^2 - 4) + B(x^3 + 2x + x^2 + 2) + (Cx + D)(x^2 - x - 2)$
- $x^2 + 5x + 6 = A(x^3 + 2x - 2x^2 - 4) + B(x^3 + 2x + x^2 + 2) + (cx^3 - cx^2 - 2cx + Dx^2 - Dx - 2D)$
- $0x^3 + x^2 + 5x + 6 = (A + B + C)x^3 + (-2A + B - C + D)x^2 + (2A + 2B - 2C - D)x$
- $A + B + C = 0 \text{ and } -2A + B - C + D = 1$
- $\frac{-2}{9} + \frac{10}{9} + C = 0 \Rightarrow c = \frac{-8}{9}$
- $-2(\frac{-2}{9}) + \frac{10}{9} - (\frac{-8}{9}) + D = 1$
- $\frac{4}{9} + \frac{10}{9} + \frac{8}{9} + D = 1 \Rightarrow D = 1 - \frac{22}{9} = -\frac{13}{9}$

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72. If $\frac{x^2+14}{(x+2)(x-1)^2} = \frac{A}{x+2} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$, then what are the values of A, B and C respectively?

A. $2, -2, -5$ B. $2, -1, 5$ C. $1, -1, 3$ D. $3, -2, 6$

Solution

Ans. B

- By cover up method It is possible to find A and C
- $letting\ x = -2 \quad A = \frac{(-2)^2+14}{(-2-1)^2} = \frac{4+14}{9} = \frac{18}{9} = 2$
- $letting\ x = 1 \quad C = \frac{(1)^2+14}{(1+2)} = \frac{1+14}{3} = \frac{15}{3} = 5$
- To find B
- Multiply both sides by LCM
- $x^2 + 14 = A(x-1)^2 + B(x+2)(x-1) + C(x+2)$
- $x^2 + 14 = A(x^2 - 2x + 1) + B(x^2 + x - 2) + C(x+2)$
- $x^2 + 14 = (A+B)x^2 + (-2A + B + C)x + A - 2B + 2C$
- $A + B = 1 \Rightarrow 2 + B = 1 \Rightarrow B = -1$

73. What is the partial fraction decomposition of $\frac{x^2+x+1}{(x+2)(x^2+1)}$?

A. $\frac{2}{3(x+2)} + \frac{2x+1}{3(x^2+1)}$

B. $\frac{3}{5(x+2)} + \frac{2x+1}{5(x^2+1)}$

C. $\frac{5}{3(x+2)} + \frac{2x+1}{3(x^2+1)}$

D. $\frac{2}{5(x+2)} + \frac{3x+1}{5(x^2+1)}$

Solution

Ans. B

- $\frac{x^2+x+1}{(x+2)(x^2+2)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+2}$
- By cover up method It is possible to find A
- $letting\ x = -2 \quad A = \frac{(-2)^2+(-2)+1}{(-2)^2+1} = \frac{4-2+1}{4+1} = \frac{3}{5}$
- To find B and C
- Multiply both sides by LCM
- $x^2 + x + 1 = A(x^2+1) + (Bx + C)(x+2)$
- $x^2 + x + 1 = A(x^2+1) + (Bx^2 + 2Bx + Cx + 2C)$
- $x^2 + +x + 1 = (A+B)x^2 + (2B + C)x + A + 2C$
- $A + B = 1 \Rightarrow \frac{3}{5} + B = 1 \Rightarrow B = 1 - \frac{3}{5} = \frac{2}{5}$

- $2B + C = 1 \Rightarrow \frac{4}{5} + C = 1 \Rightarrow c = 1 - \frac{4}{5} = \frac{1}{5}$

74. Which one of the following represents the solution set for the equation: $\frac{1}{x-2} = \frac{3}{x+2} - \frac{6x}{x^2-4}$

A. $\left\{\frac{-1}{2}, \frac{1}{2}\right\}$ B. $\{-2\}$ C. $\{-4, 2\}$ D . $\emptyset$

Solution

Ans. D.

- Domain :$x \neq 2, -2$
- B and C cannot be the answer
- $\frac{1}{x-2} = \frac{3}{x+2} - \frac{6x}{(x-2)(x+2)}$
- $1(x+2) = 3(x-2) - 6x(1)$ multiplying both sides by LCM
- $x + 2 = 3x - 6 - 6x$
- $x + 2 = -3x - 6$
- $x + 3x = -6 - 2$ collecting like terms
- $4x = -8$
- $x = -2$ but -2 *is not in the domain*

75. Which one of the following is a possible solution for $\frac{2x^2-5x+2}{2x^2-x} = 1 - \frac{1+x}{2x-1}$?

A. 0 B. 2 C. -1 D. $\frac{1}{2}$

Solution

Ans. B

- Domain: $x \neq 0, \frac{1}{2}$
- A and D cannot be the answer
- $\frac{2x^2-5x+2}{x(2x-1)} = 1 - \frac{1+x}{2x-1}$ factorizing denominators
- $2x^2 - 5x + 2 = 1\big(x(2x-1)\big) - x(1+x)$ Mult. Both sides by LCM
- $2x^2 - 5x + 2 = 1\big(2x^2 - x\big) - x - x^2$
- $2x^2 - 5x + 2 = 2x^2 - x - x - x^2$
- $x^2 - 3x + 2 = 0$ collecting like terms
- $(x-2)(x-1) = 0$
- $x = 2$ *or* $x = 1$

76. Suppose $\frac{x-8}{x^3+4x} = g(x) - \frac{2}{x}$ and $x \neq 0$. **Ans. B**

Which of the following is equal to $g(x)$?

A. $\frac{2x-1}{x^2+4}$ B. $\frac{2x+1}{x^2+4}$ C. $\frac{x+2}{x^2+4}$ D. $\frac{x-2}{x^2+4}$

Solution: $\frac{x-8}{x^3+4x} = g(x) - \frac{2}{x} \Rightarrow g(x) = \frac{x-8}{x^3+4x} + \frac{2}{x}$

$$\Rightarrow \frac{x-8+2(x^2+4)}{x(x^2+4)} = \frac{x+2x^2}{x(x^2+4)} = \frac{2x+1}{x^2+4}$$

77. Which of the following is the solution set of the inequality $\frac{1}{1-\frac{1}{x}} \leq \frac{x}{x^2-1}$?

Ans. C

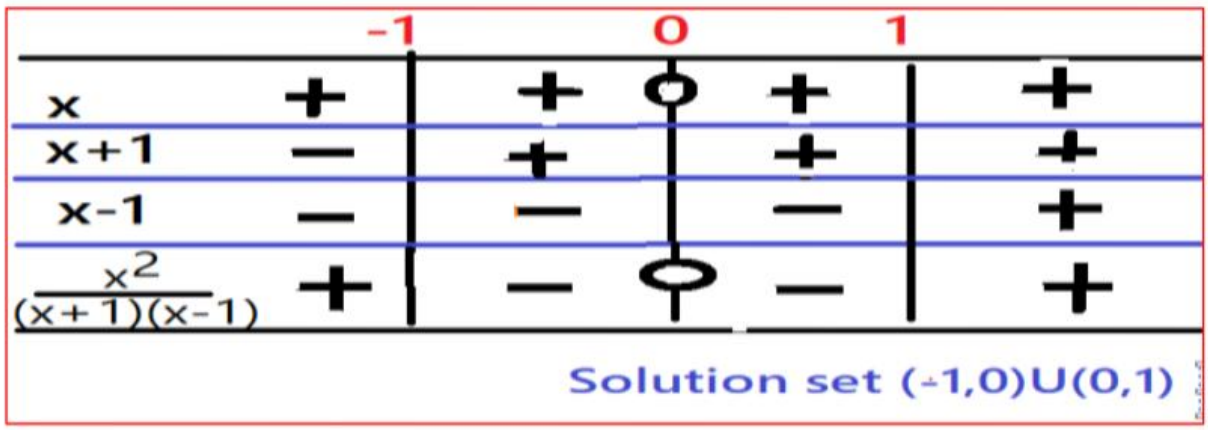
A. $(-1,1)$ B. $[-1,0) \cup (0,-1]$

C. $(-1,0) \cup (0,1)$ D. $(-\infty,-1) \cup (1,\infty)$

Solution: $\frac{1}{1-\frac{1}{x}} \leq \frac{x}{x^2-1} \Rightarrow \frac{1}{\frac{x-1}{x}} \leq \frac{x}{x^2-1} \Rightarrow \frac{x}{x-1} - \frac{x}{x^2-1} \leq 0$

$$\Rightarrow \frac{x(x+1)-x}{x^2-1} \leq 0 \Rightarrow \frac{x^2+x-x}{x^2-1} \leq 0 \Rightarrow \frac{x^2}{x^2-1} \leq 0$$

$$x-1=0 \; or \; x+1=0 \Rightarrow x=1 \; or \; x=-1$$



78. Which of the following is the universal set of the expression $\frac{1}{x^2}+\frac{x+1}{x^2+x}$? **Ans. D**

A. $\mathbb{R}/\{0,1\}$ B. $\mathbb{R}/\{0\}$

C. $\mathbb{R}/\{-1,0,1\}$ D. $\mathbb{R}/\{0,-1\}$

Solution :

- The domain of rational function is the set of real number
- except the value tha makes the denominator zero
- $\frac{1}{x^2}+\frac{x+1}{x^2+x} \Rightarrow x^2=0 \text{ or } x^2+x=0 \Rightarrow x=0 \text{ or } x+1=0$
- $x=0 \text{ or } x=-1$
- Therefore, Domain of the expression is $\mathbb{R}/\{0,-1\}$

79. Let $f(x)=\frac{x^2-x}{2x^2-x-1}$, which of the following is NOT true about f and its graph? **Ans. C**

A. $x=-\frac{1}{2}$ is a vertical asymptote

B. Thegraph of f has a hole at $(1,\frac{1}{3})$

C. f is an even function.

D. $y=\frac{1}{2}$ is a horizontal asymptote

Solution: $f(x)=\frac{x^2-x}{2x^2-x-1}, \Rightarrow \frac{x(x-1)}{2x^2-2x+x-1} \Rightarrow \frac{x(x-1)}{2x(x-1)+1(x-1)}$

$$\Rightarrow \frac{x(x-1)}{(2x+1)(x-1)} \Rightarrow \frac{x}{2x+1}$$

- The vertical asymptote is $x=-\frac{1}{2}$ and the graph of f(x) has a hole at $\left(1,\frac{1}{3}\right)$
- The horizontal asymptote is $y=\frac{1}{2}$ and $f(x)$ $is\ not\ an\ even\ function.$

80. Which one of the following is true about the graph of $f(x)=\frac{2x^3+2x^2+3x}{x^2+x}$?

A. The graph has y – intercept at (0,3). **Ans. B**

B. The vertical asymptote of the graph is only x=-1 and oblique asymptote is y=2x.

C. The graph has at least one x- intercept.

D. The vertical asymptotes of the graph are at x=0 and x=-1 but it has no horizontal asymptote.

Solution

✓ The graph has a hole at $x = 0$ and has no y-intercept

$$\frac{2x^3+2x^2+3x}{x^2+x} = \frac{x(2x^2+2x+3)}{x(x+1)} = \frac{2x^2+2x+3}{x+1}$$

✓ The graph has a hole at $\left(0, \frac{2(0)^2+2(0)+3}{0+1}\right) = (0,3)$

✓ $p(-1) \neq 0 \; and \; q(-1) = 0 \Rightarrow x = -1 \; is \; V.$

Since degree of numerator is greater than degree of denominator by one, the graph has O.A and it is obtained by dividing numerator and denominator.

$$\frac{2x^3+2x^2+3x}{x^2+x} = 2x + \frac{3x}{x^2+x}$$

Therefore the line $y = 2x$ is Oblique asymptote.

✓ $f(x) = \frac{2x^3+2x^2+3x}{x^2+x} = \frac{x(2x^2+2x+3)}{x(x+1)} = \frac{2x^2+2x+3}{x+1}$

$x - intercept \; means \; y = 0 = f(x)$

$$0 = \frac{2x^2+2x+3}{x+1}$$

$2x^2 + 2x + 3 = 0 \Rightarrow b^2 - 4ac < 0$

$\Rightarrow$ the graph has no x-intercept

✓ The graph has no Vertical Asymptote at x = 0

81. The value(s) of x where the graph of $y = \frac{x^2-1}{x^3}$ crosses its horizontal asymptote is (are) **Ans. C**

A. $x = -2$ B. $x = 0$

C. $x = -1$ and $x = 1$ D. $x = -\sqrt{2}$ and $x = 1 + \sqrt{2}$

Solution

- Degree of numerator is less than degree of numerator

$\Rightarrow y = 0$ is H. A

$\Rightarrow 0 = \frac{x^2-1}{x^3}$

$\Rightarrow x^2 - 1 = 0$

$\Rightarrow x = 1$ or $x = -1$

hence the answer is C

82. Which of the following is true about the rational function $f(x) = \frac{P(x)}{q(x)}$?

A. The domain of f(x) is the set of real numbers where q(x) is well defined and non-zero.

B. If the domain of f(x) is $\mathbb{R}$, $then\ f\ is\ polynomial.$

C. The range of f is always $\mathbb{R}$.

D. $p(x) is\ divisible\ by\ q(x).$

83. Which one of the following is a possible solution for $\frac{2x^2-5x+2}{2x^2-x} = 1 - \frac{1+x}{2x-1}$?

$A.0$ $B.2$ $C.-1$ $D.\frac{1}{2}$ Ans. B

Solution

- Domain: $x \neq 0, \frac{1}{2} \Rightarrow$ A and D cannot be the answer
- $\frac{2x^2-5x+2}{x(2x-1)} = 1 - \frac{1+x}{2x-1}$ factorizing denominators
- $2x^2 - 5x + 2 = 1\big(x(2x-1)\big) - x(1+x)$ Mult. Both sides by LCM
- $2x^2 - 5x + 2 = 1\big(2x^2 - x\big) - x - x^2$
- $2x^2 - 5x + 2 = 2x^2 - x - x - x^2$
- $x^2 - 3x + 2 = 0$ collecting like terms
- $(x-2)(x-1) = 0$
- $x = 2\ or\ x = 1$

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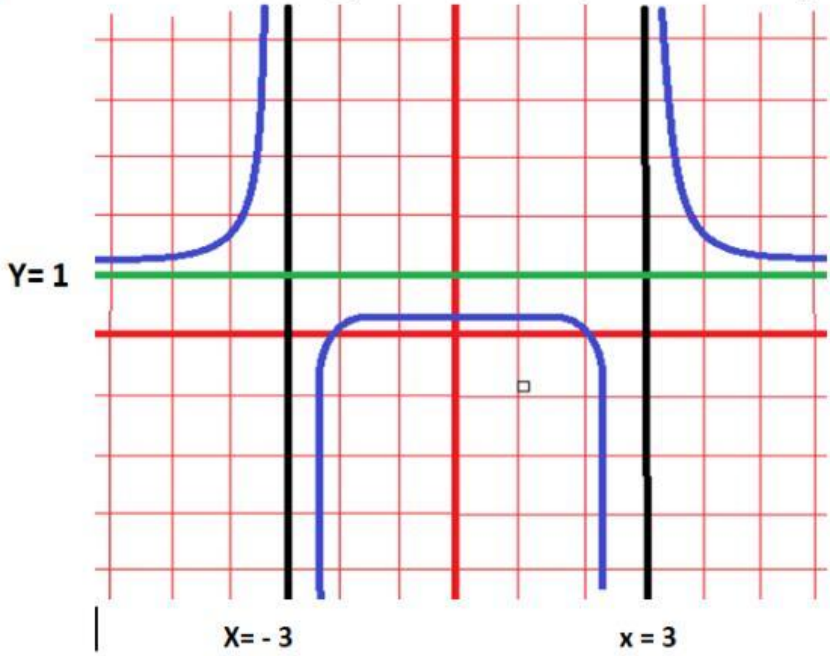
84. Which of the following function is represents the graph given below?



A. $f(x) = \frac{x^2 - 4}{x^2 + 9}$

B. $f(x) = \frac{x^2 - 9}{x^2 - 4}$

C. $f(x) = \frac{x^2 - 4}{x^2 - 9}$

D. $f(x) = \frac{x + 2}{x^2 - 9}$

Solution: **Ans. C**

- The vertical Asymptotes of the graph are $x = 3$ *and* $x = -3$.
- The horizontal Asymptote is $y = 1$.
- The $x - interceptes\ are\ (-2, 0) and\ (2, 0)$.
- The y-intercept is $(0, \frac{9}{4})$.
- Therefore, the function that satisfy this condition is $f(x) = \frac{x^2-4}{x^2-9}$.

85. Which one of the following is rational expression? **Ans. D**

A. $\log_2 x$ B. $\sqrt{\frac{x-2}{x+2}}$ C. $\frac{x+1}{\sqrt{x^2}}$ D. $1 + \frac{x^2+x}{x^2-x}$

Solution:

- $\log_2 x$ is not a rational expression.
- $\sqrt{x-2}$ *is not a polynomial function.*
- $\sqrt{x^2}$ *is not a polynomial function.*
- $1, x^2 + x$ *and* $x^2 - x$ *are polynomial.*

86. Which one of the following is NOT rational function? **Ans. C**

A. $f(x)=\frac{\sqrt{2}}{(\pi+1)^{\frac{1}{3}}}$ B. $f(x)=\frac{sin30^0+lne-3^{\frac{1}{5}}x}{|x^2+2|}$

C. $f(x)=\frac{2x^3-\sqrt{x^2}}{x+2}$ D. $f(x)=2+\frac{x^{-3}}{6}$

Solution:

- $f(x)=\frac{\sqrt{2}}{(\pi+1)^{\frac{1}{2}}}$ is a real number so it is rational expression.
- $f(x)=\frac{\sin30^0+\ln e^{-\frac{1}{35x}}}{|x^2+2|}=\frac{\frac{1}{2}-\frac{1}{35x}}{x^2+2}=\frac{35x-2}{70x(x^2+2)}$ it is a rational expression.
- $f(x)=\frac{2x^3-\sqrt{x^2}}{x+2}$, $\sqrt{x^2}$ is not apolynomial means $f(x)$ *is not rational expression.*
- $f(x)=\frac{12+x^{-3}}{6}=\frac{12x^3+1}{6x^3}$ is a rational expression.

87. Which one of the following is NOT rational expression? **Ans. B**

A. $\frac{x^3-2x^2}{x-2}$ B. $\frac{lnx+3}{3^x+1}$ C. $\frac{(x-1)(x^2+1)}{x+3}$ D. $\frac{9x^2-4}{9x^2+9x-10}$

Solution:

- A, C and D are rational expression where, B is a logarithmic expression.

88. If $p(x)=x-4, q(x)=x^2+8$ *and* $r(x)=x^2-2x-8$, then which one of the following is the most simplified form of $\frac{x}{p(x)}-\frac{q(x)}{r(x)}$?

A. $\frac{x}{x-4}-\frac{x^2+8}{x^2-2x-8}$ B. $\frac{8x}{x-4}$ C. $\frac{2}{x+2}$ D. $\frac{2x+8}{x^2-2x-8}$

Solution: **Ans. C**

$$\frac{x}{p(x)}-\frac{q(x)}{r(x)}=\frac{x}{x-4}-\frac{x^2+8}{x^2-2x-8}=\frac{x(x+2)-x^2-8}{x^2-2x-8}$$

$$=\frac{x^2+2x-x^2-8}{(x-4)(x+2)}=\frac{2x-8}{(x-4)(x+2)}=\frac{2}{x+2}$$

89. Which one of the following is a simplified form of $\frac{x-\frac{1}{x}}{1+\frac{x}{1-x}} \div \frac{x^2-2x+1}{x}$

A. $x+1$ B. $x-1$ C. $1-x$ D. $-x-1$

Solution: $\frac{x-\frac{1}{x}}{1+\frac{x}{1-x}} \div \frac{x^2-2x+1}{x} = \frac{\frac{x^2-1}{x}}{\frac{1-x+x}{1-x}} \times \frac{x}{(x-1)^2}$

$$= \frac{(x-1)(x+1)(1-x)x}{x(x-1)^2} = -x-1$$

Ans. D

90. Simplified form of $\frac{4-(a-2)^2}{a^4+2a^2+1} \div \frac{1-\frac{1}{a^2+1}}{a}$

A. $\frac{4-a}{a}$ B. $-\frac{1}{a^2(a^2+1)}$ C. $\frac{a}{a^2+1}$ D. $\frac{4-a}{a^2+1}$

Solution: $\frac{4-(a-2)^2}{a^4+2a^2+1} \div \frac{a^2+1-1}{a(a^2+1)} = \frac{4-(a^2-4a+4)}{a^4+2a^2+1} \times \frac{a(a^2+1)}{a^2}$

$$= \frac{4a-a^2}{(a^2+1)a} = \frac{4-a}{a^2+1}$$

Ans. D

91. Which one of the following is the simplest form of $\frac{4x^4-64}{2-x}$ $for\ x \neq 2$?

A. $4(x-2)(x^2-4)$ C. $4(x+2)(x^2+4)$

B. $-4(x+2)(x^2+4)$ D. $-4(x+2)(x^2-4)$

Solution:

$$\frac{4x^4-64}{2-x} = \frac{4(x^4-16)}{2-x} = \frac{-4(x^2-4)(x^2+4)}{x-2}$$

$$= \frac{-4(x-2)(x+2)(x^2+4)}{x-2}$$

$$= -4(x+2)(x^2+4)$$

Ans. B

92. What is the simplified form of $\frac{a^{-1}b^{-1}}{a^{-3}-b^{-3}}$?

A. $\frac{a^2b^2}{b^3-a^3}$ B. $\frac{a^2b^2}{b^2-a^2}$ C. $\frac{a^3-b^3}{ab}$ D. $\frac{a^3-3}{a-b}$

Ans. A

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Solution: $\dfrac{a^{-1}b^{-1}}{a^{-3}-b^{-3}} = \dfrac{\frac{1}{ab}}{\frac{b^3-a^3}{a^3b^3}} = \dfrac{a^2b^2}{b^3-a^3}$

93. Which one of the following is the universal set of the expression $\frac{1}{x^2} + \frac{x+1}{x^2+x}$?

A. $\mathbb{R}\backslash\{0\}$ **B.** $\mathbb{R}\backslash\{0,1\}$ **C.** $\mathbb{R}\backslash\{0,-1\}$ **D.** $\mathbb{R}\backslash\{0,-1,1\}$

Solution: $x^2 + x = 0 \Rightarrow x(x+1) = 0 \Rightarrow x = 0 \text{ or } x = -1$

Therefore, Domain is $\mathbb{R}/\{-1,0\}$ **Ans. C**

94. What are the respective value of A, B and C so that $\frac{x-1}{x^3+x} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$?

A. $\{-1,1.1\}$ **B.** $\{1,-2,0\}$ **C.** $\{-1,1,2\}$ **D.** $\{3,1,2\}$

Solution: **Ans. A**

$$\frac{x-1}{x^3+x} = \frac{x-1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1} = \frac{A(x^2+1)+Bx^2+Cx}{x(x^2+1)}$$

$$= \frac{Ax^2+A+Bx^2+Cx}{x(x^2+1)}$$

$$\Leftrightarrow A + B = 0 \Rightarrow C = 1, A = -1 \text{ and } B = 1$$

Therefore, $\frac{x-1}{x^3+x} = \frac{-1}{x} + \frac{-x+1}{x^2+1}$

95. If $\frac{x^2}{(x+1)(x+2)^2} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$, then what is the value of $4A + B - 10C$?

A. -15 B. -44 C. -36 D. 12

Solution: **ANS. B**

$$\frac{x^2}{(x+1)(x+2)^2} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

$$= \frac{A(x^2+4x+4) + B(x+1)(x+2) + C(x+1)}{(x+1)(x+2)^2}$$

$$= \frac{Ax^2+4Ax+4A+Bx^2+3Bx+2B+Cx+C}{(x+1)(x+2)^2}$$

$$A + B = 1 \Rightarrow A = 1 - B,$$

$$4A + 3B + C = 0$$

$$\Rightarrow 4(1-B)+3B+C=0 \Rightarrow -B+C=-4,$$
$$4A+2B+C=0 \Rightarrow 4(1-B)+2B+C=0$$
$$\mathbf{-2B+C=-4,\quad -B+C=-4}$$
$$\begin{cases} -B+C=-4 \\ -2B+C=-4 \end{cases} \Rightarrow B=0, C=-4 \; AND \; A=1$$
$$4A+B-10C=4(1)+0+10(4)=\mathbf{44}$$

96. Which one of the following represent the solution set of the equation $\frac{1}{x-2}=\frac{3}{x+2}-\frac{6x}{x^2-4}$?

A. $\{\frac{-1}{2},\frac{1}{2}\}$ B. {-4,2} C.{-2} D. $\emptyset$

Solution: $\frac{1}{x-2}=\frac{3}{x+2}-\frac{6x}{x^2-4} \Rightarrow \frac{1}{x-2}-\frac{3}{x+2}+\frac{6x}{x^2-4}=0$ **Ans. D**

$$\Rightarrow \frac{x+2-3(x-2)+6x}{x^2-4}=0$$
$$\Rightarrow \frac{x+2-3x+6+6x}{x^2-4}=0 \Rightarrow 4x+8=0 \Rightarrow x=-2.$$

➢ **Therefore, S. S = $\emptyset$**

97. What is the solution of $1-\frac{5}{x^2+4}=\frac{x-1}{x^2-x}-\frac{1}{x}$? **Ans. C**

A. $\{1,-1\}$ B. $\{2\}$ C. $\{-1\}$ D. $\{1,2,-1\}$

Solution:

$$\frac{x^2+4-5}{x^2+4}=\frac{x-1-(x-1)}{x(x-1)} \Rightarrow \frac{x^2-1}{x^2+4}=0$$
$$\Rightarrow (x-1)(x+1)=0 \Rightarrow x-1=0 \; or \; x+1=0$$
$$\Rightarrow x=1 \; or \; x=-1$$

➢ **Therefore, the solution set is {- 1 }**

98. What is the solution set of $\frac{2}{x}-\frac{x-2}{x^2-2x}=1-\frac{2x-2}{3x-2}$? **Ans. D**

A. $\{1,-2\}$ B. $\{1,2\}$ C. $\{-1\}$ D. $\{1\}$

Solution:

$$\frac{2}{x}-\frac{x-2}{x^2-2x}=1-\frac{2x-2}{3x-2} \Rightarrow \frac{2x-4-x+2}{x(x-2)}=\frac{3x-2-2x+2}{3x-2}$$

$$\frac{x-2}{x(x-2)}=\frac{x}{3x-2}\Rightarrow\frac{1}{x}-\frac{x}{3x-2}=0\Rightarrow\frac{3x-2-x^2}{x(3x-2)}=0$$

$$x^2-3x+2=0\Rightarrow x^2-2x-x+2=0$$

$$\Rightarrow x(x-2)-1(x-2)=0\Rightarrow x=1\ or\ x=2$$

Therefore, the solution set of the equation is $\{1\}$.

99. What is the solution set of $1-\frac{5}{x^2+4}=\frac{x-1}{x^2-x}-\frac{1}{x}$? **Ans. C**

A. $\{1,-1\}$ **B.** $\{2\}$ **C.** $\{-1\}$ **D.** $\{1,2,-1\}$

100. What is the solution set of $\frac{1-\frac{1}{x}}{1-\frac{1}{x^2}}=3x^2-\frac{x}{1+\frac{1}{x}}$? **Ans. B**

A. $\left\{-1,\frac{1}{3}\right\}$ **B.** $\left\{\frac{1}{3}\right\}$ **C.** $\left\{3,\frac{1}{3}\right\}$ **D.** $\left\{-\frac{1}{3}\right\}$

Solution: $\frac{1-\frac{1}{x}}{1-\frac{1}{x^2}}=3x^2-\frac{x}{1+\frac{1}{x}}\Rightarrow\frac{\frac{x-1}{x}}{\frac{x^2-1}{x^2}}=3x^2-\frac{x}{\frac{x+1}{x}}$

$$\Rightarrow\frac{x(x-1)}{(x-1)(x+1)}=3x^2-\frac{x^2}{x+1}$$

$$\Rightarrow\frac{x}{x+1}=\frac{3x^3+3x^2-x^2}{x+1}$$

$$\Rightarrow 3x^3+2x^2-x=0$$

$$\Rightarrow x(3x^2+2x-1)=0$$

$$\Rightarrow x(3x-1)(x+1)=0$$

$$\Rightarrow x=0\ or\ 3x-1=0\ or\ x+1=0$$

$$\Rightarrow x=0\ or\ x=-1\ or\ x=\frac{1}{3}$$

Therefore, the solution set is $\left\{\frac{1}{3}\right\}$

101. What is the solution set of $\frac{1}{1+\frac{1}{x}}-\frac{1}{1-\frac{1}{x}}=\frac{x+\frac{1}{x}}{x-\frac{1}{x}}$? **Ans. A**

A. $\{\}$ **B.** $\{-1\}$ **C.** $\{1\}$ **D.** $\{1,-1\}$

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Solution: $\frac{1}{1+\frac{1}{x}} - \frac{1}{1-\frac{1}{x}} = \frac{x+\frac{1}{x}}{x-\frac{1}{x}} \Rightarrow \frac{1}{\frac{x+1}{x}} - \frac{1}{\frac{x-1}{x}} = \frac{\frac{x^2+1}{x}}{\frac{x^2-1}{x}}$

$$\Rightarrow \frac{x}{x+1} - \frac{x}{x-1} = \frac{x^2+1}{x^2-1}$$

$$\Rightarrow \frac{x(x-1)-x(x+1)}{(x+1)(x-1)} - \frac{x^2+1}{x^2-1} = 0$$

$$\Rightarrow \frac{x^2-x-x^2-x-x^2-1}{(x-1)(x+1)} = 0$$

$$\Rightarrow \frac{-x^2-2x-1}{(x-1)(x+1)} = 0$$

$$\Rightarrow \frac{-x^2-x-x-1}{(x-1)(x+1)} = 0$$

$$\Rightarrow \frac{-x(x+1)-(x+1)}{(x-1)(x+1)} = 0$$

$$\Rightarrow \frac{-(x+1)(x+1)}{(x-1)(x+1)} = 0$$

$$\Rightarrow \frac{-(x+1)}{x-1} = 0 \quad \Rightarrow -x-1=0 \Rightarrow x=-1$$

- **Domain of the equation is $\mathbb{R}/\{-1,0,1\}$**
- **Therefore, the solution set is empty set.**

102. What is the solution set of $\frac{3}{x+1} - \frac{1}{x-1} = \frac{2}{x}$? **Ans. B**

A $\left\{-\frac{1}{2}\right\}$ B. $\left\{\frac{1}{2}\right\}$ C. $\{1\}$ D. $\{-1\}$

Solution:

- $\frac{3}{x+1} - \frac{1}{x-1} = \frac{2}{x} \Rightarrow \frac{3}{x+1} - \frac{1}{x-1} - \frac{2}{x} = 0$
- $\frac{3x(x-1)-x(x+1)-2(x+1)(x-1)}{x(x+1)(x-1)} = 0$

- $\frac{3x^2-3x-x^2-x-2(x^2-x+x-1)}{x(x+1)(x-1)} = 0$
- $2x^2 - 4x - 2x^2 + 2 = 0$
- $-4x + 2 = 0$
- $x = \frac{1}{2}$

103. A man can drive $25kms$ at an average speed of x km/hr. if this speed were increased by 5 $\frac{km}{hr}$ it would complete the same journey in 15minutes less time. What is the original speed?

A. $35\frac{km}{hr}$ B.$22\frac{km}{hr}$ C.$20\frac{km}{hr}$ D.$25\frac{km}{hr}$

Solution: $\frac{25\text{km}}{\frac{\text{xkm}}{\text{hr}}} = \frac{25}{\text{x}}\text{hr} \Rightarrow \frac{25\text{km}}{(\text{x}+5)\text{km/hr}} = \frac{25}{\text{x}}\text{hr} - \frac{1}{4}\text{hr}$

$$\Rightarrow \frac{25}{x+5}hr = \frac{100-x}{4x}hr \Rightarrow \frac{25}{x+5} - \frac{100-x}{4x} = 0$$

$$\Rightarrow \frac{100\text{x} - 100\text{x} + \text{x}^2 + 5\text{x} - 500}{4\text{x}(\text{x}+5)} = 0$$

$$\Rightarrow \quad \text{x}^2 + 5\text{x} - 500 = 0$$

$$\Rightarrow \quad \text{x} = \frac{-5 \pm \sqrt{25 + 4(500)}}{2}$$

$$\Rightarrow \text{x} = \frac{-5 \pm 45}{2}, \text{x} = \frac{-50}{2}\text{hr or x} = \frac{40}{2}\text{km/hr} = 20\text{km/hr}$$

104. The ratio of the area of the triangle with base 6cm to the area of the rectangle with base 3cm is 3:4. If the height of the rectangle is 2cm more than the height of the triangle, what is the area of the triangle?

A. $18cm^2$ B. $24cm^2$ C.$6cm^2$ D.12cm^2 **Ans. A**

Solution: $A_t = \frac{1}{2}bh_t \quad A_r = bh_r \Rightarrow h_r = h_t + 2$

$$\Rightarrow \frac{A_t}{A_r} = \frac{3}{4} \Rightarrow \frac{\frac{1}{2}b_t h_t}{b_r h_r} = \frac{3}{4} \Rightarrow \frac{b_t h_t}{2 \times b_r(h_t+2)} = \frac{3}{4}$$

$$\Rightarrow \frac{6h_t}{6(h_t+2)} = \frac{3}{4} \Rightarrow \frac{h_t}{h_t+2} = \frac{3}{4} \Rightarrow 3h_t + 6 = 4h_t$$

$$\Rightarrow h_t = 6cm$$

$$\Rightarrow A_t = \frac{1}{2}b_t h_t = \frac{1}{2} \times 6cm \times 6cm = 18cm^2$$

105. Two workers, Lami and Bayisa working together can complete a job in 6 days. Lami works twice as fast as Bayisa. How many days would it take each working alone to complete the job?

A. Lami in 20 days, Bayisa in 10 days
B. Lami in 24 days, Bayisa in 12 days
C. Lami in 12 days , Bayisa in 6 days
D. Lami in 9 days, Bayisa in 18days

Solution: **Ans. B**

$$\frac{1}{L} + \frac{1}{B} = \frac{1}{6} \Rightarrow \frac{1}{2B} + \frac{1}{B} = \frac{1}{6} \Rightarrow \frac{3}{2B} = \frac{1}{6} \Rightarrow 2B = 18$$

It takes 9 days for Lami and 18 days for Bayisa.

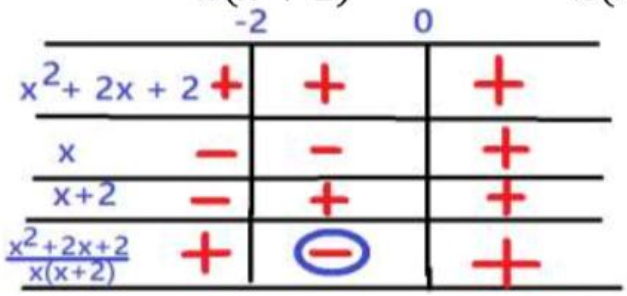
106. What is the solution set of $1 - \frac{1}{x+2} \leq -\frac{1}{x}$?

A. $(-2, 0)$ **B.** $(0, \infty)$
C. $(-\infty, -2)$ **D.** $(-\infty, -2)\mathbf{U}(0, \infty)$

Solution:

$$\frac{x+2-1}{x+2} + \frac{1}{x} \leq 0 \Rightarrow \frac{x(x+1)+(x+2)}{x(x+2)} \leq 0$$

$$\frac{x^2+x+x+2}{x(x+2)} \leq 0 \Rightarrow \frac{x^2+2x+2}{x(x+2)} \leq 0$$



S.s = (-2,0)

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107. If $f(x) = \frac{x-1}{(x-2)^2(x+1)}$, which one of the following is true about f ?

A. It has an oblique asymptote
B. the graph does not meet its asymptote
C. As $x \to -1^+, f(x) \to -\infty$
D. As $x \to 2^-, f(x) \to -\infty$

Solution: **Ans. C**

- The graph of f(x) has no an oblique asymptote.
- The graph of f(x) does not meet with its vertical asymptote.
- As $x \to -1^+, f(x) \to -\infty, As\ x \to 2^-, f(x) \to \infty$

108. Which one of the following is true about the graph of $(x) = \frac{x^3-x}{x^3(x-1)}$?

A. The vertical asymptote of the graph are $x = 0\ and\ x = 1$
B. A horizontal asymptote of the graph is $y = 1$
C. The graph intersect its horizontal asymptote at a $point\ (-1, 0)$
D. The graph intersect the vertical line $x = 1\ at\ (1, 2)$

Solution: **Ans. C**

- The vertical asymptote of the graph of f(x) is x = 0.
- The horizontal asymptote of f(x) is y = 0.
- The graph of f(x) intersect with its horizontal asymptote at (-1,0)

$$\frac{x+1}{x^2} = 0 \Rightarrow x + 1 = 0 \Rightarrow x = -1$$

- The graph of any rational function never intersect with vertical asymptote.

109. Which of the following is true about the graph of $f(x) = \frac{2x^3+2x^2+3x}{x^2+x}$?

A. The graph has y-intercept at $(0, 3)$
B. The graph has at least one x-intercept.
C. The vertical asymptote of the graph is only $x = -1$ and its oblique asymptote is $y = 2x$

D. The vertical asymptote of the graph are at x=0 and x=-1 but it has no horizontal asymptote.

Solution:

- $f(x) = \frac{x(2x^2+2x+3)}{x(x+1)} = \frac{2x^2+2x+3}{x+1}$ **Ans.C**
- The graph of f(x)has $y-$ intercept $(0,3)$ but it is a hole f(x).
- The graph f(x)has no $x-$ intercepts.
- The graph of f(x)has a vertical asymptote $x=-1$
- The graph of f(x)has an oblique asymptote $y=2x$.

110. Which of the following is true about the graph of $f(x) = \frac{x^2-1}{x-x^2}$? **Ans.D**

A. $x = 0$ *and* $x = 1$ are its vertical asymptote

B. $y = 1$ is its horizontal asymptote

C. $y = x - 1$ is its oblique asymptote

D. it is at most the same as horizontal line $y = -1$ as $x \to \pm\infty$

Solution: $\frac{(x-1)(x+1)}{x(1-x)} = \frac{-x-1}{x}$, *the vertical asymptote is* $x = 0$

Y = - 1 is a horizontal asymptote.

It has no an oblique asymptote

As y goes to – 1, then x goes to negative or positive infinity.

111. Given a rational function $f(x) = \frac{2x+4}{x-1}$ which of the following is NOT true about its graph ?

Ans. D

A. $y = 2$ is its horizontal asymptote

B. $x = 1$ is its vertical asymptote

C. $(0,-4)$ is its y- intercept

D. $(-4,0)$ is its x- intercept

Solution: The horizontal asymptote of f(x) *is* $y = 2$.

- The vertical asymptote of $f(x)$ $x = 1$
- The y intercept is at $(0,-4)$

- The x intercept of $f(x)$ is at $(-2, 0)$

112. Which one of the following is true about the graph of $f(x) = \frac{x^2+5x+6}{x^2-4} + 3$?

Ans. D

A. The graph has a hole at $x = 2$

B. The vertical asymptote of the graph are $x = -2$ *and* $x = 2$

C. The graph has $y - intercept\ at\ (0, \frac{-3}{2})$

D. A horizontal asymptote of the graph is $y = 4$

Solution: the simplified form of $f(x) = \frac{4x-3}{x-2}$

- The graph of f(x) has a hole at x=-2.
- The vertical asymptote of the graph of f(x) is x = 2.
- The graph has y- intercept at $\left(0, \frac{3}{2}\right)$.
- The horizontal asymptote of the graph of f(x) is y = 4.

113. Suppose $f(x) = \frac{Q(x)}{x(x^2-1)}$ where $Q(x)$ a quadratic function. Which one of the following is necessary true about the graph of f ?

Ans. C

A. $x = 0, x = 1\ and\ x = -1$ are vertical asymptote.

B. the graph of f does not intersect its horizontal asymptote.

C. the vertical asymptote of the graph of f is only $x = -1\ if\ Q(x) = x^2 - x$

D. the vertical asymptote of the graph of f is only $x = 1\ if\ Q(x) = 2x^2$

Solution:

- The asymptotes of the graph of f(x) depends on the function Q(x).
- Since the function Q(x)= $x^2 - x \Rightarrow x(x-1), then\ \frac{x(x-1)}{x(x-1)(x+1)} = \frac{1}{x+1}$
- $The\ vertical\ asymptote\ is\ x = -1.$

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114. Which one of the following asymptotes does NOT appear on the same graph? **Ans. C**

A. vertical asymptote and horizontal asymptote
B. vertical asymptote and oblique asymptote
C. Horizontal asymptote and oblique asymptote
D. Oblique asymptote and hole

Solution: Except horizontal asymptote and oblique asymptote all the other can exist together.

115. Which one of the following is NOT true about the graph of $g(x) = \frac{x^2-1}{x^2+1}$.

A. Range of g is $(-\infty, 1)$
B. $y = 1$ is horizontal asymptote
C. g is even function
D. as $x \to -\infty, g(x) \to 1$

Solution: The horizontal asymptote of g(x) is y = 1. **Ans. A**

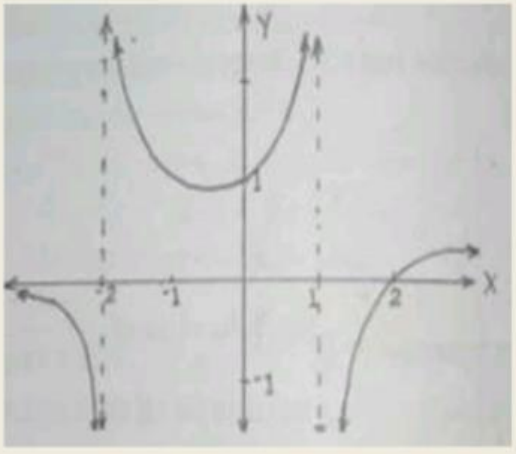
- As $x \to -\infty, g(x) \to 1$
- Let $y = \frac{x^2-1}{x^2+1}, \Rightarrow x = \frac{y^2-1}{y^2+1} \Rightarrow xy^2 + x = y^2 - 1$
- $y^2(x-1) = x+1 \Rightarrow y^2 = \frac{x+1}{x-1} \Rightarrow y = \sqrt{\frac{x+1}{x-1}}$

$\Rightarrow$ Domain of $g^{-1}(x)$ is $x > 1$ = Range of $g(x)$

$g(-x) = \frac{(-x)^2-1}{(-x)^2+1} = \frac{x^2-1}{x^2+1} = g(x)$, then $g(x)$ is an even function.

116. Which one of the following function represents the graph given below?

A. $f(x) = \frac{x+1}{(x+2)(x-1)}$
B. $f(x) = \frac{1}{(x-2)(x+1)}$
C. $f(x) = \frac{x-2}{(x+2)(x-1)}$
D. $f(x) = \frac{x^2-2}{(x+2)(x-1)}$



Solution:

Ans. C

- The vertical asymptote of the graph of f(x) is x = - 2 and x = 1.
- The horizontal asymptote of the graph of f(x) is y = 0.
- The x – intercept is at (2,0) and y – intercept is at (0, 1)

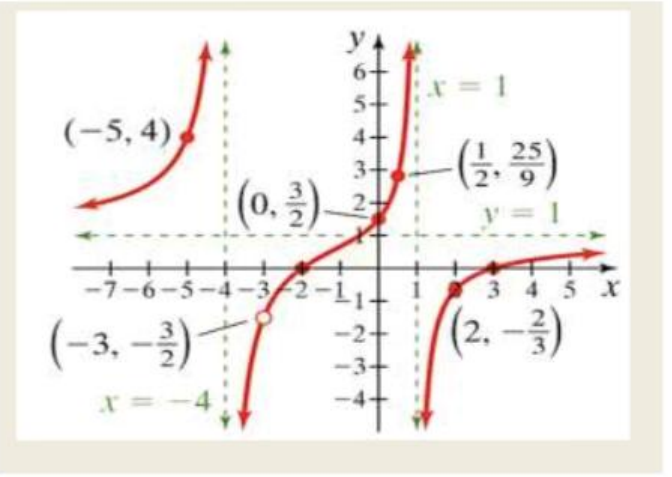
117. Which one of the following function represents the graph given below?

A. $f(x) = \frac{x^2+x+3}{x^2-x+2}$

B. $f(x) = \frac{2x^2-9x-18}{x^2+3x-4}$

C. $f(x) = \frac{x^3+2x^2-9x-18}{x^3+6x^2+5x-12}$

D. $f(x) = \frac{x^3}{(x+4)(x-1)(x+3)}$



Solution:

Ans. C

- The vertical asymptotes of the graph are x = -4 and x = 1.
- The horizontal asymptote of the graph is y = 1.
- The x – intercept is at (-2 ,0) and the y – intercept is at (0, 1.5)
- The hole of the graph of the function is at (-3, -1.5)

118. The value of x where the graph of $y = \frac{x^2-4x+3}{x^3+1}$ cross it horizontal asymptote is(are):-

Ans. C

A. $x = -2$

B. $x = 0$

C. $x = 3 \ and \ x = 1$

D. $x = \sqrt{2} \ and \ x = 1$

Solution:

- The graph of $y = \frac{x^2-4x+3}{x^3+1}$ cross its horizontal asymptote at $x^2 - 4x + 3 = 0$
- $x^2 - x - 3x + 3 = 0 \Rightarrow x(x-1) - 3(x-1) = 0 \Rightarrow x = 1 \ or \ x = 3$

119. Which of the following is the inverse of $f(x) = 2 + ln(x-1)$?

A. $g(x) = 2 + e^{x-1}$ B. $g(x) = 1 + e^{x-2}$

C. $g(x) = 2 - e^{x-1}$ D. $g(x) = 1 - e^{x-2}$

Solution: **Ans. B**

$$\text{Let } y = 2 + \ln(x-1) \Rightarrow x = 2 + \ln(y-1)$$
$$x - 2 = \ln(y-1) \Rightarrow y - 1 = e^{x-2} \Rightarrow y = 1 + e^{x-2}$$

120. Which of the following is the inverse of $f(x) = \frac{1}{1+e^{-x}}$? **Ans.D**

A. $g(x) = \ln x - \ln(1-x)$ B. $\ln x + \ln(1-x)$

C. $\ln(1+x) - \ln x$ D. $\ln(1-x) + \ln x$

Solution:

$$\text{Let } y = \frac{1}{1+e^{-x}} \Rightarrow x = \frac{1}{1+e^{-y}} \Rightarrow x = \frac{1}{1+\frac{1}{e^y}}$$
$$\Rightarrow x = \frac{e^y}{e^y+1} \Rightarrow xe^y + x = e^y \Rightarrow e^y(x-1) = -x$$
$$\Rightarrow e^y = \frac{-x}{x-1} \Rightarrow y = \ln(x) - \ln(1-x)$$

121. If $f(x) = \frac{x+1}{x-1}$ and $f(a) = 5$, then $f(2a)$ is equal to____

A. 2 B. 4 C. 6 D. 8

Solution: **Ans. A**

$$f(a) = \frac{a+1}{a-1} \Rightarrow \frac{a+1}{a-1} = 5 \Rightarrow 5a - 5 = a + 1$$
$$\Rightarrow 5a - a = 6 \Rightarrow 4a = 6 \Rightarrow a = \frac{3}{2}$$
$$\Rightarrow f(2a) = f\left(2 \times \frac{3}{2}\right) = f(3) = \frac{3+1}{3-1} = \frac{4}{2} = 2$$

122. If $f(x) = \sqrt[3]{1+e^{-x}}$, which of the following is equal to $f^{-1}(x)$?

A. $\ln\left(\frac{1}{x^3-1}\right)$ *B.* $\frac{1}{\ln(x^3-1)}$ *C.* $\ln(1-x^3)$ *D.* $(1+e^{-x})^3$

Solution: **Ans. A**

$$Let\ y = \sqrt[3]{1+e^{-x}} \Rightarrow x = \sqrt[3]{1+e^{-y}}$$
$$\Rightarrow x^3 = 1 + e^{-y} \Rightarrow x^3 = \frac{e^y + 1}{e^y}$$
$$\Rightarrow x^3 e^y = e^y + 1 \Rightarrow e^y(x^3 - 1) = 1$$
$$\Rightarrow e^y = \frac{1}{x^3 - 1} \Rightarrow y = \ln\left(\frac{1}{x^3-1}\right)$$

123. Which one of the following is true? **Ans. D**

A. A polynomial function can have infinitely many vertical asymptotes.
B. The graph of a rational function can never cross its horizontal asymptote.
C. The graph of $f(x) = \frac{3x-1}{x-1}$ has no horizontal asymptote.
D. The graph of $f(x) = \frac{x^3-x}{x^2-x}$ has no vertical asymptote.

Solution:

- A polynomial functions have no any vertical asymptotes.
- The graph of a rational function can cross the horizontal asymptote.
- The graph of $f(x) = \frac{3x-1}{x-1}$ has $y = 3$ as a horizontal asymptote.
- The graph of $f(x) = \frac{x^3-x}{x^2-x} = \frac{x(x-1)(x+1)}{x(x-1)} = x + 1$ has no any vertical asymptote.

124. Which one of the following is true about the graph of $f(x) = \frac{x^2-x}{x-x^3}$? **Ans. C**

A. $x = 0$ and $x = 1$ are its vertical asymptotes.
B. $y = 1$ is its horizontal asymptotes.
C. $y = x - 1$ is its oblique asymptote.
D. It is almost the same as the horizontal line $y = 0$ as $x \to \pm\infty$.

Solution:

- $f(x) = \frac{x^2-x}{x-x^3} = \frac{x(x-1)}{x(1-x)(1+x)} = \frac{1}{-x-1}$.
- The vertical asymptote is $x = -1$.
- The horizontal asymptote $y = 0$.
- It has no an oblique asymptote.

125. Which one of the following functions has **NO** vertical asymptote?

A. $f(x) = \ln(x+1)$ B. $f(x) = \frac{x^2+1}{x^3+8}$ C. $f(x) = \frac{x^2-9}{x-3}$ D. $f(x) = \frac{x-1}{x^3-x}$

Solution:

- $f(x) = \ln(x+1)$ has a vertical asymptote $x = -1$
- $f(x) = \frac{x^2+1}{x^2+8}$ has a vertical asymptote $x = 1$
- $f(x) = \frac{x^2-9}{x-3} = x + 3$ has no vertical asymptote
- $f(x) = \frac{x-1}{x^3-x} = \frac{x-1}{x(x-1)(x+1)} = \frac{1}{x(x+1)}$
- *it has avertical asymptote* $x = 0$ *and* $x = -1$

126. **Let $x \neq -2, x \neq 3$ and $x \neq -\frac{4}{7}$. Then $\left(\frac{2}{x+2} + \frac{5}{x-3}\right) \div \left(\frac{7x+4}{2x-6}\right)$ equals:**

A. $\frac{2}{x+2}$ B. $\frac{7x-1}{(x+2)(7x+4)}$ C. $\frac{3}{x-3}$ D. $\frac{7x+4}{x+2}$

Solution: **Ans. A**

$$\left(\frac{2}{x+2} + \frac{5}{x-3}\right) \div \left(\frac{7x+4}{2x-6}\right) \Rightarrow \frac{2(x-3)+5(x+2)}{(x+2)(x-3)} \times \frac{2(x-3)}{7x+4}$$

$$\Rightarrow \frac{2(2x-6+5x+10)}{(x+2)(7x+4)}$$

$$\Rightarrow \frac{2(7x+4)}{(x+2)(7x+4)} = \frac{2}{x+2}$$

127. What is the solution set of $\frac{2}{x-1} + \frac{4}{x} = \frac{6}{x-2}$? **Ans. C**

A. $\left\{\frac{4}{11}\right\}$ B. $\left\{\frac{11}{4}\right\}$ C. $\left\{\frac{4}{5}\right\}$ D. $\left\{\frac{5}{4}\right\}$

Solution:

$$\frac{2}{x-1} + \frac{4}{x} - \frac{6}{x-2} = 0$$

$$\Rightarrow \frac{2x(x-2)+4(x-1)(x-2)-6x(x-1)}{(x-1)(x)(x-2)} = 0$$

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$$\Rightarrow \frac{2x^2-4x+4(x^2-2x-x+2)-6x^2+6x}{(x-1)(x)(x-2)} = 0$$

$$\Rightarrow 2x^2 - 4x + 4x^2 - 12x + 8 - 6x^2 + 6x = 0$$

$$\Rightarrow -10x + 8 = 0 \Rightarrow x = \frac{4}{5}$$

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