

Experiment one

Measuring lengths Using a caliper gauge with vernier

Objective of the Experiment

- Determination of an outside, an inside-, and a depth dimension by means of a caliper gauge.
- Improvement of the measuring accuracy by means of a vernier.

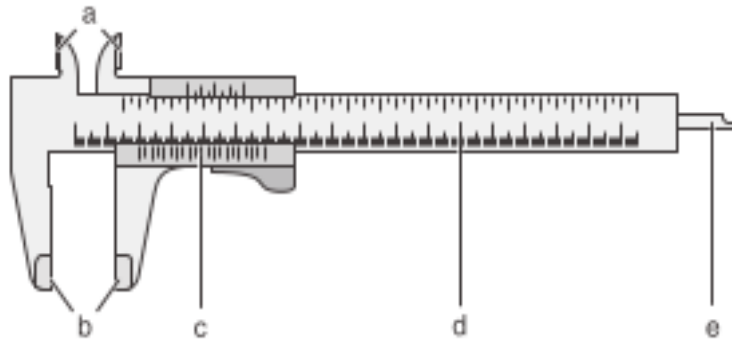


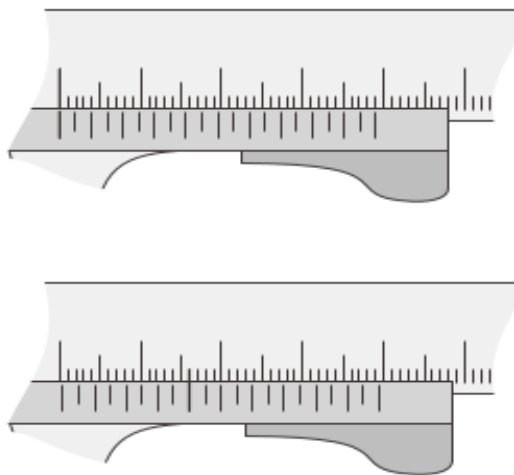
fig 1. Caliper gauge with a vernier

a acute measuring shank b long measuring shank
c slide with vernier d ruler with a millimeter scale
e feeler for depth measurements

fig. 2 Using a vernier

above: zero-position,

below: setting 0.40 mm



Principles

Measuring lengths is one of the oldest measuring difficulties. Therefore simple methods of measuring lengths are well known. For smaller lengths e. g. the caliper gauge often finds its use. It consists of a ruler with millimeter scale to which the measuring shank 1 has been attached at a right angle. The ruler simultaneously serves as a guide for the caliper gauge which carries measuring shank 2. The zero-mark of the gauge coincides with the zero-mark of the ruler if both measuring shanks are in contact.

There are two types of measuring shanks 1 and 2 available. Long measuring shanks serve to determine outside dimensions while acute measuring shanks measure inside dimensions. By means of an additional feeler, depth measurements can be performed (see Fig. 1).

In order to increase the measuring accuracy, a vernier scale has been engraved on the slide. Its long graduation marks show a distance of 3.9 mm. In zero-position the distance between the first long vernier line and the 4-mm-line of the ruler amounts to 0.1 mm, that of the second long vernier line to the 8-mm-line 0.2 mm and so on. When the slide is moved out of zero-position e. g. by 0.4 mm, the fourth long vernier line must coincide with a line of the ruler. This device allows for a reading accuracy of 0.1 mm. If also the additionally engraved short vernier lines are used, the reading accuracy is doubled once again (see Fig. 2).

Various dimensions x of a work piece are being determined several times during the test. Apart from the mean value of the sample

$$\bar{x} = \frac{1}{n} \cdot \sum_{i=1}^n x_i \dots\dots\dots(1)$$

n = number of individual measurements.

also the standard deviation of the sample is calculated.

$$S_x = \frac{1}{n-1} \cdot \sum_{i=1}^n (x_i - \bar{x})^2 \dots\dots\dots(2)$$

The latter is a measure for the spread of the individual measurements around the mean value [1]. It is being compared to the read-out accuracy of the caliper gauge.

Apparatus

1 Precision caliper gauge 311 54

additionally required:

1 Work piece with outside-, inside- and depth dimensions or

1 Saddle base 300 11

Setup and carrying out the experiment

a) Determination of an outside dimension:

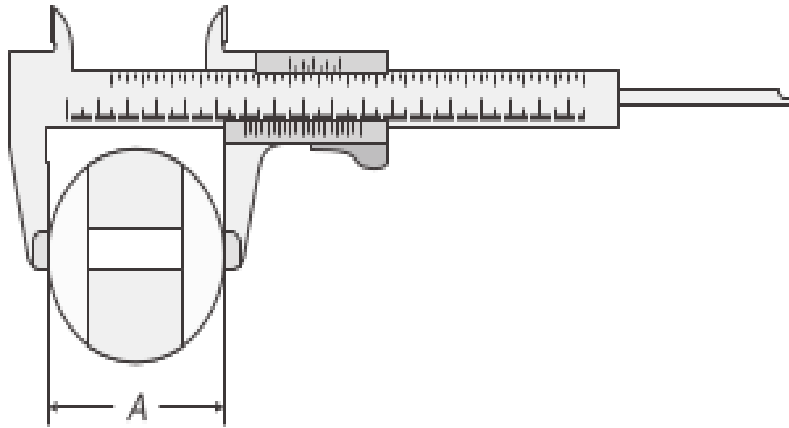


Fig. 3

a) Determination of an outside dimension with the long measuring shanks

- ✓ Loosen the catch of the gauge, bring the object to be measured between the long measuring shanks and push the measuring shanks into close contact without tilting.
- ✓ Estimate the outside dimension A on the millimeter scale and follow up with a more precise vernier reading.
- ✓ Remove caliper gauge, reset, and repeat measurement.

b) Determination of an inside dimension:

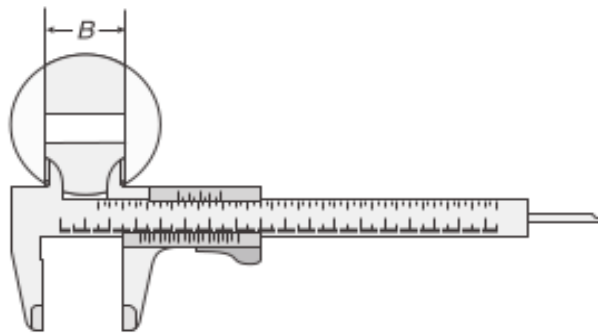


Fig. 4 Determination of an inside dimension with the acute measuring shanks

- ✓ Loosen the catch of the gauge. Guide the object to be measured over the acute measuring shanks and slide the measuring shanks apart so that they ride up on the inside plane without tilting.

- ✓ Estimate the inside dimension B on the millimeter scale and follow up with a more precise vernier reading.
- ✓ Remove caliper gauge, reset, and repeat measurement.

c) Determination of a depth:



fig. 5 Performing a depth measurement by means of a feeler

- ✓ Loosen the catch of the gauge. Set the ruler on the edge of the hole and by moving the slide lower the feeler until it makes contact with the base.
- ✓ Estimate the depth C on the millimeter scale and follow up with a more precise vernier reading.
- ✓ Remove caliper gauge, reset, and repeat measurement.

Measuring Example and Evaluation

a) Determination of an outside dimension:

Tab. 1: Individual measuring results of the outside measurement A

i	Millimeterscale	Vernier	$\frac{A_i}{\text{mm}}$
1	60	2.0	60.20
2	60	1.5	60.15
3	60	2.5	60.25
4	60	2.5	60.25
5	60	2.0	60.20

Mean value: 60.21

mm , Standard deviation: 0.04 mm, Reading accuracy: 0.05 mm,

b) Determination of an inside dimension:

Tab. 2: Individual measuring results of the inside measurement B

i	Millimeterscale	Vernier	$\frac{B_i}{\text{mm}}$
1	30	2.0	30.20
2	30	6.0	30.60
3	30	2.5	30.25
4	30	7.0	30.70
5	30	3.0	30.30

Mean value: 30.41 mm, Standard deviation: 0.22 mm, Reading accuracy: 0.05 mm

c) Determination of a depth:

Tab. 3: Individual measuring results of the depth measurement C

i	Millimeterscale	Vernier	$\frac{C_i}{\text{mm}}$
1	18	0.5	18.05
2	17	5.5	17.55
3	17	8.0	17.80
4	17	9.0	17.90
5	17	2.0	17.20

Mean value: 17.70 mm, Standard deviation: 0.33 mm, Reading accuracy: 0.05 mm

Experiment two

Determining the gravitational constant with the gravitation torsion balance after Cavendish

Objective of the experiment:

- To record the damped oscillations around the final equilibrium positions of the torsion pendulum as a function of time.
- To calculate the gravitational constant G using the end-deflection method.
- To calculate the gravitational constant G using the acceleration method.

Principles

The heart of the gravitation torsion balance according to Cavendish is a light transverse beam, horizontally suspended by a thin torsion string, which supports a small lead ball with the mass m_2 at each end at a distance d from the suspension point. These two balls are attracted by two large lead spheres with the mass m_1 . Although the force of attraction is less than 10^{-9} N, it is still possible to demonstrate this attraction using the extremely sensitive gravitation torsion balance. The motion of the small lead balls is observed and measured using the IR position detector (see Fig. 2.1).

The IR position detector has four infrared diodes which illuminate a concave mirror which is mounted in a fixed position on the transverse beam of the torsion pendulum. The reflected light is projected onto a row of photo-transistors to register the oscillations of the mass m_2 . On the basis of the motion over time, the mass m_1 and the geometry of the setup, it is possible to determine the gravitational constant using the end deflection method or – in an abbreviated measuring procedure by means of the acceleration method.

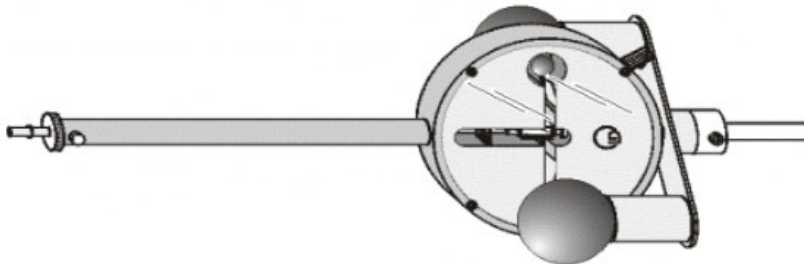
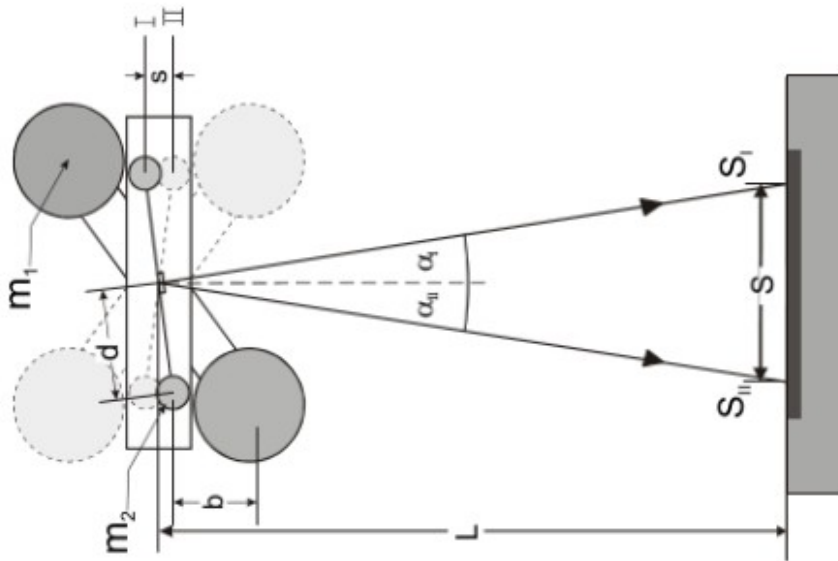


Fig. 2.1: Gravitation torsion balance according to Cavendish (left), and schematic diagram of the experimental setup (right).



Apparatus

1 Gravitation torsion balance	332 101
1 IR position detector (IRPD)	332 11
1 Optical bench, standard cross section 1 m	460 32
1 Optics rider 60/50	460 373
1 Optics rider 90/50	460 374
1 Stand rod, 25 cm	300 41
additionally required: 1 PC with Windows 98 or higher	

a) End-deflection method

The end-deflection method is based on the observation that the gravitational force between two lead spheres with the masses m_1 and m_2 at a distance b is given by (Fig. 2.1):

$$F = G \cdot \frac{m_1 m_2}{b^2} \quad \dots\dots\dots (2.1)$$

Thus, the moment of momentum M_1 acting on the torsion pendulum is

$$M_1 = 2 \cdot F \cdot d = 2 \cdot G \cdot \frac{m_1 m_2 d}{b^2} \quad \dots\dots\dots (2.2)$$

when the two large lead spheres of mass m_1 are in position I (see Fig. 1). The moment of momentum is compensated by the righting moment of the torsion cord. The torsion pendulum thus assumes the equilibrium positions I .

By swiveling the large lead spheres in position II the forces are symmetrically inverted. The moment of momentum acting on the bodies is now $M_{II} = -M_I$. The pendulum executes damped oscillations around the equilibrium positions II. For the difference of the two moments of momentum we have with the corresponding angles α_I and α_{II} :

$$D \cdot (\alpha_I - \alpha_{II}) = M_I - M_{II} = 2 \cdot M_I \dots\dots\dots(2.3)$$

The angular directional quantity D can be determined from the oscillation period T and the moment of inertia J of the torsion pendulum:

$$D = \frac{4\pi^2}{T^2} \cdot J \dots\dots\dots(2.4)$$

Safety notes

Mind the instruction sheets for the gravitation torsion balance and the infrared position detector:

- Protect the sensitive band of the gravitation torsion balance from uncontrolled mechanical loading.
- Always look the oscillation system of the gravitation torsion balance when the device is not used. In particular, make sure the oscillating system is locked during transport and assembly.
- The sensitive electronics of the IR position detector can be impaired or damaged by electrostatic discharge.
- Choose an experiment area where no electrostatic charges can be build up on either the operator or the devices.

The moment of inertia J is equivalent to the moment of inertia of the two small balls:

$$J = 2 m_2 d^2 \dots\dots\dots(2.5)$$

Thus, equation (2.4) is transformed to

$$D = \frac{8\pi^2}{T^2} \cdot m_2 \cdot d^2 \dots\dots\dots(2.6)$$

From equations (2.1), (2.3) and (2.4) we obtain

$$G = \frac{2\pi^2}{T^2} \cdot \frac{b^2 \cdot d}{m_1} (\alpha_I - \alpha_{II}) \dots\dots\dots(2.7)$$

From geometry follows the relationship (here for position)

$$\tan 2\alpha = \frac{S_1}{L} \dots\dots\dots(2.8)$$

from which follows for small angles:

$$\alpha = \frac{S_1}{2L} \dots\dots\dots(2.9)$$

By using this equation (2.8) equation (2.7) can be written as follows

$$G = \frac{2\pi^2}{T^2} \cdot \frac{b^2 \cdot d}{m_1} \cdot \frac{(S_I - S_{II})}{L} \dots\dots\dots(2.10)$$

b) Acceleration method

Directly after the large lead spheres are slewed from position I to position II, the small balls are subjected to an acceleration a_0 which follows from the equation of motion:

$$m_2 \cdot a_0 = \frac{2 \cdot G m_1 m_2}{b^2} \dots\dots\dots(2.11)$$

Thus the gravitational constant is given by:

$$G = \frac{a_0 b^2}{2 m_1} \dots\dots\dots(2.12)$$

The acceleration a_0 applied to the masses m_2 can be determined from the acceleration a_0 of the light reflection from the geometry relationship as follows:

$$a_0 = \frac{a_0' d}{2L} \dots\dots\dots(2.13)$$

As the path is given by

$$s_{(t)} = \frac{1}{2} a_0' t^2 \dots\dots\dots(2.14)$$

the acceleration a_0' can be obtained by fitting a parabola of the general form

$$s_{(t)} = A \cdot t^2 + B \cdot t + C \dots\dots\dots(2.15)$$

to the first phase of the motion. The comparison of equation (2.14) with equation (2.15) gives:

$$a_0' = 2 \cdot A \dots\dots\dots(2.16)$$

With equation (2.13) the gravitational constant is given by:

$$G = \frac{a_0' d \cdot b^2}{4 L m_1} \dots\dots\dots(2.17)$$

Setup

Important: satisfactory measuring results are only possible when the torsion balance is adjusted properly. In addition, the torsion oscillations caused by the attraction between the masses must not be disturbed by undesired pendulum motions. The torsion pendulum is extremely sensitive to shocks transmitted to the housing of the torsion balance. Changes in temperature cause convection currents in the housing of the torsion balance, which in turn results in unwanted movements of the torsion pendulum. Choose a stable experiment setup on a solid wall. Select an experiment site which is not exposed to direct sunlight or drafts.

When slewing the sphere support avoid shocks to the housing. e.g. by knocking at the setup with the lead spheres.

Fig. 2 shows the experimental setup.

Assembling the gravitation torsion balance before using it the first time:

- Assemble a bench top for the experimental setup on the wall like shown in Fig. 2 (see also instruction sheets 332 101 and 322 11).
- Setup the optical bench with the gravitation torsion balance.
- Position the gravitation torsion balance (without the large lead spheres) so that there is sufficient space to turn the supporting arm of the lead spheres.
- Loosen the arresting mechanism of the torsion pendulum and correct the alignment so that the pin at the end of the pendulum hangs in the middle of the rod hole and make sure that the torsion pendulum can swing freely.
- Allow the torsion pendulum to hang for one to two days and readjust the zero point if necessary (see instruction sheet 332 101).

Setting the distance between the gravitation torsion balance and the IR position detector for the first time:

- Clamp the IR position detector on the rear panel to stand rod which is inserted in the rider on the optical bench.
- Setup IR position detector on the optical bench in such a manner that the distance between the front window of the gravitation balance and the IR position detector is 70 cm.
- Connect power supply 12 V AC to the IR position detector and adjust the window with infrared LEDs roughly to the same level as the mirror of the gravitation torsion balance.
- The two red adjustment LEDs now light up so brightly that their mirror image can be seen in the plane of the device, either on the device itself or on a sheet of white paper which is held beside it:
 - If the image is to the left or right of the window, oscillate the gravitation torsion balance slowly so that the mirror image of the bright LEDs passes across the front panel.
 - If the image is above or below the window, project it onto the center by raising or lowering the IRPD.

Ensure that the row of photo transistors is situated in the plane which is covered by the mirror so that all of the photo transistors are used for measuring. Adjust the height by using the green and red LEDs. Assuming that the photo transistor row is in the plane illuminated by the mirror these LEDs are switched on and off depending on the luminous intensity of the photo transistors:

- Red LED flickers: illumination/adjustment sufficient
- Green LED flickers: illumination/adjustment good

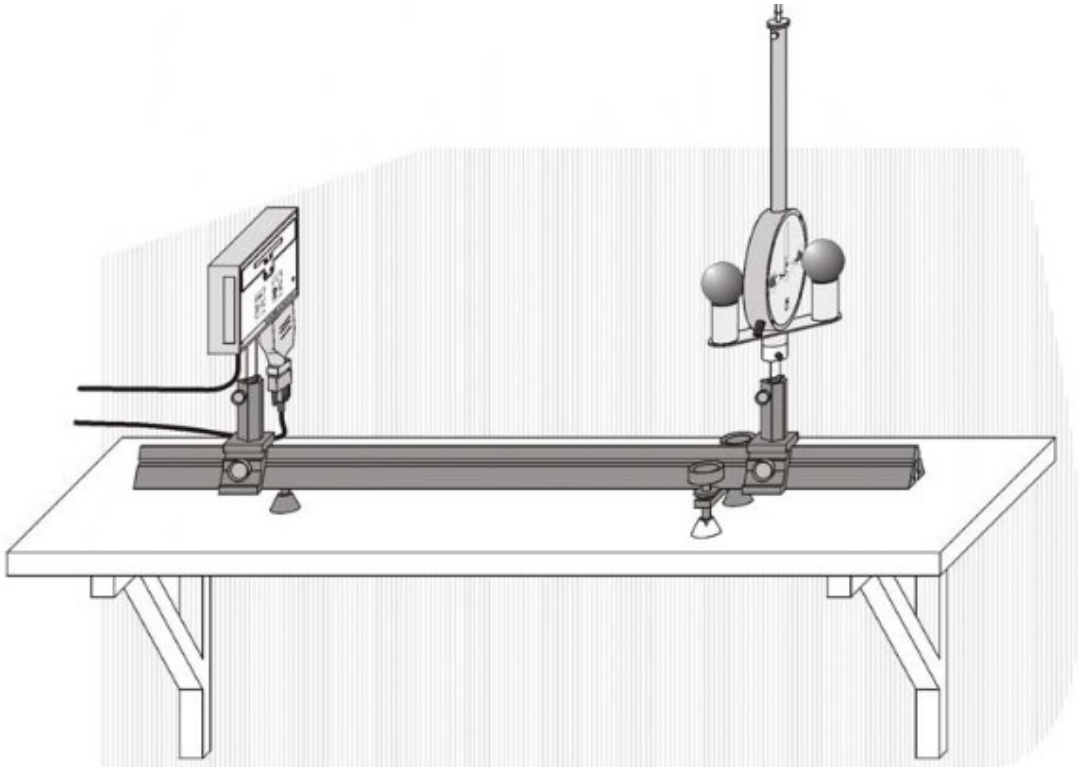


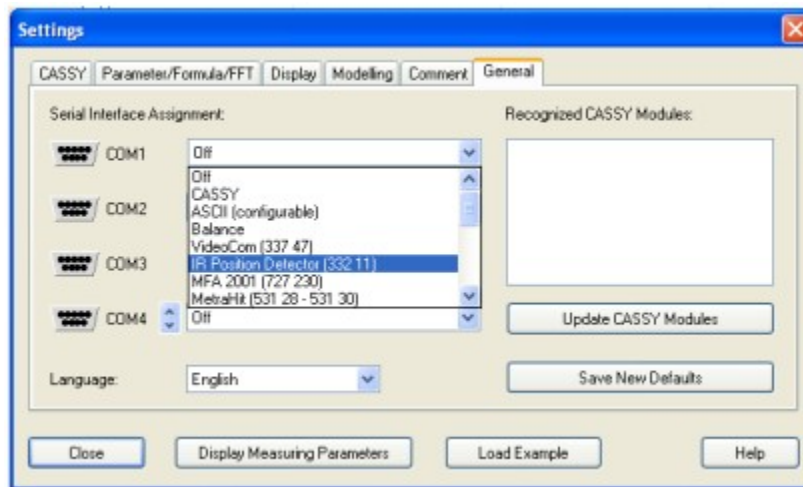
Fig. 2.2: Experimental setup: Bench top assembly of the gravitation torsion balance with electronic recording of the oscillation curve using the IR position detector.

Recording the oscillations

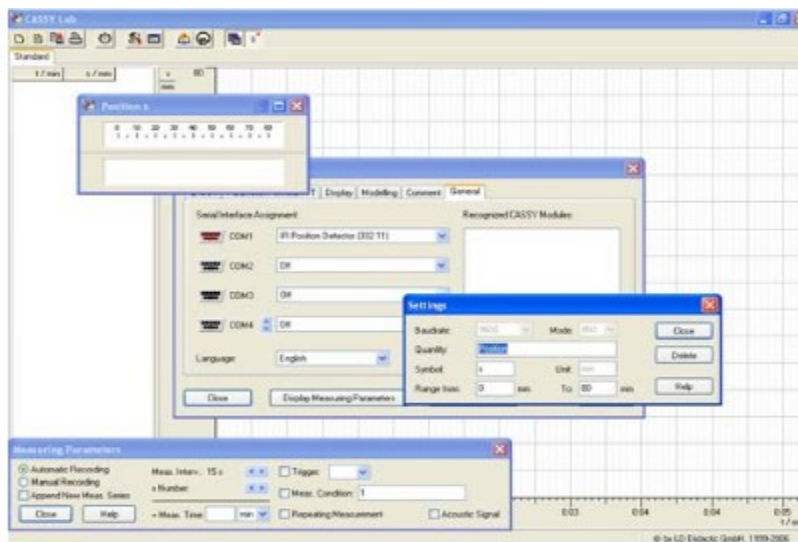
- Connect the IR detector to the computer via the RS232 port.
- If not yet installed install the software and open the software (preconfigured CASSY Lab user interface)
- Open the window “Settings” using the tool box button or function key F5 from the top button bar:



- Select the tab “General” in the window “Settings” and set the appropriate COM port by selecting the IR position detector:



After selecting the IR position detector for the appropriate Com port the software opens a table, display and various windows: indicator for the measuring quantity, i.e. the window “Position s”, the window “Settings” and the window “Measuring Parameters”:



Accept the preset values by closing all windows inside the main window. Press the button or function key F9 to start recording the oscillations of the gravitation torsion balance. Note: The button works as a toggle switch. The data acquisition can be stopped by pressing or F9.

These adjustments of the experimental setup have to be performed only once because the rest position is retained after the gravitation torsion balance has been locked.

Carrying out the experiment

First Preparation

- Allow the setup to stand for at least two hours undisturbed by shocks, so that the pendulum can come to rest in one of the equilibrium position.

Note: Typical equilibrium positions for a good adjustment are approximately 20 mm and 50 mm, respectively. If this is not fulfilled the gravitation torsion balance has to be rotated by a small angle. It might be necessary to repeat this step several times. If the arresting screw has been loosened after a long period of disuse, the torsion pendulum may require more time to settle into a stable equilibrium position.

- Check the stability of the zero point by recording the base line. Started data acquisition by the button .
- Measure the zero-point fluctuations for at least 10 minutes.

a) End-deflection method

- Wait until the system settles into a stable equilibrium position (see above first preparation).
- You may clear the baseline measurement with the button or the function key F4.
- Start data acquisition by pressing the button or F9.
- Move the supporting arm with the lead spheres rapidly (but carefully!) from position I to position II.
- Swivel the lead spheres from position II back to position I and repeat the measurement for the oscillations around the equilibrium position I.
- Stop the data acquisition by pressing the button or F9.

Note: You may save your measurements by pressing the button or using the function key F2.

b) Acceleration method

- Wait until the system settles into a stable equilibrium position (see above first preparation).
- You may clear the baseline measurement with the button or the function key F4.
- Start the data acquisition by pressing the button or F9.
- Move the supporting arm with the lead spheres rapidly (but carefully!) from position I to position II and record the first phase of the motion.
- Stop the data acquisition by pressing the button or F9.

Note: As this experiment uses the IR position detector for recording the oscillations of the gravitation torsion balance the acceleration method is automatically included when performing the end-deflection method. The “measurement interval” is set to an appropriate value, e.g. 15 s.

Measuring example

Fig. 3 shows an illustration of damped oscillations of the gravitation torsion balance and the two corresponding final positions. While swiveling the lead spheres from position II back to position I the data acquisition was not switched off.

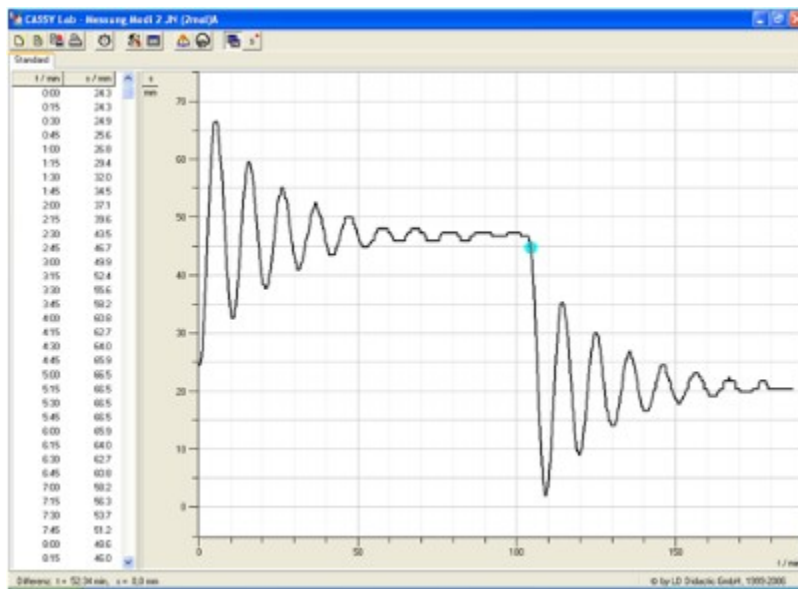


Fig. 3: Oscillations at the equilibrium positions I and II of the gravitation torsion balance according to Cavendish.

Evaluation

The software allows an easy evaluation of the measured data. To access the data evaluation tools click with right mouse button into the display (plot) to open the pop up menu.

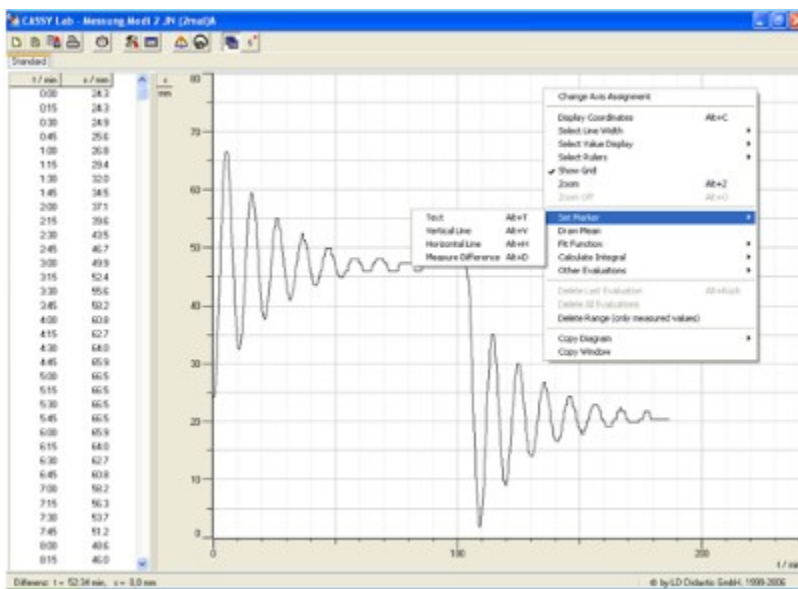


Fig. 4: Accessing the data evaluation tools by clicking with the right mouse button into the display.

a) End-deflection method

Note: The zoom tool (Alt-Z) might be useful to enlarge the plot of the recorded data. Alternatively, the scale of the display can be changed by clicking with the right mouse button into the x- or y-axis area.

Determining the equilibrium positions I and II

The two stable final equilibrium positions I and II of the two suspended small lead balls can be determined as follows:

- Calculate the mean average of e.g. position I by selecting the tool “Draw the Mean”
- Select the end part of the recorded oscillation by dragging over data the mouse pointer (selected data become blue).
- Plot the result of the calculation into the display by selecting the tool “Text” form the “Set Marker” popup menu (or Alt-T). Alternatively, the result can be dragged from the status line (left window bottom) into the display (Fig. 5).
- Repeat the evaluation for position II.

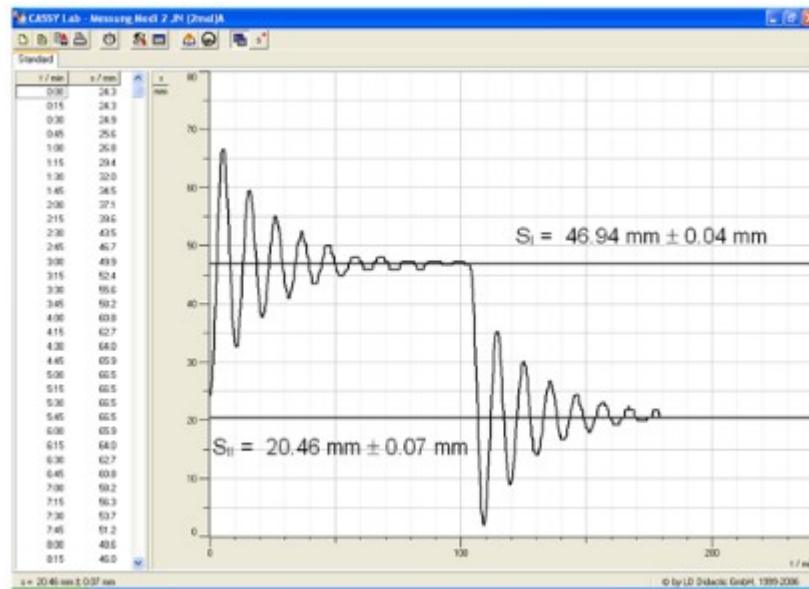


Fig. 5: Determining the equilibrium positions I and II by calculating the mean values S_I and S_{II} at the end of the oscillations.

Determining the oscillation period

- Select the “Measure Difference tool” by Alt-D (or by clicking the right mouse button into the display and choosing the sub menu “Select Marker”)
- Measure the period e.g. over 5 oscillations by clicking at the plot when the oscillation first passes the mean value S_I and then click at the plot when 5 oscillations have been elapsed.
- The result of the evaluation can be displayed using e.g. Alt-T. Alternatively, the result can be dragged from the status line into the display (Fig. 6).
- Repeat the evaluation for the oscillation around the stable position II.

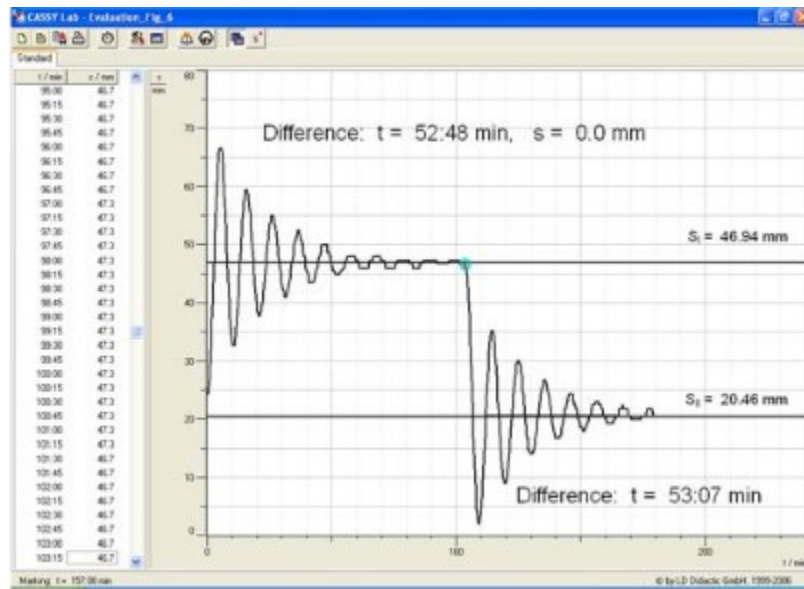


Fig. 6: Determining the oscillation period over 5 periods for the oscillations at positions I and II. (Note: The text of mean values for S I and S II of Fig. 5 was changed for sake of clarity.)

b) Acceleration method

- Zoom the data around the first phase of the motion, e.g. at position II, and fit a parabola to the recorded data.
- The result of the evaluation can be displayed in the plot by Alt-T.

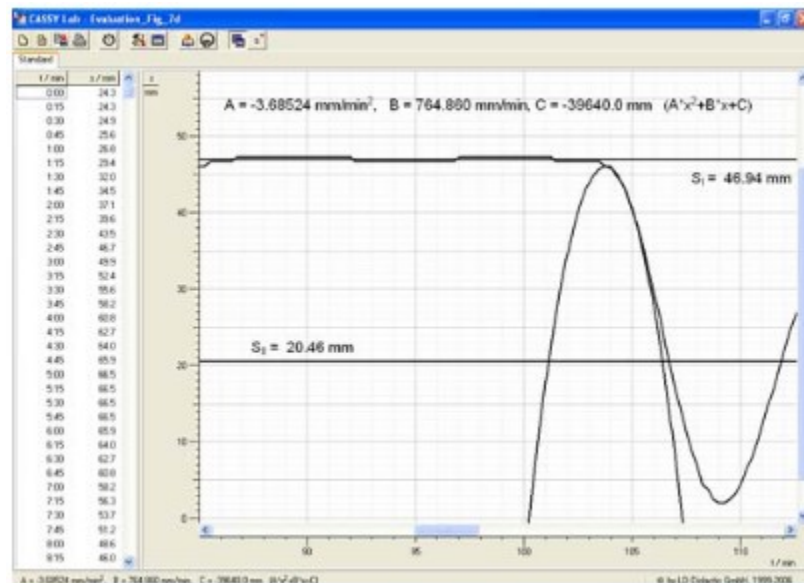


Fig. 7: Determining the acceleration for the first phase of the oscillation after swiveling the lead spheres from position II back to position I (for smaller zoom range see Fig. 8).

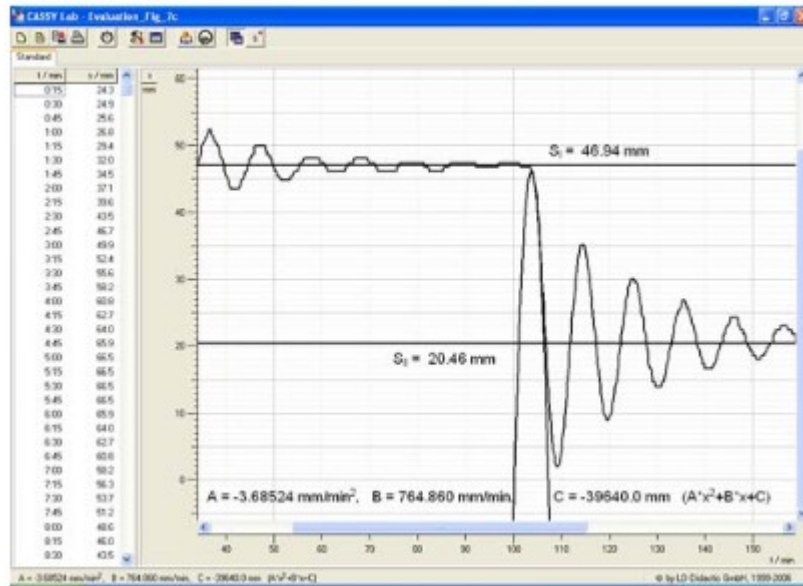


Fig. 7: Determining the acceleration for the first phase of the oscillation after swiveling the lead spheres from position II back to position I (for larger zoom range see Fig. 7).

Results

a) End-deflection method

The calculation of the sample measurement of Fig. 5 and Fig. 6 shows:

Period of oscillation $T = 634 \text{ s}$

The difference between the two final positions:

$$S = S_{II} - S_I = 26.5 \text{ mm}$$

Substituting these values into equation (IX) we obtain:

$$G = \frac{2\pi^2}{634^2 \text{ s}^2} \cdot \frac{0.047^2 \text{ m}^2}{1.5 \text{ kg}} \cdot \frac{0.05 \text{ m}}{0.7 \text{ m}} \dots\dots\dots(2.18)$$

$$G = 6.71 \cdot 10^{-11} \frac{\text{m}^3}{\text{kg s}^2} \dots\dots\dots(2.19)$$

$$\text{literature value } G = 6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg s}^2} \dots\dots\dots(2.20)$$

b) Acceleration method

From matching a parabola to first phase of the oscillation after swiveling the lead spheres from position II back to position I (see Fig. 7) we obtain:

$$A = 1.03 \times 10^{-6} \frac{\text{m}}{\text{s}^2}$$

$$a_0' = 2.05 \times 10^{-6} \frac{m}{s^2}$$

$$G = 1.33 \times 10^{-6} \frac{m}{s^2} \cdot \frac{0.05m \times 0.047^2 m^2}{4 \times 1.5 kg \times 0.7 m}$$

$$G = 5.4 \times 10^{-11} \frac{m^3}{kgs^2}$$

Supplementary information

For a detailed treatment of the geometrical relations and the resulting their errors refer to the alternative light pointer method .

EXPERIMENT 3

Determining the Force along the plane and force normal to the plane by using Inclined plane

Objective of the experiment

- ➔ Measuring the force F_1 along the plane and the force F_2 normal to the plane of a body on an inclined plane as a function of the angle of inclination α .
- ➔ Comparing the measured forces F_1 and F_2 with the forces calculated through vectorial resolution of the force of gravity G .

Principles

The motion of a body on an inclined plane can be described most easily when the force exerted by the weight G (force of gravity) on the body is vectorially resolved into a force F_1 along the plane and a force F_2 normal to the plane. The force along the plane acts parallel to a plane inclined at an angle α , and the force normal to the plane acts perpendicular to the plane (see Fig. 1). For the absolute values of the forces, we can say

$$F_1 = G \cdot \sin \alpha \quad \dots\dots\dots(3.1)$$

and

$$F_2 = G \cdot \cos \alpha \quad \dots\dots\dots(3.2)$$

The experiment verifies this resolution. Here, the two forces F_1 and F_2 are measured for various angles of inclination α using precision dynamo meters. We can vary the angle of inclination α by moving a support with the height $h = 5$ cm to various distances s between the pivot of the inclined plane and the support point (see Fig. 1). We can say

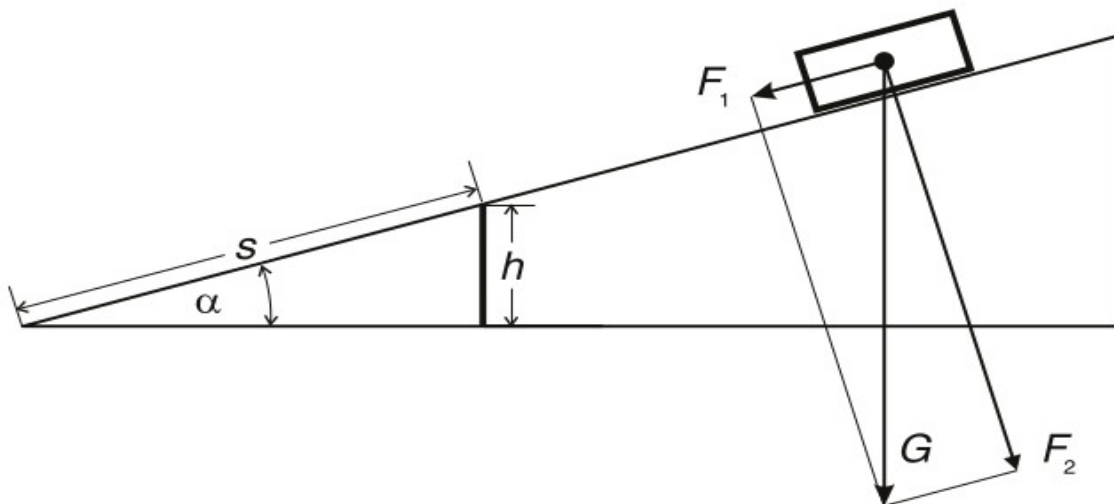


Fig. 3.1 Vectorial resolution of the force of gravity G into the force F_1 along the plane and the force F_2 normal to the plane on an inclined plane

$$\sin \alpha = \frac{h}{s} \dots\dots\dots(3.3)$$

and

$$\cos \alpha = \sqrt{1 - \left(\frac{h}{s}\right)^2} \dots\dots\dots(3.4)$$

(3.1) and (3.3) give us the force along the plane

$$F_1 = G \cdot \frac{h}{s} \dots\dots\dots(3.5)$$

and (3.2) and (3.4) give us the force normal to the plane

$$F_2 = G \cdot \sqrt{1 - \left(\frac{h}{s}\right)^2} \dots\dots\dots(3.6)$$

Apparatus

- 1 Inclined plane with trolley and screw model 341 21
- 1 Precision dynamo-meter, 1.0 N 314 141

Setup and carrying out the experiment

a) Correcting the zero point of the dynamo-meter

- Lay out dynamo-meter F_1 horizontally and correct the zero point.
- Hold dynamo-meter F_2 vertically downward and correct the zero point.

b) Determining the force of gravity

- Suspend the trolley freely from dynamo-meter F_2 using the folding metal hook and determine the weight G of the trolley.

c) Determining the force along the plane and the force normal to the plane

- Set up the inclined plane and position the support (e) at $s = 50$ cm.
- Place the trolley (b) on the inclined plane and hook it to dynamo-meter F_1 (c); support the dynamo-meter with block (d).
- Carefully arrange dynamo-meter F_2 (a) as nearly perpendicular as possible to the inclined plane and lift the trolley until it is just barely touching the plane surface.
- Read off and write down forces F_1 and F_2 .
- Move the ramp support (b) to the positions $s = 40, 30, 20, 15$ and 10 cm one after another; each time arrange the dynamo-meter perpendicular to the inclined plane and read off and write down forces F_1 and F_2 .

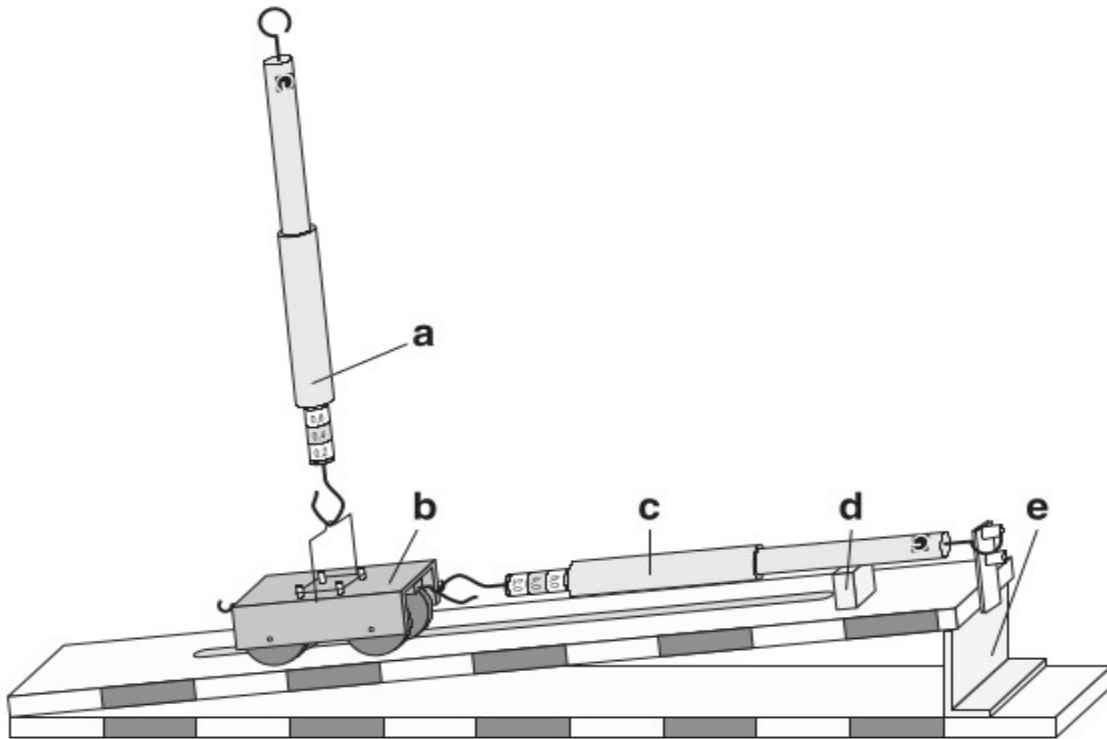


Fig. 3.2 Experiment setup for determining the force along the plane and force to the plane

Measuring example

$h = 5 \text{ cm}$ and $G = 1.07 \text{ N}$

Table 1: Position s of ramp support and forces F_1 and F_2 on the inclined plane

$\frac{s}{\text{cm}}$	$\frac{F_1}{\text{N}}$	$\frac{F_2}{\text{N}}$
50	0.10	1.01
40	0.12	0.98
30	0.18	0.97
20	0.27	0.97
15	0.35	0.95
10	0.59	0.81

Evaluation and results

Tables 2 and 3 enable a comparison of the measured forces and those calculated using (V) and (VI). Fig. 3 shows the results plotted in a graph.

For the force normal to the plane we see a systematic deviation between the measured and calculated values. This is due to the fact that the trolley is still partially supported when the force is measured.

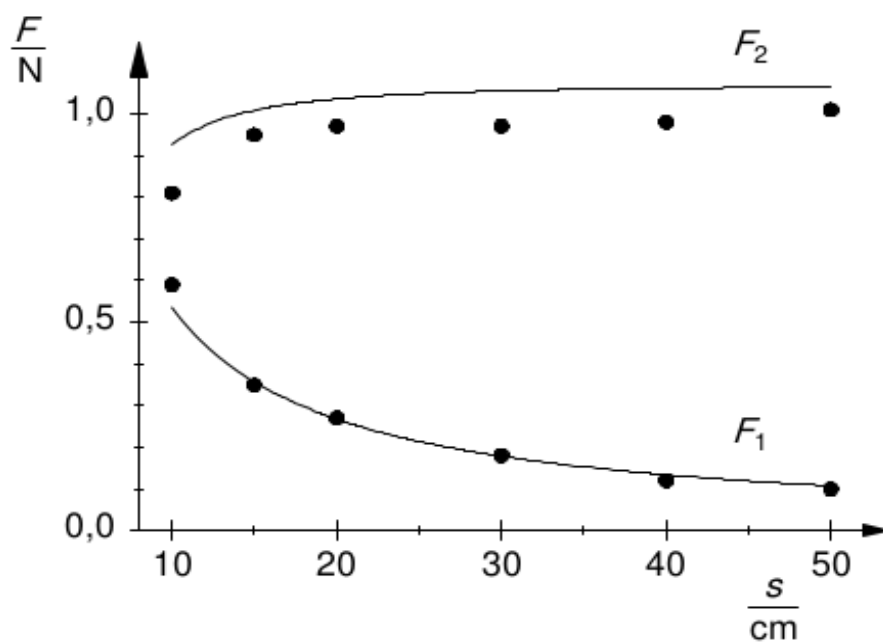
Table. 2: Measured an calculated force F 1 along the plane

$\frac{s}{\text{cm}}$	$\frac{F_1}{\text{N}}$ measured	$\frac{F_2}{\text{N}}$ calculated
50	0.09	0.107
40	0.12	0.134
30	0.18	0.178
20	0.27	0.268
15	0.33	0.357
10	0.53	0.535

Table 3: Measured an calculated force F 2 normal to the plane

$\frac{s}{\text{cm}}$	$\frac{F_1}{\text{N}}$ measured	$\frac{F_2}{\text{N}}$ calculated
50	1.01	1.065
40	0.98	1.062
30	0.97	1.055
20	0.97	1.036
15	0.95	1.009
10	0.81	0.927

Fig. 3 Measured (solid line) and calculated (points) values for the force F_1 along the plane and the force F_2 normal to the plane



EXPERIMENT FOUR

Free fall:

time measurement with the
contact plate and the counter S

Objects of the experiment

- Measuring the falling times of a ball between the holding magnet and the contact plate for recording the path-time diagram point by point.
- Confirming the proportionality between the falling distance and the square of the falling time.
- Determining the acceleration of gravity.

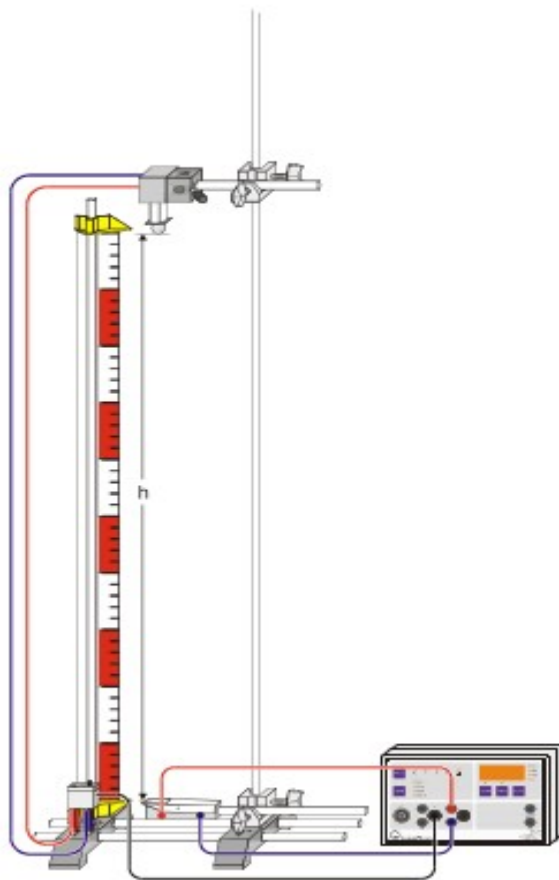


Fig. 4.1 Experimental setup for determining the acceleration of gravity with the contact plate and the counter S

Principles

When a body falls in the gravitational field of the earth from a height h to the ground, it experiences a constant acceleration g as long as the falling distance is small and friction can be neglected. This motion is called free fall. If the body starts at the time $t_0 = 0$ with the initial velocity $v_0 = 0$, the distance it cover in the time t is

$$h = \frac{1}{2} \cdot g \cdot t^2 \dots\dots\dots(4.1)$$

Thus the free fall is an example of a uniformly accelerated motion. In the experiment, the free fall is investigated on a steel ball which is suspended from an electromagnet (see Fig. 4.1). Due to its gravitational force

$$F = m \cdot g \dots\dots\dots(4.2)$$

m: mass of the ball

it falls downward in a uniformly accelerated motion as soon as the electromagnet is switched off. In this moment the electronic time measurement is started. Having covered the falling distance h, the ball hits a contact plate and stops the measurement at the falling time t. The measuring results for various falling distances are entered in a path-time diagram as pairs of values. As the ball is at rest at the beginning of the time measurement, E. g. (I) can be used to determine the acceleration of gravity g.

Apparatus

1 large contact plate, incl. steel ball	336 23
1 holding magnet with multi-clamp	336 21
1 holding magnet adapter with a release mechanism	336 25
1 counter S	575 471
2 stand bases MF	301 26
3 stand rods, Ø 10 mm, 25 cm	300 46
1 stand rod, Ø 12 mm, 150 cm	301 01
2 Leybold multi-clamps	301 21
1 scale with pointers	311 23
connecting leads	

Setup

The experimental setup is illustrated in Fig. 1.

- Set the two stand bases MF up so that the bores are in front and establish a rigid connection between then stand bases using two short stand rods.
- Clamp the scale in the left stand base MF and the long stand rod in the right one. The scale and the stand rod should not touch the ground.
- Mount the contact plate at the lower end of the stand rod and the holding magnet at the upper end of the stand rod.
- If necessary press the contact plate into the zero position (horizontally aligned, i.e. the switch is closed).

- Align the scale and the contact plate so that the height of the black impact surface is exactly 0 cm.
- Connect the connecting leads to the holding magnet and plug the free ends in the bores of the left stand base MF.
- Connect the holding magnet adapter with release mechanism to the free ends of the connecting leads and, on the other side, to the input E of the counter S.
- Using connecting leads, connect the two 4 mm sockets of the contact plate to the two 4 mm sockets of the input F of the counter S (right socket to ground).
- Connect the counter to the mains by means of the plug-in power supply.
- Arrange soft material around the contact plate as a cushion for the ball when it bounces from the contact plate.
- Suspend the ball from the holding magnet, and align the holding magnet so that the falling ball hits exactly the black impact surface.
- Suspend the ball from the holding magnet anew, and turn the knurled screw back until the ball just adheres to the magnet.
- Adjust the distance $h = 100$ cm between the lower edge of the ball and the contact plate.

Carrying out the experiment

- Set the operating mode of the counter to $t_{E \rightarrow F}$ by pressing the MODE key several times.
- If necessary, press the contact plate back into the zero position.
- Press the START key so that the associated status LED shines.
- Press the key of the holding magnet adapter quickly to start the free fall of the ball.
- When the ball has hit the contact plate, read the falling time and take it down.
- Reduce the falling distance h by 5 cm by lowering the holding magnet, press the contact plate into its zero position, and reset the counter S to zero by pressing the Start key.
- Suspend the ball anew, and repeat the measurement.
- Reduce the falling distance in steps of 5 cm, each time repeating the measurement.

Measuring example

Table 1: falling times t measured for various falling distances h

$\frac{h}{\text{cm}}$	$\frac{t}{\text{ms}}$	$\frac{h}{\text{cm}}$	$\frac{t}{\text{ms}}$
100	458	50	328
95	448	45	311
90	437	40	292
85	424	35	273
80	411	30	256
75	398	25	233
70	384	20	209
65	374	15	184
60	357	10	149
55	343	5	106

Evaluation

a) Falling distances $h = 10 \text{ cm}$, 40 cm and 90 cm : With the values from Table 1 we obtain:

$$\frac{t(40 \text{ cm})}{t(10 \text{ cm})} = \frac{0,292 \text{ s}}{0,149 \text{ s}} = 1.96 \approx 2$$

$$\frac{t(90 \text{ cm})}{t(10 \text{ cm})} = \frac{0.437 \text{ s}}{0.149 \text{ s}} = 2.93 \approx 3$$

For falling distances with the ratio $9 : 4 : 1$, the ratio of the falling times is $3 : 2 : 1$.

That means, the falling distance is proportional to the square of the falling time.

b) complete evaluation:

Fig. 2 shows the path-time diagram of the ball based on the values from Table 1. The ball experiences a uniform acceleration due to its gravitational force. Therefore the falling distance h covered is not a linear function of the time t . This is confirmed by a fit of the measured values to a parabola.

A linearization is obtained in Fig. 3 by plotting the falling distance against the square of the falling time (compare Table 2). Eq. (I) is confirmed by the agreement of the fitted straight line through the origin with the measured values. The slope A of the straight line gives

$$g = 2 \cdot A = 9.43 \frac{\text{m}}{\text{s}^2}$$

Literature value of the acceleration of gravity for Europe: $g = 9.81 \frac{\text{m}}{\text{s}^2}$

Table 2: Values of t^2 calculated for various falling distances h

$\frac{h}{\text{cm}}$	$\frac{t^2}{\text{s}^2}$	$\frac{h}{\text{cm}}$	$\frac{t^2}{\text{s}^2}$
100	0.210	50	0.108
95	0.201	45	0.097
90	0.191	40	0.085
85	0.180	35	0.075
80	0.169	30	0.066
75	0.158	25	0.054
70	0.147	20	0.044
65	0.140	15	0.034
60	0.127	10	0.022
55	0.118	5	0.011

Result

In a free fall, the falling distance h is proportional to the falling time t . From the factor of proportionality the acceleration of gravity g can be calculated.

Supplementary information

In the evaluation, the fact that the ball falls with a delay of a few milliseconds after pressing the Start key was not taken into account. This effect is the greater, the lower the knurled screw of the holding magnet has been turned. Moreover, the contact plate stops the time measurement after the ball has hit it with a certain delay. If, for example a time delay of 7.5 ms is taken into account in the present measuring data, the overall agreement of the measured value with the literature value of the acceleration of gravity is even better.

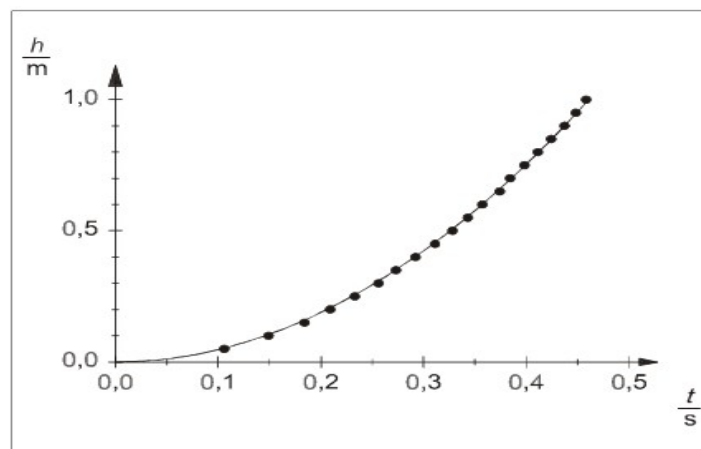


Fig. 2 Path-time diagram of the free fall of the ball

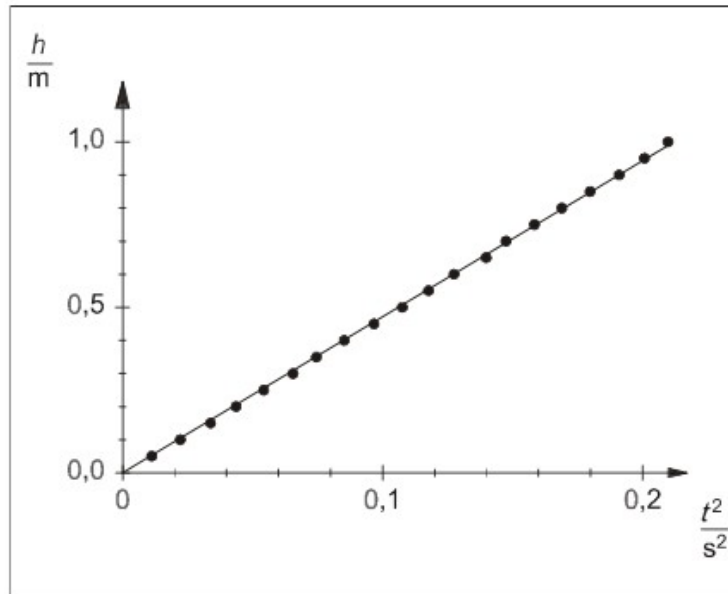


Fig. 3 Falling distance as a function of the square of the falling time

EXPERIMENT FIVE

Static friction, sliding friction and rolling friction

Objective of the experiment

- Investigating the static and sliding friction as a function of the area, the weight and the material.
- Comparison of static and sliding friction as a function of the weight and determining the coefficient of friction.
- Comparison of rolling and sliding friction as a function of the weight and determining the coefficient of friction.

Principles

In discussing friction between solid bodies, we distinguish between static friction, sliding friction and rolling friction. Static friction force is the minimum force required to set a body at rest on a solid base in motion. Analogously, dynamic (kinetic), or sliding friction force is the force required to maintain a uniform motion of the body. Rolling friction force is the force which maintains the uniform motion of a body which rolls on another body.

To begin, this experiment verifies that the static friction force F_H and the sliding friction force F_G are independent of the size of the contact surface and proportional to the resting force G on the base surface of the friction block. Thus we can say:

$$F_H = \mu_H \cdot G \quad \dots\dots\dots(1)$$

and

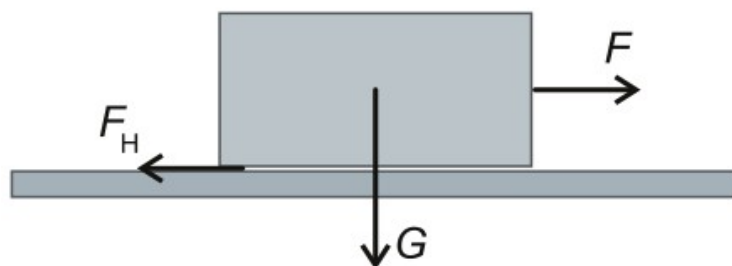
$$F_G = \mu_G \cdot G \quad \dots\dots\dots(2)$$

The coefficients of friction μ_H and μ_G depend on the material of the contact surfaces. As the static friction force is always greater than the sliding friction force, we can always say

$$\mu_H > \mu_G \quad \dots\dots\dots(3)$$

To distinguish between sliding and rolling friction, the friction block is placed on top of multiple stand rods laid parallel to each other. The rolling friction force

$$F_R = \mu_R \cdot G \quad \dots\dots\dots(4)$$



is measured as the force which maintains the friction block in a uniform motion on the rolling rods. The sliding friction force F_G is measured once more for comparison, whereby this time the friction block is pulled over the stand rods arranged as a fixed base. The experiment confirms the relationship

$$\mu_G > \mu_R \dots\dots\dots(5)$$

Setup

- Prepare clean, dry and smooth experiment surfaces (e. g. laboratory bench) for the friction experiments.
- If the resulting frictional forces are too slight, use different base surfaces.

Carrying out the experiment

a) Static and sliding friction as a function of the area, the weight and the material

- Place the small block on the experiment surface with the plastic side down.
- Using the dynamo-meter, measure the maximum horizontal pulling force at which the body remains stationary on the experiment surface as the static friction force F_H .
- Measure the horizontal pulling force which maintains a uniform motion on the experiment surface as the sliding friction force F_G .
- Place the wooden block on the base surface with the wide wooden side and then the narrow wooden side down and repeat your measurements for F_H and F_G .
- Repeat the measurements with the large block for friction experiments.
- Repeat the measurement on other surfaces as desired.

Apparatus

1 Pair of wooden blocks for friction experiments	315 36
1 Set 7 weights, 0.1 – 2 kg, with hook	342 10
1 Dynamo-meter, 10.0 N	314 47
6 Stand rods 10 cm	300 40

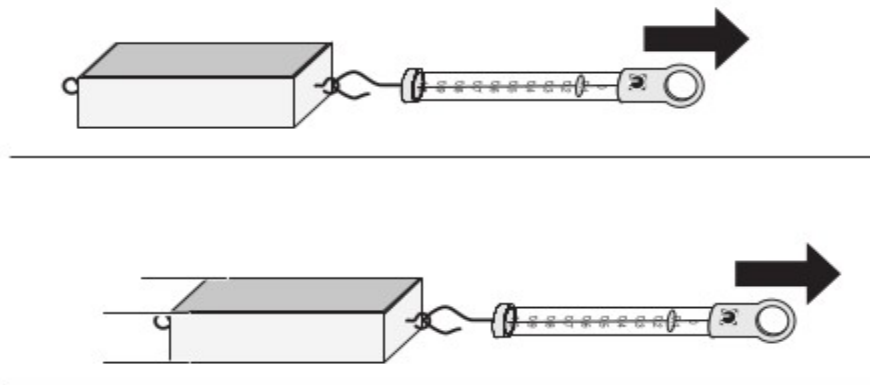


Fig. 1 Measuring the static friction force F_H (top) and the sliding friction force F_G (bottom).

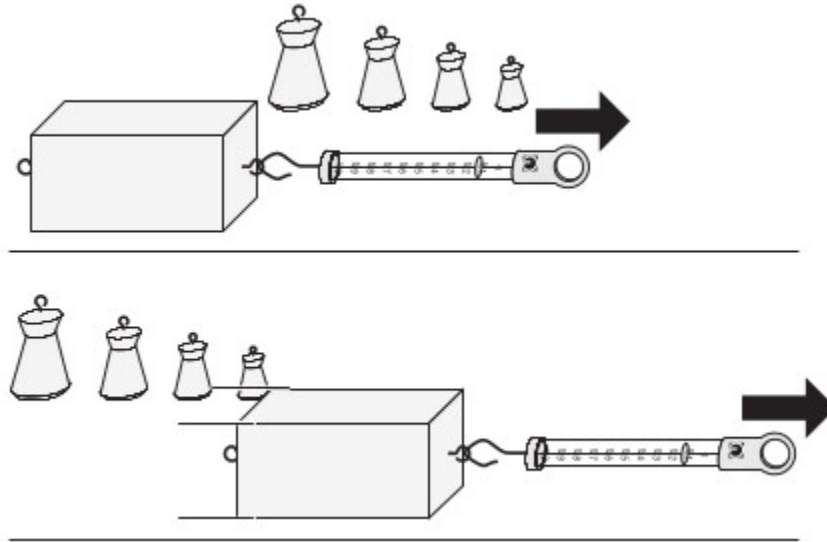


Fig. 2 Measuring the static friction force F_H (top) and the sliding friction force F_G (bottom) as a function of the force of gravity G .

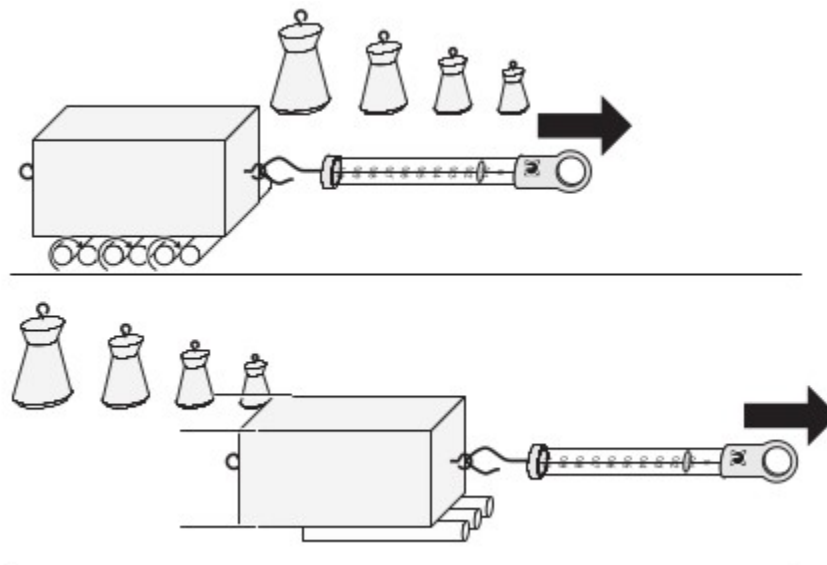


Fig. 3 Measuring the rolling friction force F_R (top) and the sliding friction force F_G (bottom) as a function of the force of gravity G .

b) Static and sliding friction force as a function of the force of gravity

- Place the large block on the experiment surface with the plastic side down and measure the static and sliding friction force.
- Increase the weight of the block by adding in turn the weights 0.1 kg, 0.2 kg, 0.5 kg and 1.0 kg and repeat the measurements.
- Carry out the same measurements for the wooden side of the block as well.

c) Rolling and sliding friction force as a function of the force of gravity

- Lay the stand rods next to each other and place the large block on the rods with the plastic side down.
- Measure the horizontal pulling force which maintains a uniform motion on the rolling rods as the rolling friction force F_R .
- Increase the weight of the block by adding in turn the weights 0.1 kg, 0.2 kg, 0.5 kg and 1.0 kg and repeat the measurements.
- Align the block parallel to the rod axes and measure the sliding friction force.

Measuring example

$$G_1 = 1.5\text{N} \quad \text{and} \quad G_2 = 3\text{N}$$

a) Static and sliding friction as a function of the area, the weight and the material

Experiment surface: plastic-coated bench top

Tab. 1: Static friction force F_H and sliding friction force F_G as a function of the force of the area, the weight and the material

$\frac{G}{N}$	Material	$\frac{A}{\text{cm}^2}$	$\frac{F_H}{N}$	$\frac{F_G}{N}$
1.5	Plastic	12×6	0.8	0.6
1.5	Wood	12×6	0.3	0.3
1.5	Wood	12×3	0.3	0.3
3.0	Plastic	12×6	1.6	1.1
3.0	Wood	12×6	0.5	0.5

b) Comparison of static and sliding friction force

Experiment surface: plastic-coated bench top

Table 2: Static friction force F_H and sliding friction force F_G as a function of the force of gravity G

	Plastic side		Wooden side	
$\frac{G}{N}$	$\frac{F_H}{N}$	$\frac{F_G}{N}$	$\frac{F_H}{N}$	$\frac{F_G}{N}$
3	1.6	1.1	0.5	0.4
4	2.2	2.0	0.8	0.6
5	3.1	2.8	0.9	0.8
8	5.0	4.6	1.9	1.3
13	8.3	8.0	3.0	2.0

c) Comparison of sliding and rolling friction

Table 3: Sliding friction force F_G and rolling friction force F_R as a function of the force of gravity G

$\frac{G}{N}$	$\frac{F_G}{N}$	$\frac{F_R}{N}$
3	3.0	0.1
4	4.3	0.2
5	5.2	0.2
8	9.0	0.3
11		0.4
13		0.5
18		0.6
23		0.7

Evaluation and results

a) Static and sliding friction as a function of the area, the weight and the material

As the measuring results in Table 1 show, both the static friction force and the sliding friction force depend on the material properties of the friction surfaces and on the weight (force of gravity) of the blocks. However, the friction forces are independent of the size of the friction area.

b) Comparison of static and sliding friction force

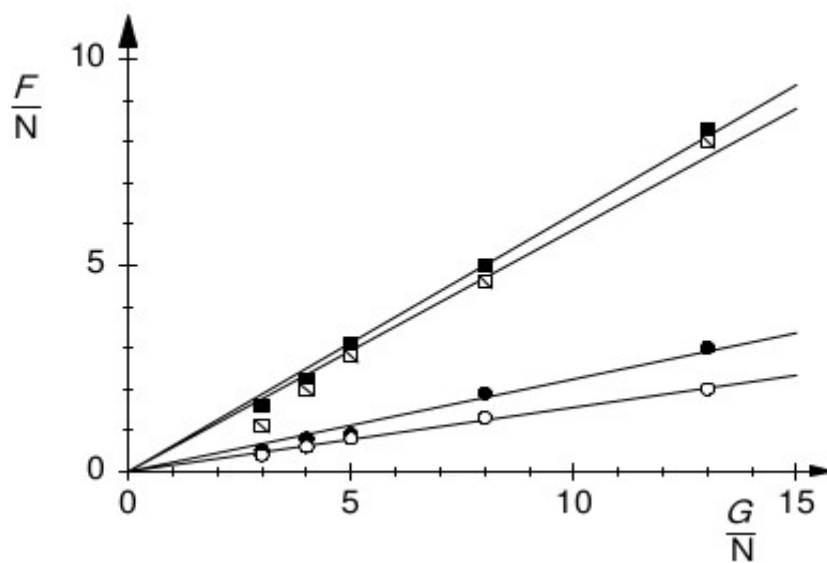


Fig. 4 Static friction force F_H and sliding friction force F_G as a function of the force of gravity G

Closed squares:

static friction for plastic Open squares: sliding friction for plastic

Closed circles: static friction for wood

Open circles: sliding friction for wood

Experiment surface: plastic-coated bench-top

Fig. 4 shows the results of the measurements. The slope of the line through the origin is equivalent to the coefficients of friction μ_H and μ_G calculated using (I) and (II) (see Table 2).

Material	μ_H	μ_G
Plastic	0.63	0.59
Wood	0.22	0.15

c) Comparison of sliding and rolling friction

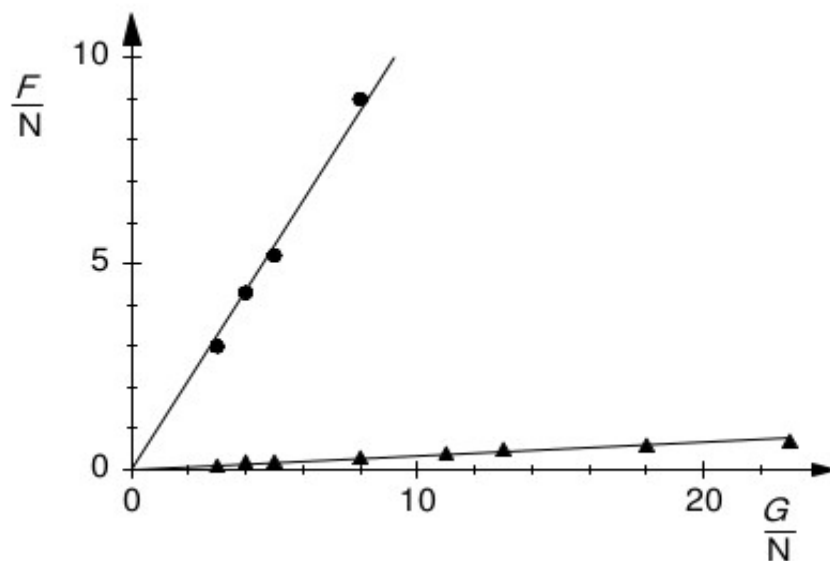


Fig. 5 Sliding friction force F_G (circles) and rolling friction force F_R (triangles) as a function of the force of gravity G

Fig. 5 shows the results of the measurements. The slope of the line through the origin is equivalent to the coefficients of friction μ_H and μ_G calculated using (I) and (II) (see Table 3).

Table 5: Rolling friction coefficient μ_R and sliding friction coefficient μ_G

Material:	μ_G	μ_R
Plastic	1.09	0.03

EXPERIMENT SIX

Centrifugal force of an orbiting body

Measuring with the centrifugal force apparatus

Objects of the experiment

- Determination of the centrifugal force as function of the angular velocity for different radii.
- Determination of the centrifugal force as function of the angular velocity for two different masses.

Principles

According to Newton's law of inertia a body on which no force is acting is either at rest or moving in a straight line with constant velocity. If, therefore, a body executes a curvilinear motion e.g. a circular motion forces must act upon the body to produce the change of direction. These forces are well known from the cases of a vast vehicle which takes a curved path and the merry-go-around. When measuring radial force (depending on the reference frame centripetal force or centrifugal force) the usual spring balances would alter because of their extension the radius of rotation of the body undergoing the measurement. In this experiment, therefore, following a suggestion of Prof. Schürholz the measurement of the radial force is done with a torsion fiber with which the distance of the body from the axis remains the same at rest and at rotation. If the body is rotating in a circle the radial force tilts a mirror whereby the change in the arc radius caused by the tilt is negligible. The tilt of the mirror is proportional to the radial force and can be detected by a light pointer. The arrangement is calibrated using a precision dynamo-meter while the centrifugal force apparatus is idle. The centrifugal or centripetal force F is given by

$$F = m \cdot \omega^2 \cdot r \dots\dots\dots(1)$$

In this experiment F is determined as function of the angular velocity ω for two different radii r and two different masses m . The angular velocity is determined from the orbit period T of the light pointer. T is measured manually by a stop clock.

Apparatus

1 Centrifugal force apparatus.....	347 22
1 Experiment motor	347 35
1 Control unit for experiment motor.....	347 36
1 Lamp, 6 V/30 W	450 51
1 Lamp housing	450 60
1 A spherical condenser	460 20

1 Transformer	521 210
1 Vertical scale, 1 m long.....	311 22
1 Stand base, V-shape, 20 cm.....	300 02
1 Stand rod, 47 cm.....	300 41
1 Saddle base	300 11
1 Leybold multi-clamp	301 01
1 Stop clock	313 07
1 Precision dynamo-meter, 1.0 N.....	314 14

Setup

- Mount the experiment motor on the table like shown in Fig. 1.
- Insert the centrifugal force apparatus completely into the chuck. Thus an easy control of the zero position of light pointer is always possible during the experiment by just unfixing the centrifugal force apparatus and turn it to a position where the light pointer can be observed on the vertical scale.
- Position the vertical scale at a distance e.g. of about 80 cm from the experiment motor.
- Arrange the lamp with the a spherical condenser with a slit in the diaphragm holder in such a manner that on the vertical scale the light pointer becomes visible.

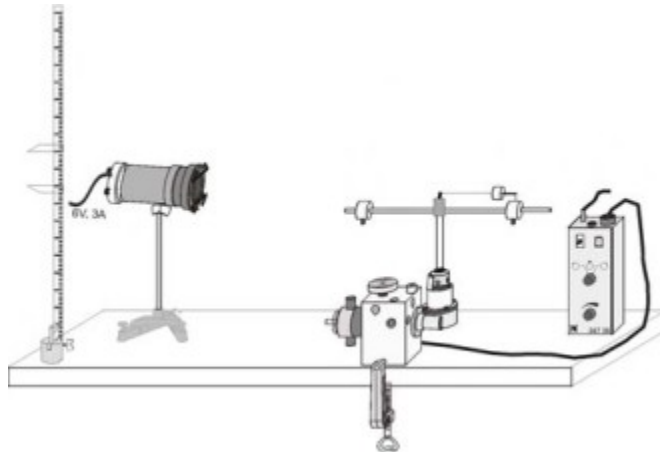


Fig. 1: Experimental setup(schematically).

Carrying out the experiment

The experiment can be performed in three steps.

- Measuring the centrifugal force as function of the angular velocity for two different the radii
 - Set up the centrifugal force apparatus for the mass $m = 12,5 \text{ g}$ with the wire $r = 10 \text{ cm}$.
 - Set the deflection of the light pointer to a small value, e.g. 2 cm, by increasing the speed of the experiment motor.

- Measure the period T with the stop clock. To increase accuracy measure the T as mean average of e.g. 10 periods.
- Repeat the these measurements for different speeds of the motor. It is recommended to increase the speed of the experiment motor each time in such a manner that the step of light pointer is increased by e.g. 2 cm.
- Repeat the experiment for the same mass with the wire $r = 20$ cm.

b) Measuring the centrifugal force as function of the angular velocity for two different the masses

Set up the centrifugal force apparatus for the body of mass $m = 25$ g with the wire $r = 10$ cm.

- Set the deflection of the light pointer to a small value, e.g. 2 cm by increasing the speed of the experiment motor.
- Measure the period T with the stop clock. To increase accuracy measure the time for e.g. 10 periods.
- Repeat the these measurements for different speeds of the motor. It is recommended to increase the speed of the experiment motor each time in such a manner that the step of light pointer is increased by 2 cm.

Hint: For a comparison the experiment can be repeated also for the body with $m = 25$ g with $r = 20$ cm.

c) Calibration of the centrifugal force apparatus

- Remove the body and the wire form the centrifugal apparatus.
- Set up the centrifugal force apparatus in such a manner that the light pointer can be observed on the vertical scale while the apparatus is at rest.
- Measure statically the deflection of the light pointer by using the precision dynamo-meter.

Hint: The zero point of the light pointer can easily be checked during and after the experiment by loosening the centrifugal force apparatus in the chuck.

Measuring example

a) Measuring the centrifugal force as function of the angular velocity for two different the radii

Table 1: Deflection s of the light pointer and period T for two different radii r for the mass $m = 12.5$ g

$\frac{s}{\text{cm}}$	$\frac{T}{s} (r = 10 \text{ cm})$	$\frac{T}{s} (r = 20 \text{ cm})$
2	1.43	2.06
4	1.10	1.52
6	0.93	1.25
8	0.79	1.12
10	0.71	1.00
12	0.64	0.91
14	0.61	0.86
16	0.56	0.80
18	0.54	0.74
20	0.51	0.73
22	0.48	0.69
24	0.46	0.65
26	0.45	0.64
28	0.42	0.62
30	1.43	0.60

b) Measuring the centrifugal force as function of the angular velocity for two different the masses

Table 2: Deflection s of the light pointer and period T for two different masses m ($r = 10 \text{ cm}$). see Table 1.

$\frac{s}{\text{cm}}$	$\frac{T}{s} (m = 12,5 \text{ g})$	$\frac{T}{s} (m = 25 \text{ g})$
2	1.43	1.89
4	1.10	1.47
6	0.93	1.26
8	0.79	1.10
10	0.71	0.99
12	0.64	0.91
14	0.61	0.84
16	0.56	0.80
18	0.54	0.75
20	0.51	0.72
22	0.48	0.69
24	0.46	0.66
26	0.45	0.63
28	0.42	0.61
30	1.43	0.60

c) Calibration of the centrifugal force apparatus

Table 3: Force F as function of the deflection s of the light pointer

$\frac{F}{N}$	$\frac{s}{cm}$
0.0	0.0
0.1	2.0
0.2	5.0
0.3	7.5
0.4	10.0
0.5	13.0
0.6	15.0
0.7	17.5
0.8	20.5
0.9	22.5
1.0	25.0

Evaluation and results

Fig 2. shows that the deflection s of the light pointer is proportional to the force F acting on the mirror:
 $F \sim S$

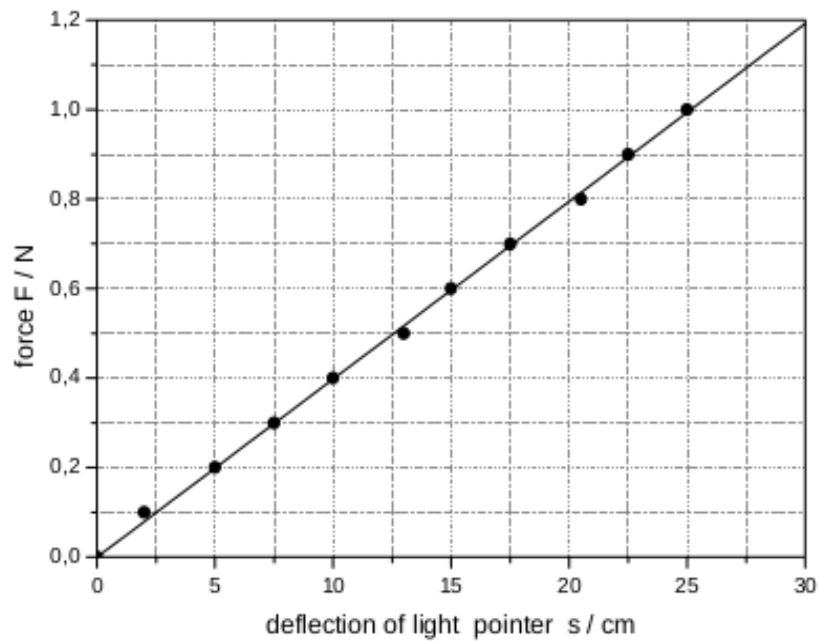


Fig. 2: Measured force F as function of the deflection s of the light pointer (centrifugal apparatus at rest).

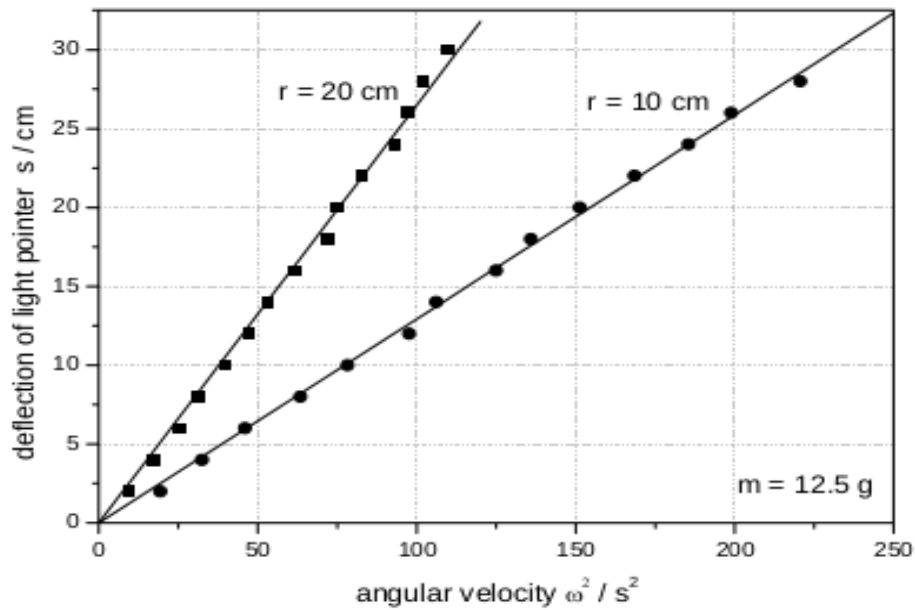


Fig. 3: Measured the deflection s of the light pointer as function of the square of the angular velocity for $m = 12.5 \text{ g}$.

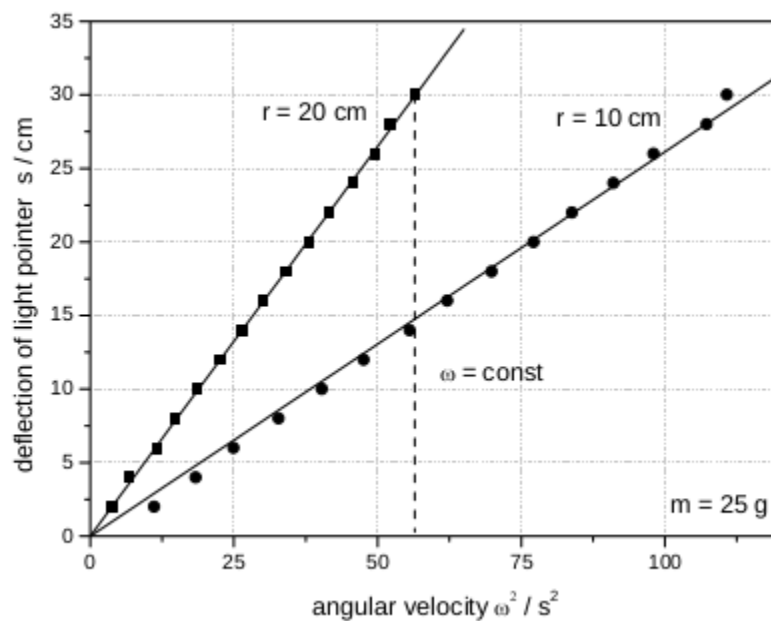


Fig. 4: Measured the deflection s of the light pointer as function of the square of the angular velocity ω for $m = 25 \text{ g}$.

Fig. 3 to Fig. 5 show also that the centrifugal force F is proportional to the square of angular velocity

$$F \sim \omega^2 \quad \dots\dots\dots(3)$$

Fig. 3 and Fig. 4 shows that the centrifugal force F is proportional to the radius r . For a constant angular velocity ω the force F increases in proportion to the radius:

$$F \sim r \quad \dots\dots\dots(4) \quad 41$$

Fig. 5 shows that the centrifugal force F is proportional to the mass m of the body in the centrifugal force apparatus. For a constant angular velocity ω the force F increases in proportion to the mass:

$$F \sim m \dots\dots\dots(5)$$

Collecting together the results of equation (3), (4) and (5) gives equation (1).

Supplementary information

From a stationary observer the body must be drawn with the radial force towards the axis of rotation. Instead of flying off in a straight line along the tangent the body has been forced to travel along a circular path due to the radial force (centripetal force). The experiment are interpreted differently for an observer who rotates with the body. This observer concludes that a force acts on the body in order to keep it in its place. This centrifugal force is equal and opposite to the centripetal force.

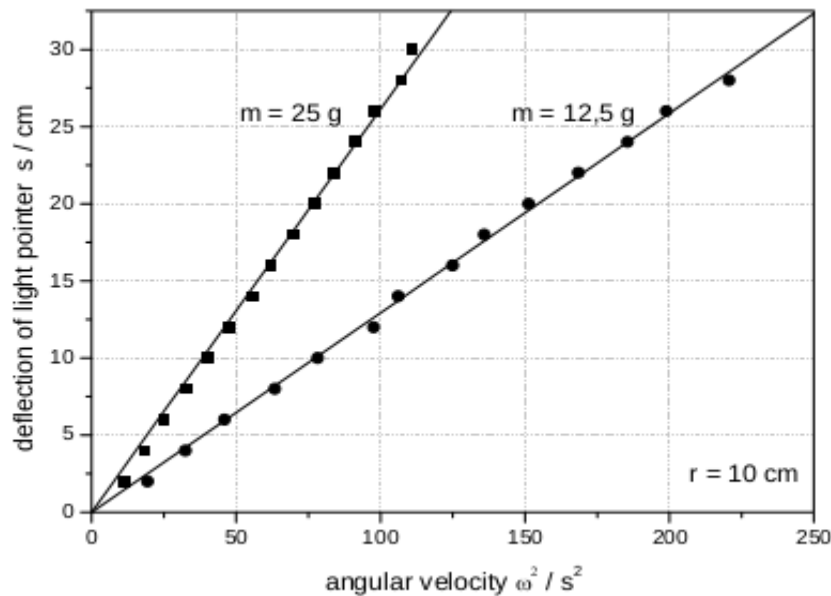


Fig. 5: Measured the deflection s of the light pointer as function of the square of the angular velocity ω for $r = 10$ cm.

EXPERIMENT SEVEN

Definition of moment of inertia

Objects of the experiment

- Measuring the period of oscillation of a thin transverse rod with weights on a torsion axle as a function of the distance of the weights from the torsion axle.
- Confirming the proportionality between the moment of inertia of the weights and the square of the distance.
- Determining the restoring torque of the torsion axle.

Principles

The moment of inertia is a measure of the inertia that a body exhibits when a torque acts on it causing a change of its rotational motion. It corresponds to the inertial mass in the case of translational motions. In rotational oscillations, for example, the period of oscillation T is the greater, the greater the moment of inertia J of the oscillating system is. More specifically:

$$T = 2\pi\sqrt{\frac{J}{D}} \dots\dots\dots(1)$$

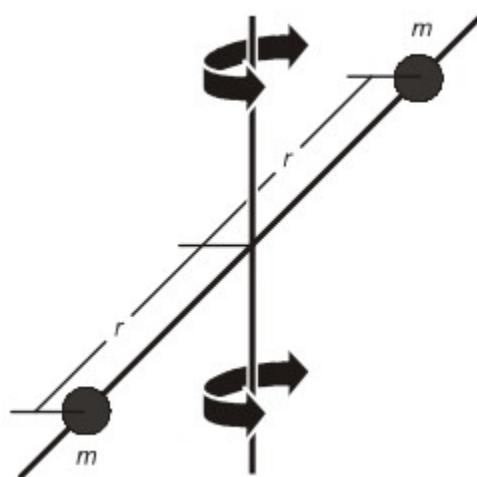
D: restoring torque

The moment of inertia of a point like mass m moving on a circular path with radius r is

$$J = m \cdot r^2 \dots\dots\dots(2)$$

The moment of inertia of two equal masses m that are rigidly connected and have the same distance r from the axis of rotation is

$$J = 2 \cdot m \cdot r^2 \dots\dots\dots(3)$$



In both cases, the moment of inertia is proportional to the square of the distance r . In the experiment, the rigid connection between the two masses is established by means of a thin rod whose middle is fixed to the torsion axle (see Fig. 1). After deflection from the equilibrium position, the system oscillates with the period of oscillation T . From e.q. (I) it follows that

$$J = D \cdot \left(\frac{T}{2\pi}\right)^2 \dots\dots\dots(4)$$

However, the moment of inertia is composed of the moment of inertia J_2 of the two weights and the moment of inertia J_0 of the rod:

$$J = 2 \cdot m \cdot r^2 + J_0 \dots\dots\dots(5)$$

Therefore the period of oscillation T_0 of the rod without weights is measured in another measurement, which leads to

$$D \cdot \left(\frac{T}{2\pi}\right)^2 = 2m \cdot r^2 + D \cdot \left(\frac{T_0}{2\pi}\right)^2 \quad \text{or} \quad T^2 = 8 \frac{m \cdot \pi^2}{D} \cdot r^2 + T_0^2 \dots\dots\dots(6)$$

Thus a linear relation between the square of the period of oscillation T and the square of the distance r is obtained. From the slope of the straight line,

$$a = \frac{8m \cdot \pi^2}{D} \dots\dots\dots(7)$$

the restoring torque D can be calculated if the mass m is known.

Apparatus

1 torsion axle.....	347 80
1 stand base, V-shape, 20 cm.....	300 02
1 stop clock I, 30 s / 15 min	313 07

Setup and carrying out the experiment

The experimental setup is illustrated in Fig. 2.

- Fix the middle of the transverse rod to the torsion axle and arrange the weights symmetrically at a distance of 30cm from the torsion axle.
- Mark the equilibrium position on the table.
- Rotate the transverse rod to the right by 180° and release it.
- Start the time measurement as soon as the transverse rod passes through the equilibrium position and stop the measurement after five oscillations.
- Repeat the measurement four times, alternately deflecting the rod to the left and to the right.
- Calculate the period of oscillation T from the mean value of the five measured values.

- One after another reduce the distance to 25 cm, 20 cm, 15 cm, 10 cm and 5 cm, each time repeating the measurement.
- Remove the weights, and repeat the measurement.

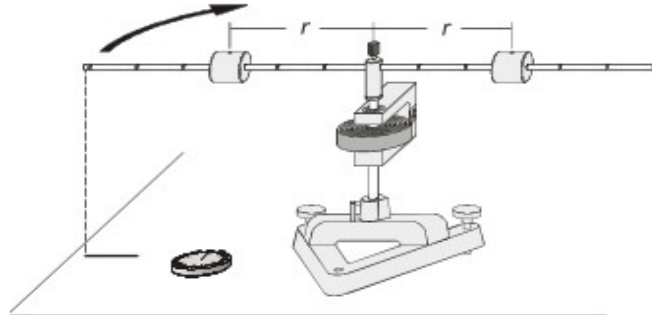


Fig. 2 Experimental setup for determining the period of oscillation.

Measuring example

individual weight to be put on the rod: $m = 0.24 \text{ kg}$

Table 1: measured duration of five oscillations for various distances r between the torsion axle and the weights and mean value of the period of oscillation T calculated from the measured values

$\frac{r}{\text{cm}}$	$\frac{5 \cdot T}{\text{s}}$					$\frac{T}{\text{s}}$
30	45.2	45.1	45.0	44.9	45.2	9.016
25	38.0	38.1	38.0	38.2	38.0	7.612
20	31.5	31.6	31.6	31.7	31.3	6.308
15	25.5	25.4	25.2	25.6	25.4	5.084
10	19.6	19.5	19.8	19.7	19.7	3.932
5	15.0	15.5	15.3	15.7	15.4	3.076
Without weights	13.1	13.3	13.3	13.4	13.4	2.660

Evaluation

From Table 1 we obtain $T_0 = 2.660 \text{ s}$ and therefore $T^2_0 = 7.076 \text{ s}^2$. The squares of the oscillation periods of the system with weights are listed in Table 2.

Table 2: list of the values of r^2 and T^2

$\frac{r}{\text{cm}}$	$\frac{r^2}{\text{cm}^2}$	$\frac{T^2}{\text{s}^2}$
30	900	81.288
25	625	57.943
20	400	39.791
15	225	25.847
10	100	15.461
5	25	9.462

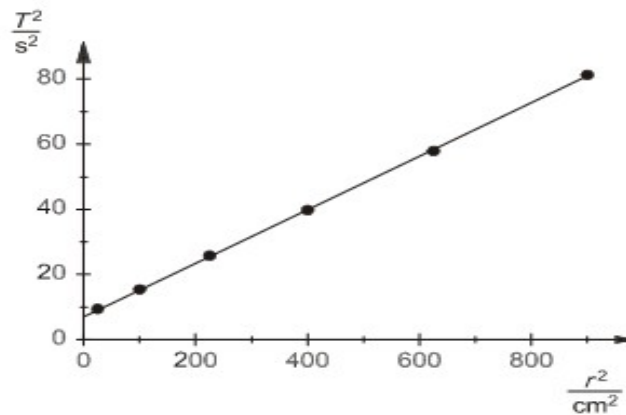


Fig. 3 Square of the period of oscillation T^2 as a function of the square of the distance r^2

In Fig. 3, the relation between T^2 and r^2 is shown graphically. Within the accuracy of measurement,

the values drawn in the figure lie on a straight line with the slope $a = 821 \frac{\text{s}^2}{\text{m}^2}$

As the mass m is known ($m = 0.240 \text{ kg}$), the restoring torque D can be calculated from the slope according to (VII). We obtain $D = 0.023 \text{ Nm}$.

Result

The period of oscillation of a transverse rod with weights on a torsion axle is the greater, the greater the distance of the weights from the axis of rotation is because the moment of inertia of the system increases. The moment of inertia of a “point like” mass with respect to an axis of rotation is proportional to the square of the distance from the axis of rotation.

EXPERIMENT EIGHT

Steiner's theorem

(parallel axis theorem)

Objects of the experiment

- Determining the moment of inertia of a circular disk for various distances between the axis of rotation and the axis of symmetry.
- Confirming Steiner's theorem (parallel axis theorem).

Principles

The moment of inertia of an arbitrary rigid body whose mass elements Δm_i have the distances r_i from the axis of rotation A is

$$J_A = \sum_i \Delta m_i \cdot r_i^2 \dots\dots\dots(1)$$

If the axis of rotation does not pass through the centre of mass of the body, application of Eq. (I) leads to an involved calculation. Often it is easier to calculate the moment of inertia J_S with respect to the axis S, which is parallel to the axis of rotation and passes through the centre of mass of the body.

For deriving the relation between J_A and J_S , the plane perpendicular to the axis of rotation where the respective mass element Δm_i is located is considered (see Fig. 1). In this plane, the vector a points from the axis of rotation to the centre-of-mass axis, the vector r_i points from the axis of rotation to the mass element Δm_i , and the vector s_i points from the centre-of-mass axis to the mass element. Thus

$$r_i = a + s_i$$

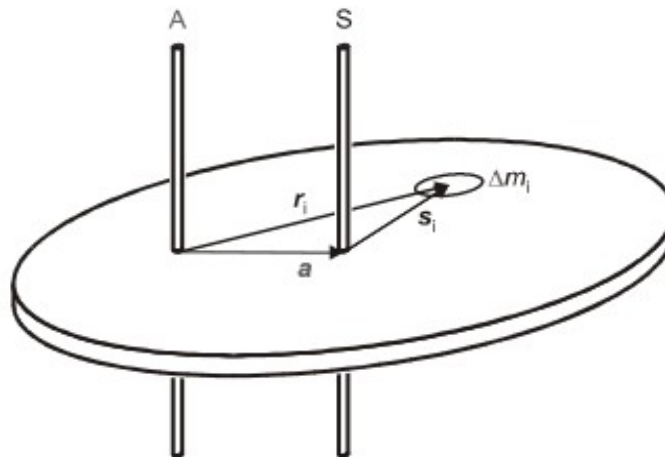


fig. 1 Schematic illustration referring to the derivation of Steiner's theorem (parallel axis theorem)

and the squares of the distances in Eq. (I) are

$$r_i^2 = (a + s_i)^2 = a^2 + 2as_i + s_i^2 \dots\dots\dots(2)$$

Therefore the sum in Eq. (1) can be split into three terms:

$$J = \left(\sum_i \Delta m_i \right) \cdot a^2 + 2 \cdot \left(\sum_i \Delta m_i \cdot s_i \right) \cdot a + \sum_i \Delta m_i \cdot s_i^2 \dots\dots\dots(3)$$

In the first sum and, $\sum_i \Delta m_i = M$

is the total mass of the body. In the last sum and,

$$\sum_i \Delta m_i \cdot s_i^2 = J_s \dots\dots\dots(4)$$

is the moment of inertia of the body with respect to the centre-of-mass axis. In the middle sum and,

$$\sum_i \Delta m_i \cdot s_i = 0$$

because the vectors s_i start from the axis through the centre of mass. Thus Steiner's theorem follows from Eq. (4):

$$J_A = M \cdot a^2 + J_s \dots\dots\dots(5)$$

This theorem will be verified in the experiment with a flat circular disk as an example. Its moment of inertia J_A with respect to an axis of rotation at a distance a from the axis of symmetry is obtained from the period of oscillation T of a torsion axle to which the circular disk is attached. We have

$$J_A = D \cdot \left(\frac{T}{2\pi} \right)^2 \dots\dots\dots(6)$$

D: restoring torque of the torsion axle

Apparatus

1 torsion axle.....	347 80
1 circular disk for the torsion axle	347 83
1 stand base, V-shape, 20 cm	300 02
1 stop clock I, 30 s / 15 min	313 07

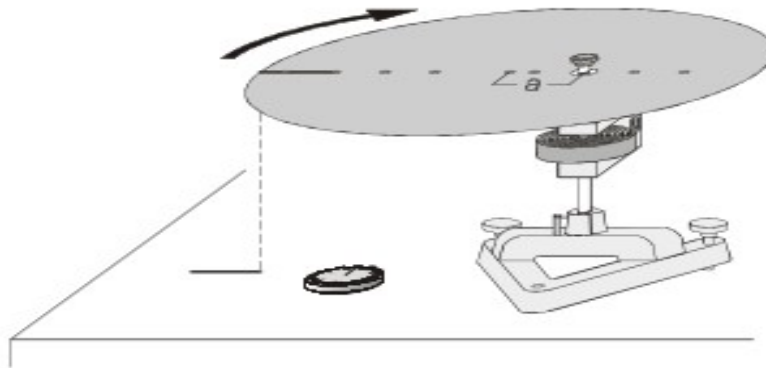
Setup and carrying out the experiment

The experimental setup is illustrated in Fig 2.

- Fix the centre of the circular disk to the torsion axle and mark the equilibrium position on the table.
- Rotate the circular disk by 180° from the equilibrium position and release it.
- Start the time measurement as soon as the circular disk passes through the equilibrium position and stop the measurement after five oscillations.
- Repeat the measurement four times alternately deflecting the disk to the left and to the right.
- Calculate the period of oscillation T from the mean value of the five measurements.

- Mount the circular disk on the torsion axle so that its centre is at a distance of 2 cm from the axle, and, if necessary, mark the equilibrium position anew.
- Measure the time of five oscillations five times alternately deflecting the disk to the right and to the left.
- Calculate the period of oscillation T .
- Repeat the measurement for other distances a from the axis of symmetry.

Fig. 2 Experimental setup for the experimental confirmation of Steiner's theorem (parallel axis theorem)



Measuring example

Mass of the disk: $M = 704 \text{ g}$ Radius of the disk: $R = 20 \text{ cm}$

Table 1: measured time of five oscillations for various distances a between the axis of rotation and the axis of symmetry and oscillation periods T calculated from the mean value of the measured values

$\frac{a}{\text{cm}}$	$\frac{5 \cdot T}{\text{s}}$					$\frac{T}{\text{s}}$
0	24.7	24.7	24.6	24.5	24.2	4.91
2	25.0	24.6	24.7	24.5	24.7	4.94
4	25.5	25.9	25.6	25.7	25.6	5.13
6	26.7	26.7	26.7	26.5	27.0	5.34
8	28.1	28.4	28.1	28.2	28.5	5.65
10	29.9	30.0	30.3	29.9	30.2	6.01
12	32.6	32.6	32.4	32.4	32.8	6.51
14	35.0	35.2	34.9	35.1	35.6	7.03
16	37.3	37.3	38.3	37.9	37.6	7.54

Evaluation

With the aid of Eq. (VI), the moment of inertia J_A can be calculated from the values of the period of oscillation listed in Table 1. The restoring torque D required for the calculation was determined in the experiment.

$$D = 0.023 \frac{Nm}{rad}$$

The results are listed in Table 2.

Table 2: list of the squares a^2 of the distance a and the moments of inertia J_A calculated from the period of oscillation T

$\frac{a}{cm}$	$\frac{a^2}{cm^2}$	$\left(\frac{T}{2\pi}\right)^2$ $\frac{s^2}{s^2}$	$\frac{J_A}{g \cdot m^2}$
0	0	0.610	14.0
2	4	0.618	14.2
4	16	0.667	15.3
6	36	0.723	16.6
8	64	0.809	18.6
10	100	0.916	21.1
12	144	1.074	24.7
14	196	1.253	28.8
16	256	1.439	33.1

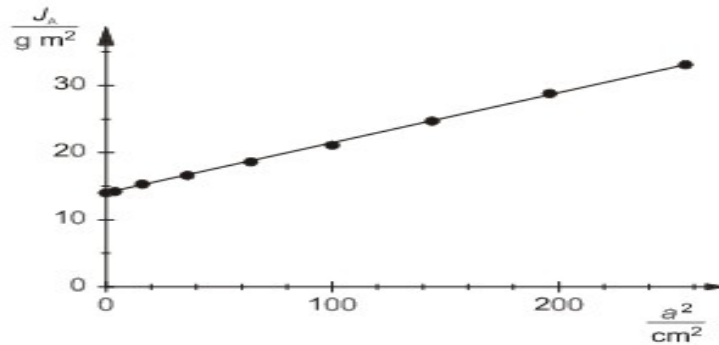


Fig. 3 Moment of inertia J_A as a function of the square of the distance a between the axis of rotation and the axis of symmetry

Eq.(5) describes a linear relation between J_A and a^2 with the slope M and the intercept of the ordinate J_S . This relation is confirmed by Fig. 3. The straight line drawn in the figure has the slope $M = 740 \text{ g}$ and the intercept of the ordinate $J_S = 14 \text{ gm}^2$.

Result

The moment of inertia of a body with respect to an arbitrary axis can be calculated from the moment of inertia with respect to the centre-of-mass axis, the total mass and the distance between the two axes.