
Addis Ababa City Administration
Education Bureau
Condensed Mathematics Curriculum
From Grade 9-12
Designed For Grade 12 Tutorial
Program

October, 2025

Addis Ababa, Ethiopia

Preface

Dear teachers,

Addis Ababa City Education Bureau has prepared this condensed Mathematics curriculum highlighting on the core lesson notes and selected some key questions based on the syllabus and past exam papers. In addition, this curriculum is delivered within the first month. Thus, it would give the students a solid foundation about *The Number System, Solution of Equation, Further on Sets, Relations and Functions*

As a professional, feel free to modify, add extra explanations, or include examples that you think will help students especially, if you know where the students usually struggle. Moreover, you can bring in your own teaching experiences, employ different techniques to keep the students engaged for the 8-hour tutorial classes.

Generally, the program emphasizes ensuring students' engagement in ample practices and completing the sessions with greater confidence that would enhance their readiness for the university entrance examination.

Contents

Lesson 1	1
1.1.Radicals and simplification of radicals	1
1.2. Equations involving exponents and radicals.....	2
Systems of linear equations in two variables.....	2
Lesson 2	5
2.1.Solving absolute value and quadratic equations	5
2.2. Operations on sets and its application.....	7
Lesson 3	10
3.1. Functions and composition of functions	10
3.2. Types of functions (1-1, onto and 1-1 correspondence) and special types of functions(power function, sgn function and greatest integer function).....	13
Lesson 4	15
4.1. Apothem, side, perimeter and area of a regular polygons	15
4.2. Theorem on polynomial functions and its graph	18

Lesson 1

1.1.Radicals and simplification of radicals

☞ By the end of this lesson students would be able to simplify radical expressions

Key concepts:

1. For $a > 0$, and q and p any integers $a^{\frac{p}{q}} = \sqrt[q]{a^p}$
2. For $a \geq 0$, and q and p positive integers $\sqrt[n]{\sqrt[m]{a}} = \sqrt[n \times m]{a}$
3. $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$
4. $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$
5. $(\sqrt{a} + \sqrt{b})(\sqrt{a} + \sqrt{b}) = \sqrt{a}(\sqrt{a} + \sqrt{b}) + \sqrt{b}(\sqrt{a} + \sqrt{b})$
$$= a + 2\sqrt{ab} + b$$
6. Rationalizing factor for $\frac{1}{\sqrt{a}+\sqrt{b}}$ is $\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}-\sqrt{b}}$ and for $\frac{1}{\sqrt{a}-\sqrt{b}}$ is $\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}}$

Examples

1. Simplify the following radical expressions

a. $3\sqrt{10} + 5\sqrt{10} - 2\sqrt{10}$

Solution a.

$$\begin{aligned} & 3\sqrt{10} + 5\sqrt{10} - 2\sqrt{10} \\ &= \sqrt{10}(3 + 5 - 2) \\ &= 6\sqrt{10} \end{aligned}$$

b. $3\sqrt{12} - 2\sqrt{20} + 2\sqrt{45} - 2\sqrt{3}$

Solution b.

$$\begin{aligned} &= 3\sqrt{12} - 2\sqrt{20} + 2\sqrt{45} - 2\sqrt{3} \\ &= 3\sqrt{3 \times 4} - 2\sqrt{4 \times 5} + 2\sqrt{9 \times 5} - 2\sqrt{3} \\ &= 2 \times 2\sqrt{3} - 2 \times 2\sqrt{5} + 2 \times 3\sqrt{5} - 2\sqrt{3} \\ &= 2\sqrt{3} + 2\sqrt{5} \end{aligned}$$

2. Rationalize the denominator and write its simplified form of the following expressions.

a. $\frac{\sqrt{7}+3}{\sqrt{5}+\sqrt{2}}$ solution; $\frac{\sqrt{7}+3}{\sqrt{5}+\sqrt{2}} = \frac{\sqrt{7}+3}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{(\sqrt{7}+3)(\sqrt{5}-\sqrt{2})}{3}$

b. $\frac{3\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}$ solution; $\frac{3\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = \frac{3\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} \times \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} = \frac{18+6\sqrt{6}+3\sqrt{2}+6}{18-12} = \frac{24+6\sqrt{6}+3\sqrt{2}}{6}$

Exercise

1. Simplify the following radical expressions

a. $\sqrt[3]{\sqrt{64}} - 8^{2/3}$

b. $4\sqrt{3} \times 2\sqrt{27}$

c. $\frac{\sqrt[5]{192}}{\sqrt[5]{6}}$

3. Rationalize the denominator and write its simplified form of the following expressions.

a. $\frac{2}{\sqrt{10}}$

b. $\frac{2}{\sqrt{3}-\sqrt{2}}$

c. $\frac{3}{\sqrt{5}-1}$

Exam type questions

1. The simplified form of the expression $\frac{6\sqrt{20}-3\sqrt{45}}{3\sqrt{75}}$ is

A. $\sqrt{5}$ B. $\frac{\sqrt{15}}{15}$ C. $\frac{1}{5}$ D. 5

2. The simplified form of the expression $\frac{2\sqrt{72}}{3} - \frac{3\sqrt{128}}{4} + 5\sqrt{\frac{1}{2}}$ is:

A. 1 B. $18\sqrt{2}$ C. $\frac{\sqrt{2}}{2}$ D. $\sqrt{2}$

3. Which of the following expression is used as the rationalizing factor of $\frac{7}{3+\sqrt{5}}$?

A. $\frac{\sqrt{3}-5}{\sqrt{3}-5}$ B. $\frac{\sqrt{3}+5}{\sqrt{3}+5}$ C. $\frac{\sqrt{3}+\sqrt{5}}{\sqrt{3}+\sqrt{5}}$ D. $\frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}-\sqrt{5}}$

1.2. Equations involving exponents and radicals

Systems of linear equations in two variables

☞ By the end of this lesson students would be able to:

- Solve equations involving exponents and radicals
- Systems of linear equations in two variables

Key concepts:

1. For $a > 0$, $a^x = a^y$ if and only if $x = y$
2. For $a > 0$, $x^a = y^a$ if and only if $x = y$
3. $a^n \times a^m = a^{n+m}$
4. $\frac{a^n}{a^m} = a^{n-m}$
5. $(a^m)^n = a^{mn}$
6. The solution set for the system of linear equation $\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$ is the intersection point of the lines $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ if they intersect each other.
7. For the system of linear equation $\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$, it has;
 - a. One (unique) solution if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
 - b. Infinite solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
 - c. No solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Examples

1. Solve the following exponential equations
 - a. $5^{x+2} = 125 \times 5^{-x}$
 - b. $2^{2x+1} + 2^x = 136$

Solution;

$$\begin{aligned}
 \text{a. } 5^{x+2} &= 125 \times 5^{-x} \\
 &= \frac{5^{x+2}}{5^{-x}} = 125 \times \frac{5^{-x}}{5^{-x}} \\
 &= 5^{2x+2} = 125 = 5^3 \\
 &= 2x + 2 = 3 \\
 x &= \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{b. } 2^{2x+1} + 2^x &= 136 \\
 \text{let } 2^x &= a. \text{ The above eqn. will become,} \\
 &= 2a^2 + a - 136 = 0. \text{ Solving this we get;} \\
 a &= 8 = 2^x \\
 x &= 3
 \end{aligned}$$

2. Solve the following system of linear equations

$$\text{a. } \begin{cases} 4x + 6y = 12 \\ 2x + 3y = 8 \end{cases}$$

Solution: To solve such equation we can employ different techniques. Let me apply cancelation rule. That is multiply the second equation by -2 and add up them together. We get;

$$\begin{array}{r}
 \begin{cases} 4x + 6y = 12 \\ -4x + 6y = -16 \end{cases} \\
 \hline
 0 + 0 = -4 \text{ which is false}
 \end{array}$$

Therefore, the system has no solution.

Exercise

1. Solve the following exponential equations

$$\text{c. } (3x + 1)^3 = (2x - 4)^3$$

$$\text{d. } 9^{2x+2} \times \left(\frac{1}{81}\right)^{x+2} = 243^{-3x-2}$$

2. Solve the following system of linear equations

$$\text{a. } \begin{cases} -3x + y = 5 \\ 3x + y = 5 \end{cases}$$

$$\text{b. } \begin{cases} 4x + 6y = 12 \\ 2x + 3y = 6 \end{cases}$$

3. For what values of a and b does the system of linear equation $\begin{cases} 4x + 3y = 8 \\ 3x - by = a \end{cases}$ has no solution?

Exam type questions

1. The solution set of the equation $25^{2x-1} \times \left(\frac{1}{125}\right)^{x+2} = 625$ is:
A. {12} B. {8} C. {4} D. {2}
2. The solution set of the equation $4^{2x+2} = 4^{2x-1} + 126$ is:
A. {6} B. $\{\frac{3}{4}\}$ C. {3} D. {}
3. What is the value of the y-coordinate of the solution to the following system of linear equation? $\begin{cases} x + y = 3 \\ 3x - 2y = 5 \end{cases}$
A. 11 B. -4 C. $\frac{11}{5}$ D. $\frac{4}{5}$
4. Simon has twin younger brothers. The sum of the ages of the three brothers is 48 and the difference between his age and the age of one of his younger brothers is 3. How old is Simon?
A. 15 B. 18 C. 21 D. 25

Lesson 2

2.1.Solving absolute value and quadratic equations

☞ By the end of this lesson students would be able to:

- Solve absolute value equations
- Solve quadratic equations

Key concepts:

1. $|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$

2. $\sqrt{x^2} = |x|$

3. The solution set of $|x| = a$ is:

- $x = a$ or $x = -a$, if $a > 0$
- $x = 0$, if $a = 0$
- $\{ \}$, if $a < 0$

4. The value of x for the quadratic equation $ax^2 + bx + c = 0$ is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

5. The quadratic equation $ax^2 + bx + c = 0$

- has **one** real roots, if $b^2 - 4ac = 0$
- has **two** real root, if $b^2 - 4ac > 0$
- has **no** real roots, if $b^2 - 4ac < 0$

6. The quadratic equation $ax^2 + bx + c = 0$ has two roots r_1 and r_2 and they are

written as $r_1 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$ and $r_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$

✍ Their sum: $r_1 + r_2 = -\frac{b}{a}$

✍ Their product: $r_1 \times r_2 = \frac{c}{a}$

Examples

1. Solve the following absolute value equations

a. $|2x - 3| = 7$

b. $|6x - 2| + |3x - 1| = 12$

Solution

a. $|2x - 3| = 7$

$$2x - 3 = 7 \text{ or } 2x - 3 = -7$$

$$2x = 7 + 3 \text{ or } 2x = -7 + 3$$

$$2x = 10 \text{ or } 2x = -4$$

$$x = 5 \text{ or } x = -2$$

b. $|6x - 2| + |3x - 1| = 12$

$$2|3x - 1| + |3x - 1| = 12$$

$$3|3x - 1| = 12$$

$$|3x - 1| = 4$$

$$3x - 1 = 4 \text{ or } 3x - 1 = -4$$

$$x = \frac{5}{3} \text{ or } x = -1$$

2. Solve the following quadratic equation using factorization or quadratic formula

$$x^2 - 12x + 20 = 0$$

Solution. $a = 1, b = -12, c = 20$. Substitute these values into the following equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow x = \frac{12 \pm \sqrt{(-12)^2 - 4 \times 1 \times 20}}{2 \times 1} \rightarrow x = \frac{12 \pm 8}{2}, x = 10 \text{ or } x = 2$$

Exercise

1. Solve the following absolute value equations

c. $|4x + 1| = x + 3$

d. $|2 - 3x| = |x - 5|$

e. $\sqrt{(5 - 3x)^2} = 11$

2. Solve the following quadratic equation using factorization or quadratic formula.

$$9 + 30x + 25x^2 = 0$$

3. The sum of the roots of a quadratic equation $2x^2 + (k + 1)x + 4 = 0$ is -3 . What is the value of k ?
4. What value of k so that the equation $3x^2 - 6x + k = 0$ will have exactly one real solution?

5. One of the roots of a quadratic $x^2 - 8x + 4k = 0$ is three times the other. So what is the value of k ?

Exam type questions

- What is the solution set of $|x^2 + 3x - 2| = 2$?
A. $\{-3, 1, 0, -4\}$
B. $\{3, -1, 0, -4\}$
C. $\{3, 1, 0, 4\}$
D. $\{-3, -1, 0, -4\}$
- If one of the roots of the equation $x^2 - 4x + k = 0$ exceeds the other by 2, then find the roots and the value of k ?
A. -1
B. 1
C. 0
D. 3
- If a and b are the two roots of the quadratic equation $2x^2 + 3x - 5 = 0$, then what is the value of $(a - 1)(b - 1)$?
A. -1
B. 1
C. 0
D. 3
- For what non-negative value of k that the equation $x^2 - (k + 1)x + 4 = 0$ has exactly one real root?
A. 5
B. 1
C. 3
D. 6

2.2. Operations on sets and its application

☞ By the end of this lesson students would be able to:

- Apply operations on sets
- Solve word problems associated with the concept of sets

Key concepts:

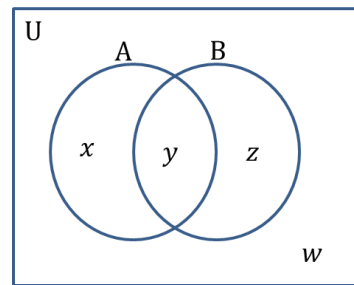
- $A \cup B = \{x | x \in A \text{ or } x \in B\}$
- $A \cap B = \{x | x \in A \text{ and } x \in B\}$
- $A - B = \{x | x \in A \text{ and } x \notin B\}$
- $A' = \{x | x \in U \text{ and } x \notin A\}$
- $A \Delta B = \{x | x \in (A \cup B) \text{ and } x \notin (A \cap B)\}$
- **Considering the Venn diagram below:**

i. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
 $= x + y + y + z - y = x + y + z$

ii. $n(A - B) = n(A) - n(A \cap B)$
 $= x + y - y = x$

iii. $n(A \cup B)' = n(U) - n(A \cup B)$
 $= x + y + z + w - (x + y + z) = w$

iv. $n(A \Delta B) = n(A \cup B) - n(A \cap B)$
 $= x + y + z - y = x + z$



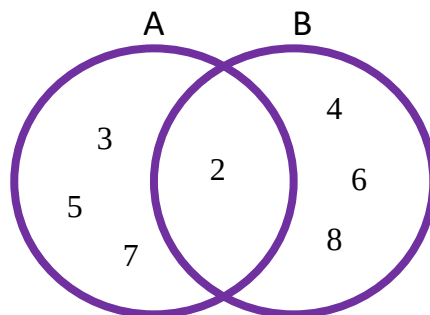
Examples

1. Let $A = \{x \mid x \text{ is a prime number less than } 10\}$, $B = \{x \mid x \text{ is an even natural number less than } 10\}$, then find:

- $A - B$
- $B - A$
- $A \triangle B$

Solution

Use Venn diagram to solve this type of question

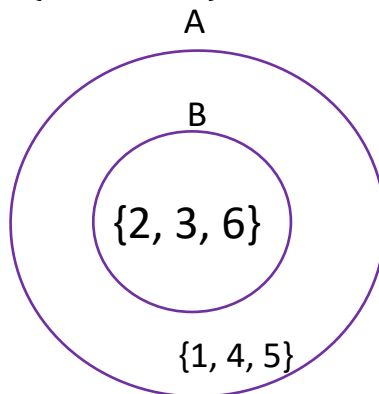


$$A = \{2, 3, 5, 7\}, B = \{2, 4, 6, 8\}$$

- a. $A - B = \{3, 5, 7\}$ b. $B - A = \{4, 6, 8\}$ c. $\{3, 4, 5, 6, 7, 8\}$

2. If $B \subseteq A$, $A \cap B' = \{1, 4, 5\}$ and $A \cup B = \{1, 2, 3, 4, 5, 6\}$, then find the set B.

$$B = \{2, 3, 6\}$$



Exercise

- Let $A = \{3, 7, a^2\}$ and $B = \{2, 4, a + 1, a + b\}$ be two sets and all the elements of the two sets are integers. If $A \cap B = \{4, 7\}$. Then
 - Find the value of a and b
 - Find $A \cup B$
- Among a group of 100 students, 72 students like English subject and 43 students like mathematics subject. How many students like;
 - English subject only?
 - Mathematics only?
- A school has 300 students. 150 students play basketball, 120 play football, and 80 plays both. How many students play neither sport?

4. Among 150 students in a school, 28 students failed in English, 32 students failed in mathematics and 15 students failed in both English and Mathematics.
- How many students failed in either English or in Mathematics?
 - How many students passed both in English and in Mathematics?

Exam type questions

- In a group of 300 people, 150 can speak French and 200 can speak Arabic. How many people can speak both French and Arabic if they can speak either of the language?
A. 40 B. 50 C. 20 D. 30
- Let $A = \{8, 6, b^2\}$ and $B = \{25, 7, b + 3, a + b\}$ be two sets and all the elements of the two sets are integers. If $A \cap B = \{25, 6\}$, then what is the value of a ?
A. -1 B. -5 C. 2 D. 11
- If $n(U) = 16$, $n(A) = 7$ and $n(B) = 12$, what is the greatest $n(A \cap B)$ and least $n(A \cup B)$ respectively?
A. 4, 7 B. 7, 12 C. 3, 7 D. 3, 12

Lesson 3

3.1. Functions and composition of functions

☞ By the end of this lesson students would be able to:

- Define a function
- Find the compositions of two functions

Key concepts:

- A **function** is a relation such that no two ordered pairs have the same first-coordinates and different second-coordinates.
- If $f: A \rightarrow B$ is a function, then, for any $x \in A$ the image of x under f , $f(x)$ is called the functional value of f at x
- Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be functions, then the function h defined by $h(x) = g(f(x))$ for $x \in A$ and $f(x)$ is in the domain of g is called the composition of g by f , denoted by gof , that is $(gof)(x) = g(f(x))$
- Given a function f , if its inverse is also a function, then we say that f is invertible and we denote its inverse by f^{-1}
- Given a function $y = f(x)$, if f is invertible then to find its inverse.
 - i. interchange x and y in the expression $y = f(x)$
 - ii. solve y in terms of x , (if possible);
 - iii. write f^{-1} as $y = f^{-1}(x)$
- $I(x) = x$ is called identity function
- If $f(x)$ and $g(x)$ are inverse to each other, then $f(g(x)) = g(f(x)) = x = I(x)$
- The graph of $f^{-1}(x)$ is obtained from the graph of $f(x)$ by reflecting it along the line $y = x$.

Examples

1. Identify whether the following are functions or not

a. $R_1 = \{1, -2), (3, 4), (5, 5), (-1, -2)\}$

b. $R_2 = \{(x, y): x \text{ is the square of } y\}$

Solution;

R_1 is a function, because every first element is paired with exactly one element of the second element.

R_2 is also a function, because the square of a number is exactly one number.

2. If $f(x) = 3x - 5$ and $g(x) = 4x - 3$, then find $f(g(x))$

Solution;

$$f(g(x)) = 3(g(x)) - 5 = 3(4x - 3) - 5 = 12x - 9 - 5 = 12x - 14$$

Exercise

1. Identify whether the following are functions or not

a. $R = \{(x, y): x + y^2 = 1\}$

b. $R = \{(x, y): y < x\}$

c. $R = \{(x, y): y \text{ is the father of } x\}$

d. $R = \{(x, y): x = |y|\}$

2. If $f = \{(2, 1), (3, -1), (-2, 3), (0, 7), (-1, -1)\}$ and

$g = \{(7, -1), (1, 0), (4, -2), (3, 5), (-1, -3)\}$, then find $f \circ g$ and $g \circ f$

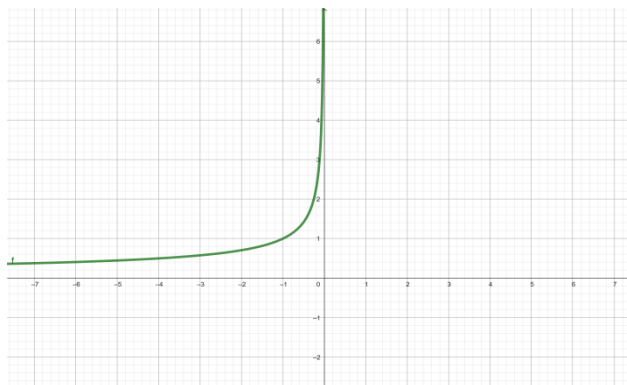
3. If $f(g(x)) = 3x^2 - 4$ and $g(x) = 1 - 2x$, then find $f(x)$

4. Find the inverse of the following functions

b. $g(x) = 4x - 1$

c. $h(x) = \frac{2x-1}{x+1}$

5. sketch the graph of f^{-1} , the graph of is given below:



Exam type questions

1. If $f(x) = ax - b$ and $f^{-1}(x + 1) = \frac{1}{2}x + 2$, then what are the values of a and b respectively.
A. 2 and 3
B. 1 and 3
C. 5 and 7
D. 1 and 8
2. Which one of the following relations is a function?
A. $R = \{(x, y): x = |y|, \text{ for } x, y \in \mathbb{R}\}$
B. $R = \{(x, y): x^2 + y^2 = 9, \text{ for } x, y \in \mathbb{R}\}$
C. $R = \{(x, y): y = x^{-0.5}, \text{ for } x, y \in \mathbb{R}\}$
D. $R = \{(x, y): x = 2, \text{ for } x, y \in \mathbb{R}\}$
3. Let $f(x) = 4x + 1$ and $g(x) = 3x + b$. What is the value of b , so that $f(g(x)) = g(f(x))$?
A. $\frac{2}{5}$
B. $\frac{1}{3}$
C. $\frac{4}{7}$
D. $\frac{2}{3}$
4. If $f(x)$ and $g(x)$ are inverse to each other, then which one of the following **is not necessarily true**?
A. $f(g(x)) = g(f(x))$
B. $f(x)g(x) = 1$
C. $g(f(x)) = x$
D. $x - f(g(x)) = 0$
5. Given that $f(x) = \frac{1}{2}x - a$ and $g(x) = 3x + b$. If $g(g(2)) = 10$ and $f^{-1}(2) = 14$, then find the values of a and b respectively
A. 5 and -3 B. -5 and 3 C. -5 and -3 D. 5 and 3

3.2. Types of functions (1-1, onto and 1-1 correspondence) and special types of functions(power function, sgn function and greatest integer function).

☞ By the end of this lesson students would be able to:

- Identify types of functions
- Work with a special types of functions

Key concepts:

1. **One-to-one function (Injective function):** If each element in the domain of a function has a distinct image in the co-domain, then the function is said to be one -to- one function.
2. **Onto function (Surjective Function):** A function is called an onto function if each element in the co-domain has at least one pre-image in the domain
3. **One-to-one correspondence (Bijective function):** A function $f:R \rightarrow R$ is said to be a one-to-one correspondence if f is both one-to-one and onto.
4. **Power function:** is a function of the form $f(x) = ax^n$, where a and n are any real numbers and $a \neq 0$.
5. The function defined by $f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$ is called the **Signum function**.

It is denoted by $sgnx$

6. The function defined by $f(x)$ =the greatest integer that is less than or equal to x , denoted by $f(x) = \lfloor x \rfloor$ is called the **greatest integer (floor or step)** function.

Examples

1. Solve the following equations $sgn(x^2 + x + 4) = 1$

$$\text{We know that } sgnx = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$$

So $sgn(x^2 + x + 4) = 1$ means $x^2 + x + 4 > 0$ which is always true.

Therefore the solution set is $\mathbb{R}$

2. What is the range of the function $f(x) = 6 - \operatorname{sgn} x$?

$$\text{We have } 6 - \operatorname{sgn} x = 6 - \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases} = 6 - 1, 6 - 0, 6 - (-1). \text{ Which is } 5, 6, 7$$

Therefore the range will be $\{5, 6, 7\}$. The domain is $\mathbb{R}$

Exercise

1. Solve the following equations

$$\lfloor x^2 - 1 \rfloor = 3$$

2. What is the solution set of the equation $\lfloor x + 7 \rfloor = -8$?

3. Let $f(x) = \lfloor x \rfloor + \operatorname{sgn}(2x)$, then $f(-0.5)$ is equal to:

Exam type questions

1. Which one of the following functions is one to one correspondence?

A. $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = \tan x$

B. $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = 2^x$

C. $f: [0, \infty) \rightarrow [0, \infty), f(x) = x^2$

D. $f: [0, \infty) \rightarrow [0, \infty), f(x) = x + 5$

6. Which one of the following is one to one correspondence function from $A = [0, 1]$ to $[1, 2]$

A. $f(x) = x$

B. $f(x) = \frac{1}{3}x + 1$

C. $f(x) = 2x + 1$

D. $f(x) = x^2 + 1$

7. Which of the following functions is one to one correspondence?

A. $f: [0, \infty) \rightarrow \mathbb{R}, f(x) = |x|$

B. $f: \mathbb{R} \rightarrow [0, \infty), f(x) = x^2$

C. $f: \mathbb{R} \rightarrow [0, \infty), f(x) = 3^x$

D. $f: (0, \infty) \rightarrow \mathbb{R}, f(x) = \log_2 x$

8. Which one of the following is true about signum, absolute value and greatest integer functions?

A. $\operatorname{sgn}(x) = \pm|x|$, for all $x \in \mathbb{R}$

C. $|x| = x \operatorname{sgn}(x)$, for all $x \in \mathbb{R}$

B. $\operatorname{sgn}(x) \leq \lfloor x \rfloor$, for all $x \leq 0$

D. $\operatorname{sgn}(x) \leq \lfloor x \rfloor$, for all $x \geq 0$

Lesson 4

4.1. Apothem, side, perimeter and area of a regular polygons

☞ By the end of this lesson students would be able to:

- Find apothem, side, perimeter and area of regular polygons

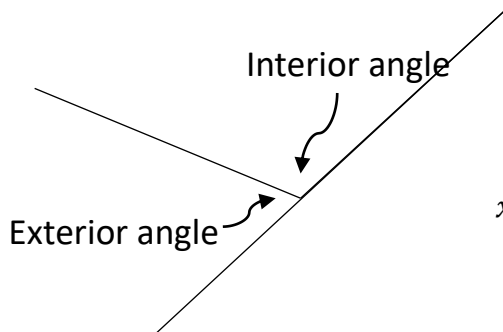
Key concepts:

1. **n -sided regular** polygon has **n lines of symmetry**.
2. The sum of the measure of interior angles of n -sided polygon is equal to **$(n-2) \times 180^\circ$** .
3. The measure of each interior angle of n -sided regular polygon is determined by
$$\frac{(n-2) \times 180^\circ}{n}$$
4. The measure of each exterior angle of n -sided regular polygon is $\frac{360^\circ}{n}$
5. The measure of each central angle of n -sided regular polygon is $\frac{360^\circ}{n}$
6. For n -sided regular polygon inscribed in a circle of radius r , length of side s , apothem a , perimeter P and area A are determined by:
 - a. $S = 2r \sin\left(\frac{180^\circ}{n}\right)$
 - b. $P = 2nr \sin\left(\frac{180^\circ}{n}\right)$
 - c. $a = 2r \cos\left(\frac{180^\circ}{n}\right)$
 - d. $A = \frac{1}{2} aP$

Examples

1. The ratio of the interior angle to the exterior angle of a regular polygon is 7:2. Find the number of sides of the polygon.

Solution;



The problem can be interpreted as;

$$7x + 2x = 180^\circ$$

$$9x = 180$$

$$x = 20^\circ \text{ that is one of the interior angle is } 20 * 7 = 140^\circ$$

Using the formula $\frac{(n-2) \times 180^\circ}{n} = 140$, we get $n = 9$

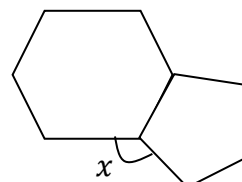
2. This figure shows a regular pentagon and regular hexagon connected to each other at one side.

Calculate the value of x .

The one on the left side is regular hexagon and the other is regular pentagon.

So their each interior angle is 120 and 108 for regular hexagon and regular pentagon respectively.

$$\text{So } 120 + 108 + x = 360 \rightarrow x = 132$$

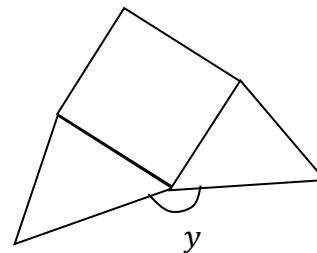


Exercise

1. The sum of the interior angles of a polygon is four times the sum of its exterior angles. Find the number of sides of the polygon.

2. The diagram is made from a square and two equilateral triangles.

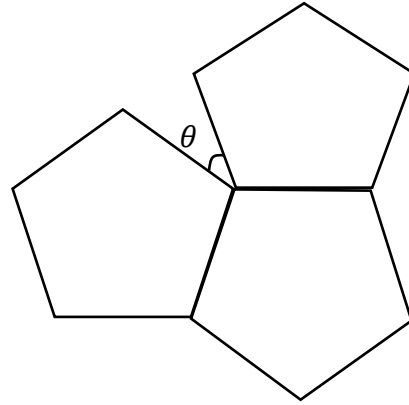
Calculate the value of y .



Exam type questions

1. For n -sided regular polygon, the measure of each interior angle is equal to 4 times the measure of each exterior angle. What is the number of sides of a polygon?
2. Given a regular hexagon with side length 10 cm, find its area using the apothem formula.
3. The diagram shows 3 identical regular pentagons.

What is the value of θ ?



4.2. Theorem on polynomial functions and its graph

☞ By the end of this lesson students would be able to:

- Apply polynomial remainder theorem
- Apply polynomial factor theorem
- Identify the behaviors of the graph of polynomial function

Key concepts:

- If $f(x)$ and $d(x)$ are polynomials such that $d(x) \neq 0$, and the degree of $d(x)$ is less than or equal to the degree of $f(x)$, then there exists a unique polynomials $q(x)$ and $r(x)$ such that

$$\begin{array}{ccccccc} f(x) & = & q(x) & \cdot & d(x) & + & r(x) \\ \downarrow & & \downarrow & & \downarrow & & \downarrow \\ \text{Dividend} & & \downarrow & & \text{Divisor} & & \text{Remainder} \\ & & \text{Quotient} & & & & \end{array}$$

Where $r(x) = 0$ or the degree of $r(x)$ is less than the degree of $d(x)$.

- Let $f(x)$ be a polynomial of degree greater than or equal to 1 and let c be any real number. If $f(x)$ is divided by the linear polynomial $(x - c)$, then the remainder is $f(c)$
- Let $f(x)$ be a polynomial of degree greater than or equal to one, and let c be any real number, then $x - c$ is a factor of $f(x)$ if $f(c) = 0$.

Properties of the graph of polynomial function

- Degree n polynomial function has at most $n - 1$ turning points.
- Degree n polynomial function has at most n zeros.
- The y - intercept of polynomial function is its constant term.
- If a is the zero of polynomial function with odd multiplicity, then the graph crosses x - axis at $x = a$.
- If a is the zero of polynomial function with even multiplicity, then the graph touches x - axis at $x = a$, but it doesn't cross x - axis at this point.

- The graph of odd degree polynomial function with positive leading coefficient, moves far down from the left and move far up from the right
- The graph of odd degree polynomial function with negative leading coefficient, moves far up from the left and move far down from the right
- The graph of even degree polynomial function with positive leading coefficient, moves far up from the left and from the right
- The graph of even degree polynomial function with negative leading coefficient, moves far down from the left and from the right.

Examples

1. For the following pairs of polynomials, find polynomials $q(x)$ and $r(x)$ such that

$$f(x) = q(x)d(x) + r(x)$$

$$f(x) = -x^3 + 4x^2 - x - 6; \quad d(x) = x^2 + x - 3$$

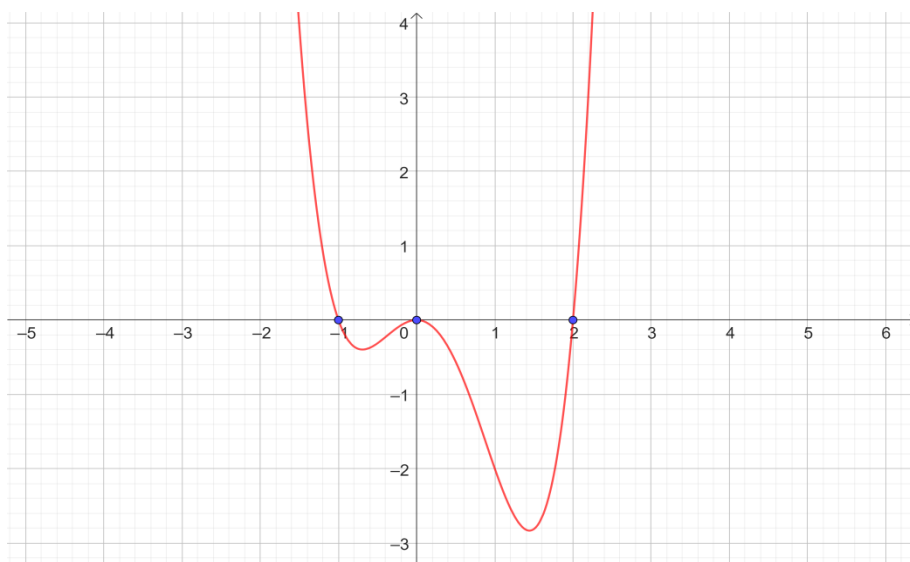
$$\begin{array}{r}
 \text{Divisor} \rightarrow x^2 + x - 3 \overline{) \begin{array}{l} -x^3 + 4x^2 - x - 6 \leftarrow \text{Dividend} \\ -(-x^3 + -x^2 + 3x) \\ \hline 5x^2 - 4x - 6 \\ - (5x^2 + 5x - 15) \\ \hline -9x + 9 \leftarrow \text{Remainder} \end{array} } \\
 \begin{array}{l} -x + 5 \leftarrow \text{Quotient} \end{array}
 \end{array}$$

2. When $f(x) = 5x^3 - kx^2 + 8x - 1$ is divided by $d(x) = x - 1$, the remainder is 10. Find the value of k .

This means $f(1) = 10 \rightarrow 5(1)^3 - k(1)^2 + 8 \times 1 - 1 = 10$. When the value of k is solved it gives $k = 2$

exercice

1. If $x + 3$ is a factor of $f(x) = x^4 + 2kx^3 - x^2 - 5kx + 6$, then find the value of k .
2. For the following of graph of polynomial function, determine
 - a. Number of turning points
 - b. Degree of the function
 - c. The sign of leading coefficient
 - d. The constant term of the function
 - e. The zeros of the function with their multiplicity (whether it is odd or even)



Exam type questions

1. Which one of the following is a true statement?
 - A. If you divide $5x^3 + 4x^2 + x + 9$ by $x + 2$, then the remainder is -17
 - B. $x - 1$ is the factor of $x^n + 1$, for a natural number n
 - C. If $x^4 + 4x^2 + 3$ is divided by $x - 1$; the constant term of the quotient is 3
 - D. $x - 2$ is the factor of $x^4 + 2x^3 - 3x^2 - 4x + 4$
2. Which one of the following is **NOT** true about the graph of the polynomial function
$$f(x) = 2x^5 - 7x^3 + 2x - 1$$
 - A. It has at most 4 turning points and intersects the x-axis almost 5 times
 - B. As we move far from the origin it rises upward to the right and falls downward to the left
 - C. It is a smooth and continuous curve
 - D. It passes through the origin
3. Let $f(x) = x^3 + 5x^2 - x + 8$ and $g(x) = x^2 + 2$. If $f(x)$ is divided by $g(x)$, then
 - A. the remainder is $x + 5$
 - B. the quotient is $3x + 2$
 - C. the constant term of the quotient is -2
 - D. the degree of the remainder is 1