ADDIS ABABA UNIVERSITY
COLLEGE OF NATURAL AND COMPUTATIONAL SCIENCES
DEPARTMENT OF MATHEMATICS
Mathematics for Natural Sciences (Math 1011)
Mid-Term Examination - December 19, 2019

Name _____ **ID. No.** _____

Select Campus (AAiT, CHS, CNCS, EiABC) Section _____ Time allowed: $2\frac{1}{2}$ Hours

Instructions: This examination has 10 short answer questions and 5 workout problems. Attempt each question on the space provided for each question. Using **Calculators** and **Mobiles** are strictly forbidden.

For Instructor's use only:

PART I	PART II	TOTAL

Part I: (Short Answer) *Give the most simplified answers on the space provided.*

(1) Let p and q be propositions. If $p \Rightarrow q$ is False, then the truth value of $((\neg p) \wedge q) \Leftrightarrow (p \vee q)$ is _____.

(2) If $\frac{x+yi}{i} = 7 + 9i$, where x and y are real numbers, then the value of $(x + yi)(x - yi)$ is equal to _____.

(3) Let the set $S = \{x \in \mathbb{R} : |x - 2| < 3\}$. Then the $lub(S)$= _____ and $glb(S)$= _____.

(4) The simplified form of the set $(2, 4]^c \cap ((-\infty, 5) \cap [0, \infty))$ as an interval or union of intervals is equal to _____.

(5) Let the set $B = \{y : y \in \mathbb{N} \text{ and } 1 \leq y \leq \sqrt{5}\}$. Then the power set of B, $P(B)$ is equal to _____.

(6) The smallest positive integer m for which $(\frac{1+i}{1-i})^m = 1$ is equal to _____.

(7) Let $GCF(x, 400) = 20$ and $LCM(x, 400) = 1200$. Then the value of x is equal to_____.

(8) Determine the truth value of the following propositions
(a) $(\exists x \in \mathbb{R})(\exists y \in \mathbb{R})[|x| + y = 0]$ is _____.

(b) $(\exists x \in \mathbb{N})(\forall y \in \mathbb{R})[x \leq y^2]$ is _____.

(9) In a class of 40 students, 12 registered for both Mathematics and Physics, 22 registered for Physics. If the students of the class registered for at least one of the two subjects, then how many students registered only for Mathematics? _____.

(10) The values of real numbers x and y that satisfy the equation $x - yi = |x + yi|$ respectively are equal to _____ and _____.

Part II: ***(Workout Problems)*** *Attempt each of the following questions by showing all the necessary steps.*

(1) Using the principle of mathematical induction, prove that
$2^1 + 2^2 + 2^3 + \ldots + 2^n = 2^{n+1} - 2$. for any $n \in \mathbb{N}$. [4 Pts.]

(2) Let U be a universal set and A and B be any two subsets of U. Prove formally that
$A - (A - B) = A \cap B$. [4 Pts.]

(3) Find the three cube roots of the complex number $z = -8i$. [4 Pts.]

(4) Without using truth table, show that $(q \wedge (\neg q \vee p)) \vee (p \wedge \neg(q \wedge p)) \equiv p$ for any propositions p and q. [4 Pts.]

(5) Determine the validity of the argument: [4 Pts.]
If there was a great run, then traveling was difficult.
If they arrived on time, then traveling was not difficult.
They arrived on time.
Therefore, there was no great run.