GRADE 12

UNIT-6

Magnetism

Objective of this lesson

♣ **Upon completion of this unit we should be able to**

- Gain knowledge and understanding of the fundamental concepts magnetism.
- Define basic terms in magnetism like magnetic force, magnetic field and magnetic flux
- State basic law Ampere's law
- Develop skills and techniques of solving theoretical problems concerning magnetic force and field.

What is magnetism?

Magnetism is the branch of physics which studies about magnet and the description of how atoms of materials respond to a magnetic field.

- **Note**: - Magnet is a substance which can attract some magnetic materials. Magnet exhibits such a property due to the arrangements of its elemental magnets (spinning electrons).

Depending the arrangements formed by this unpaired electrons as they exposed to external magnetic field, materials can be classified as **Paramagnetic**, **diamagnetic**, and **ferromagnetic**

- **Paramagnetic:-** forms magnetic field the same direction as the external field as far as the field exists. E.g Al, Li, Ca, …..
- **diamagnetic:-** such substances has a tendency to form magnetic field in the opposite direction as the external field

  E.g silver, gold bismuth, copper

- **Ferromagnetic**:- such substance has a tendency to form magnetic field in the same direction as the external field or When a ferromagnetic substance is placed in an external magnetic field, its atomic moments tend to line up with the field. This alignment process (the arrangement of those unpaired charges) will be last long even in the absence of external field.

  E.g:- Iron, Cobalt, Nickel, Gadolinium, Fe_2O_3

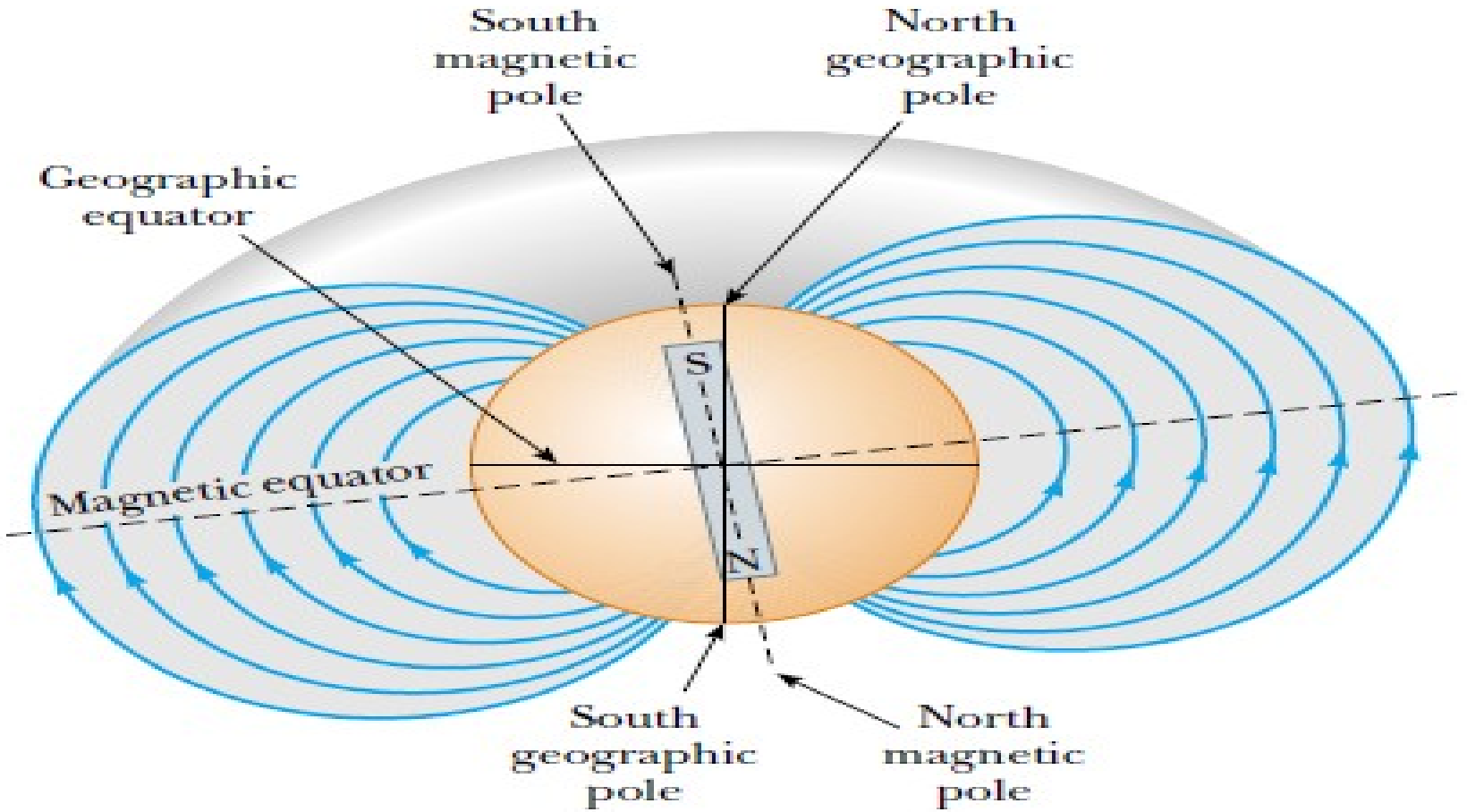
Magnets naturally may be formed due to the existence of ferromagnetic substances. Loadstone (leading stone) nowadays known to be Magnet named due to the place where it was first found in (Magnesia) was purely an iron ore known to be magnetite (Fe_2O_3).

This stone is known since the Greek philosophers, but it is well known and becomes familiarize after the 11th -13th century due to the rise of Navigation by huge ships. Loadstone/magnet/ which carved as needle shape is called compass. Compass is the basic tool for every navigator.

NOTE:- One of the amazing phenomenon of nature is Compass or any suspended magnet always rest in the North-South position. (why?)

A Compass or any suspended magnet always rest in the North-South position is due to the magnetic nature of the earth itself.

Our earth has magnetic nature. There are different hypothesis one was given by Sir William Gilbert. “earth’s Magnetic Nature arise due to a huge permanent bar magnet found deep inside our earth, inverting its polarities.”


South magnetic pole
North geographic pole
Geographic equator
S
N
Magnetic equator
South geographic pole
North magnetic pole

Around magnet there are two basic regions and these are *magnetic poles* and *magnetic fields*.

Magnetic poles:- are regions found at the two ends of magnet where there is a high concentration of magnetic force.

* There are *only two poles* (North-N) and (South-S) after Pierre de Maricourt named them for the first time.

* **Note**: - a single magnetic pole has never been isolated. That is, magnetic poles are always exist in pairs.

Magnetic Field: - is a region around magnet where we feel (detect) magnetic force.

* Magnetic field is represented by imaginary lines known as magnetic force of lines.

Property of field lines

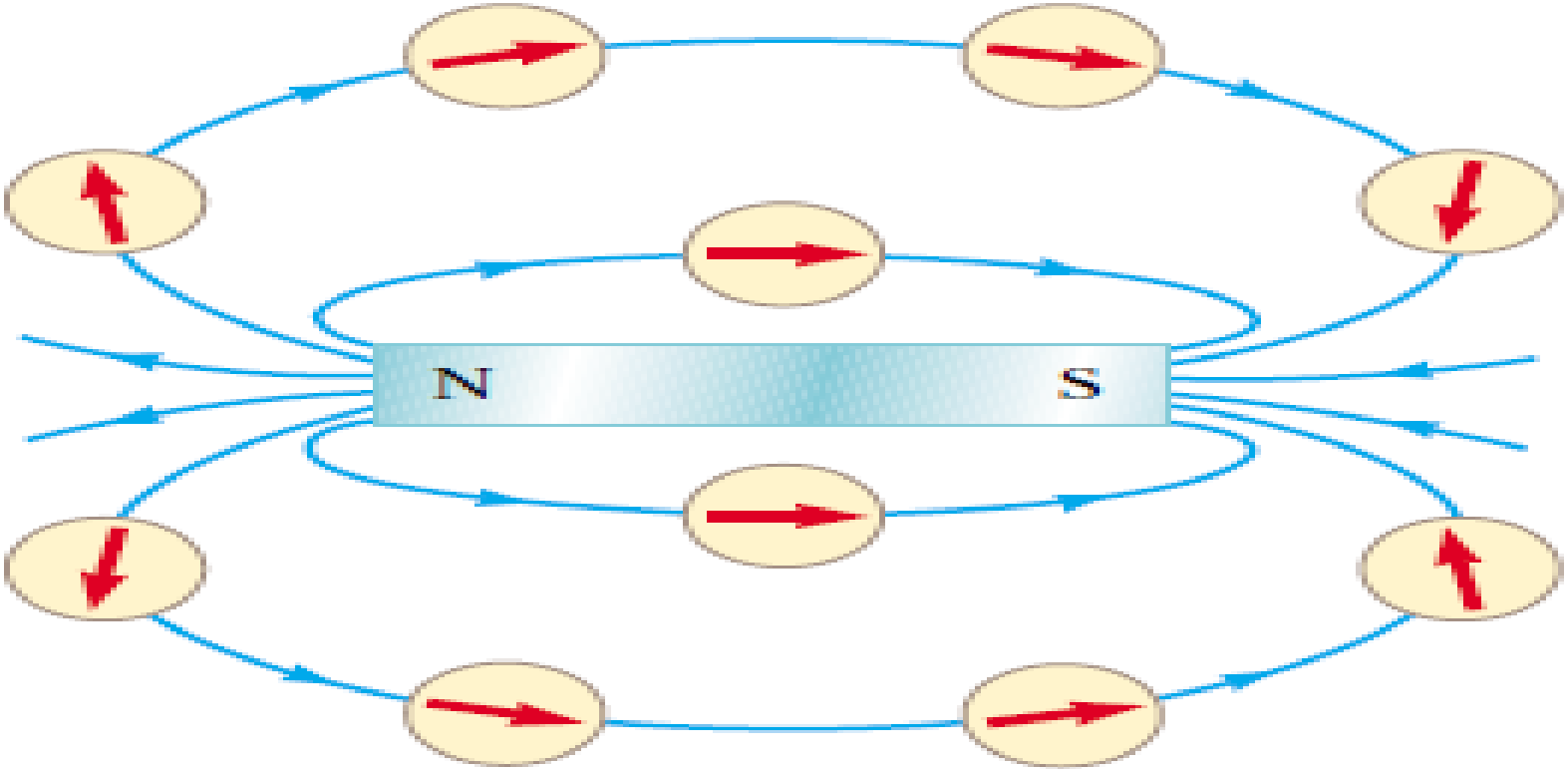
Using compass just nearer to a magnet, the field lines has the following properties,

i) They run out from North pole and run in to South pole outside the magnet.

ii) they never cross to each other

iii) they are denser at the poles

iv) magnetic force is tangent to the field lines.


N
S

Recall:-

- I) around any charged body there is electric field (E) and i.e. $\boxed{\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}}$

- In magnetism, the very same concept is stated as

  * Around any moving charged body there is magnetic field (B), and i.e. $\boxed{\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{V}\mathrm{x}\,\hat{r}}{r^2}}$ we will discuss it latter. Where

μ_0 = permiabilty of free space $\mu_0 = 4\pi x 10^{-7} Tm/A$

ii) If a charged particle is immersed in an electric field, the field will exert electric force and it is

$$F = Eq$$

In magnetism, the very same concept is stated as

* If a charged body is immersed in magnetic field and moving non-parallel to field, the field will exert magnetic force on to the charge. i.e.

$$\vec{F} = q\vec{V}x\vec{B} \Rightarrow F = Bvq\sin\theta$$

Where:- B= magnetic field strength, q= charge, V= speed of the charge, F= magnetic force and

θ = is the angle between the velocity of the charge and the magnetic field.

$F = Bvq \sin\theta$ $F_{max} = Bvq$ as $B \perp V$ & $\theta = 90^0$ and $F_{min} = 0$ as $B \parallel V$ and $\theta = 0,\ 180^0$.

$$B = \frac{F}{qV\sin\theta} \rightarrow B = \frac{F}{qV} \rightarrow [B] = \frac{[F]}{[q][V]} \rightarrow$$

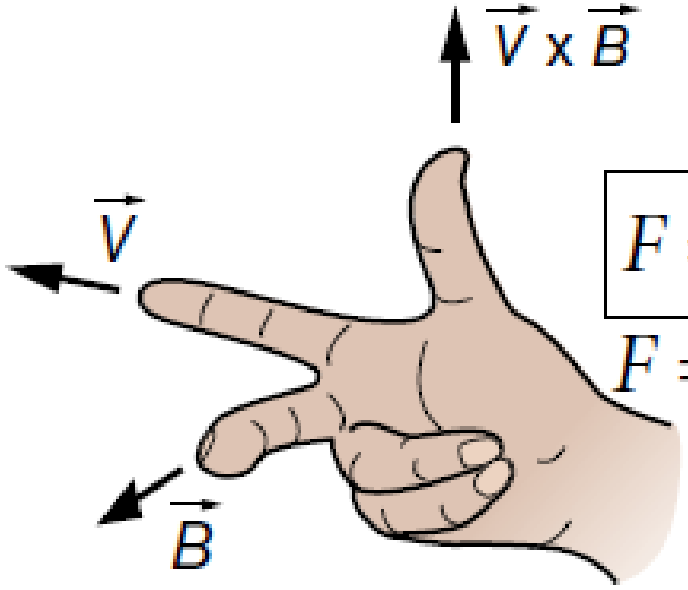
$$[B] = \frac{Newton\ (N)}{(coulomb)(^{meter}/_{second})} \Rightarrow [B] = \frac{NS}{Cm}\ or\ \frac{N}{\frac{C}{s}m} = \frac{N}{A.m} = Tesla\ (T)$$

Note:- The SI unit of magnetic field is Tesla in honor of the extra ordinary physicist Nicola Tesla.

$$1T = \frac{NS}{Cm}\ or\ \frac{N}{\frac{C}{s}m} \quad \& \quad 1T = \frac{N}{A.m}$$

** There is also non-SI unit known as Gauss (G) i.e. $1G = 10^{-4}T$

NOTE:- Magnetic force is always perpendicular to both the magnetic field and the velocity of the particle, and can be determined using right hand rule.

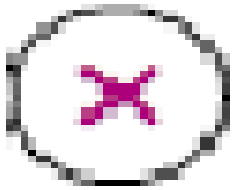


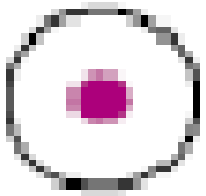
$$F = q\vec{V} \times \vec{B}$$

$$F = qvB\sin\theta$$

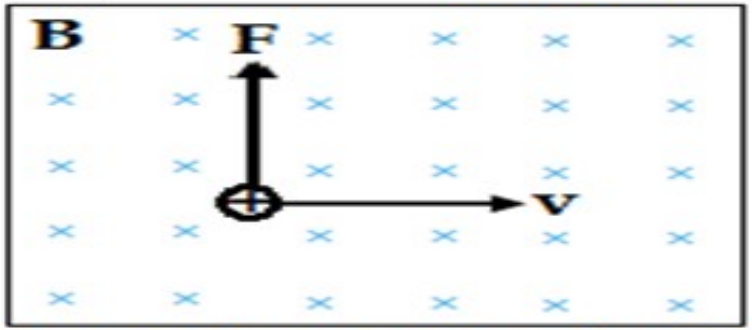
where θ is the angle between $\vec{v}$ and $\vec{B}$

conventional symbol of direction of magnetic field

 in to the page

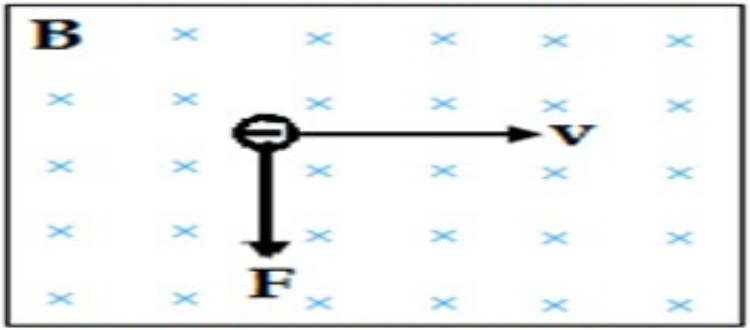
 Out of the page

positive charge



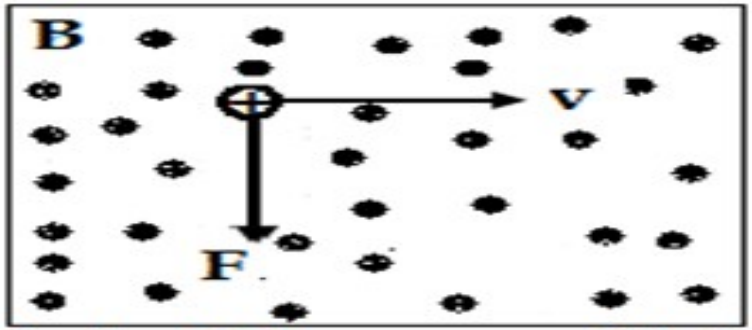
In to the page field

Negative charge



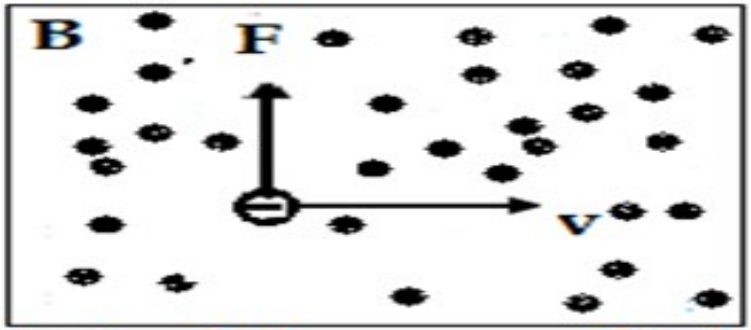
In to the page field

positive charge



Out of the page field

Negative charge

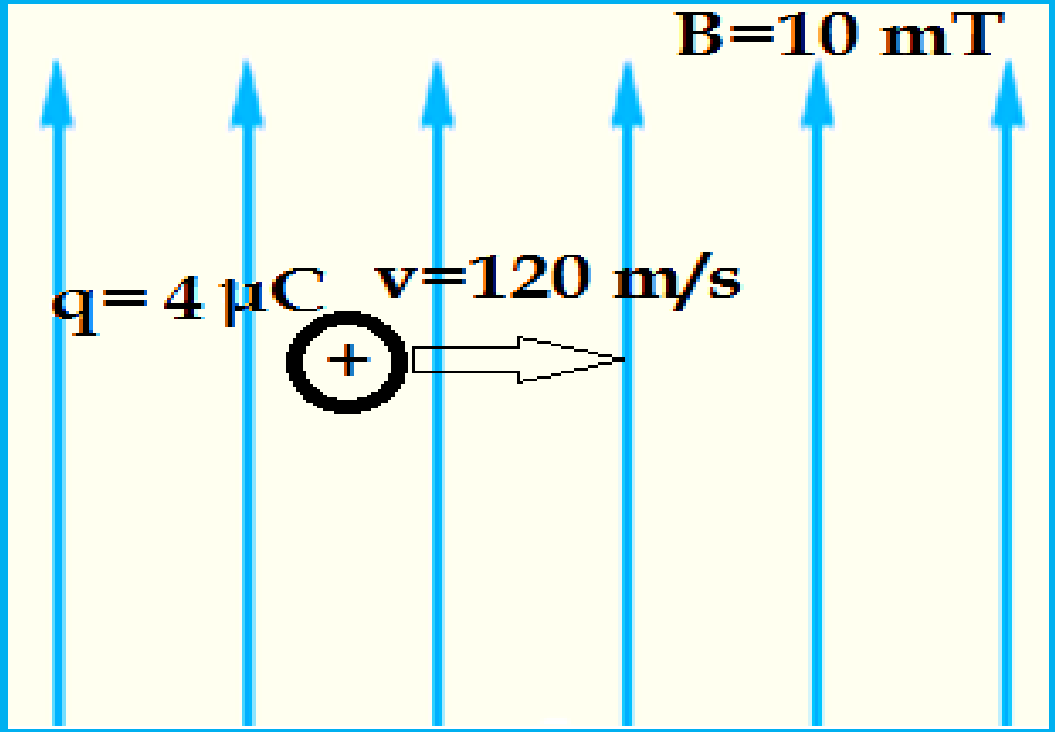


Out of the page field

Exercise:-

Suppose a 4μC positive charge moving with a velocity of 120 m/s along the + x direction is Immersed in a uniform magnetic field of 10mT acting to the + y direction, what is the magnitude and direction of the magnetic force exerted on it?

Answer



Answer

Given:- q=4μC, V=120 m/s, B= 10mT and θ=90

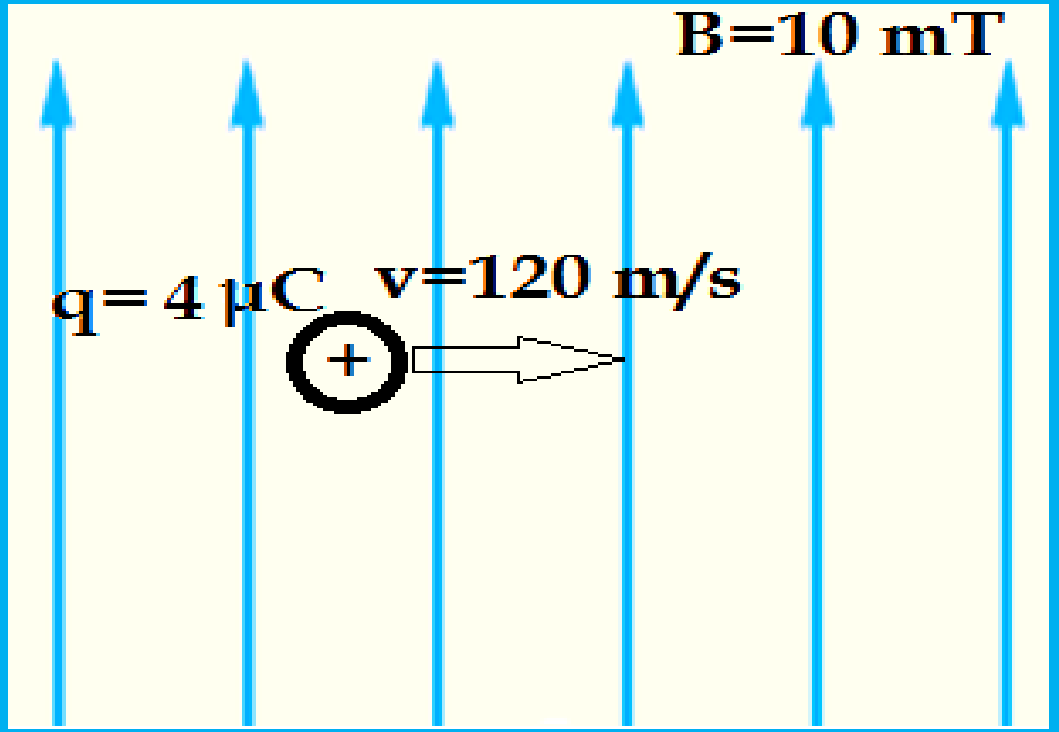
Req:- F=?

Solution:-

$$F = qVB \sin\theta$$

$$F = (4\mu)(120)(10m)$$

$$F = 4.80 \times 10^{-6} N$$



Magnetic flux $/(\emptyset_B)$ or $\emptyset$ /

It is a scalar quantity which measures the number of magnetic field lines which penetrate a given crossectional area.

$$\emptyset_B = \vec{B}.\vec{A} = BA\cos\theta$$

Where θ = the angle between the magnetic field and the normal line (n) to the crossectional area.

As magnetic field (B) is perpendicular to the surface area (A), the angle (θ) is zero. Since the normal line is parallel to the field.

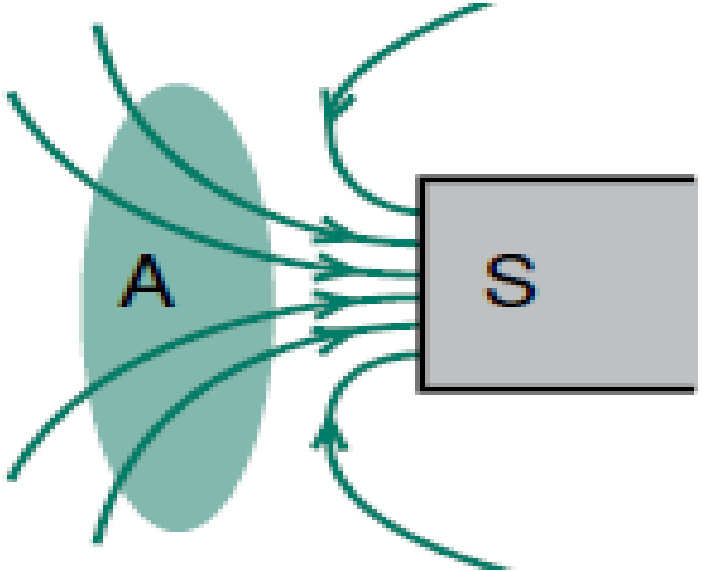
$$\emptyset_B = \vec{B}.\vec{A} = BA\cos\theta \Rightarrow \quad \theta = 0 \Rightarrow \emptyset = \emptyset_B = BA\cos 0$$

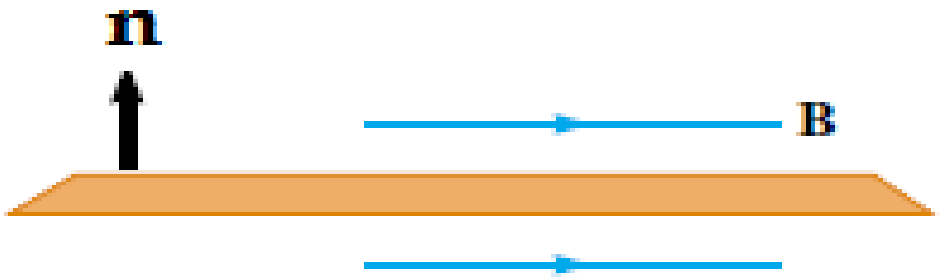
$$\emptyset = \boldsymbol{BA}$$

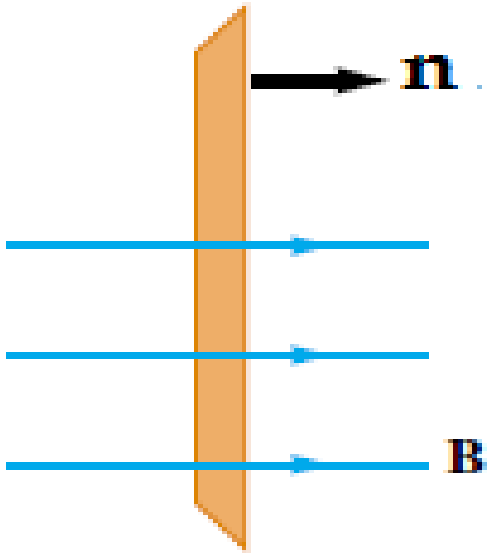
$$[\emptyset] = [\boldsymbol{B}][\boldsymbol{A}]$$

$$[\emptyset] = \boldsymbol{T.m^2 = weber\ (wb)}$$

NOTE:- The SI unit of magnetic flux is Weber (Wb) $\quad 1Wb = 1T.m^2 \Rightarrow$ $1T = 1\frac{Wb}{m^2}$ recall $1T = \frac{NS}{Cm}$ and $\quad 1T = \frac{N}{mA}$ Therefore, $\quad 1T = 1\frac{Wb}{m^2} = 1\frac{NS}{Cm} = \frac{N}{mA}$

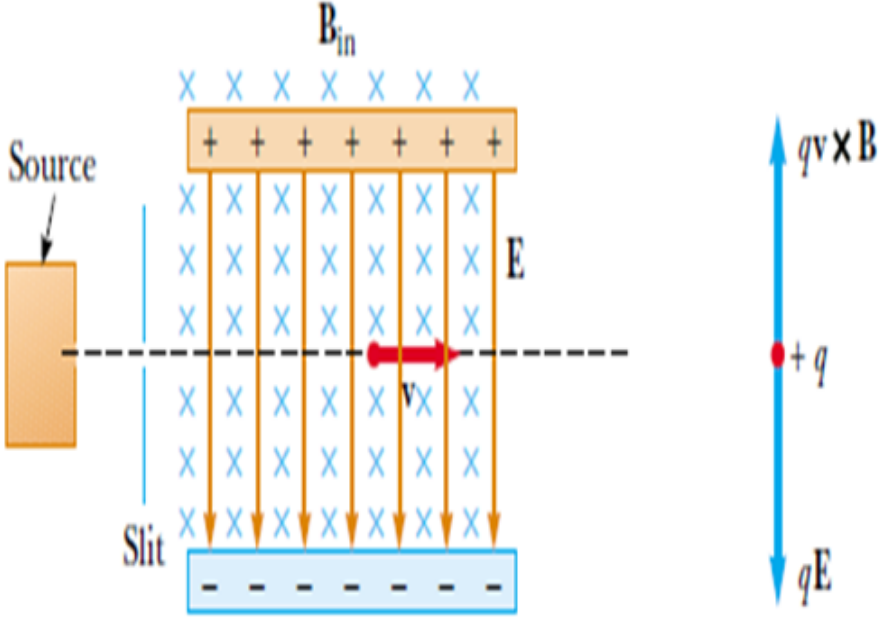

A
S


n
B


n
B

Motion of charged particle in combined /mixed/ field

Mixed field consist both electric and magnetic field, some very important electrical devices uses this mixed field as the main component; the common are velocity selector, J.J Thompson's apparatus and mass-spectrometer.



i) <u>Velocity selector</u>:-a device which uses a combination of electric and magnetic fields to trap particles moving at different speeds, apply this concept as the net force is zero.

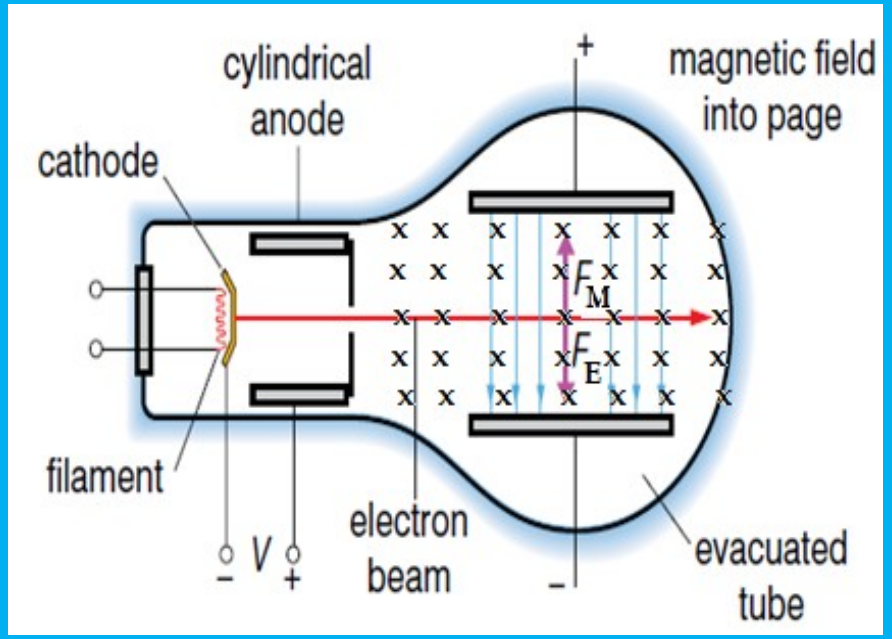
$$\vec{F}_E = \vec{F}_B \Rightarrow \quad Eq = Bvq \sin\theta$$

Hence, $V = \dfrac{E}{B \sin\theta}$ since $v \perp B \Rightarrow \theta = 90$

$$V = \frac{E}{B}$$

ii) J.J. Thompson's apparatus

J.J. Thompson used balanced electric and magnetic fields to measure the charge to mass ratio for an electron. His apparatus is shown in figure below.



$\frac{1}{2}mv^2 = qV \Rightarrow$ Where V= potential difference and v= speed of the particle, but

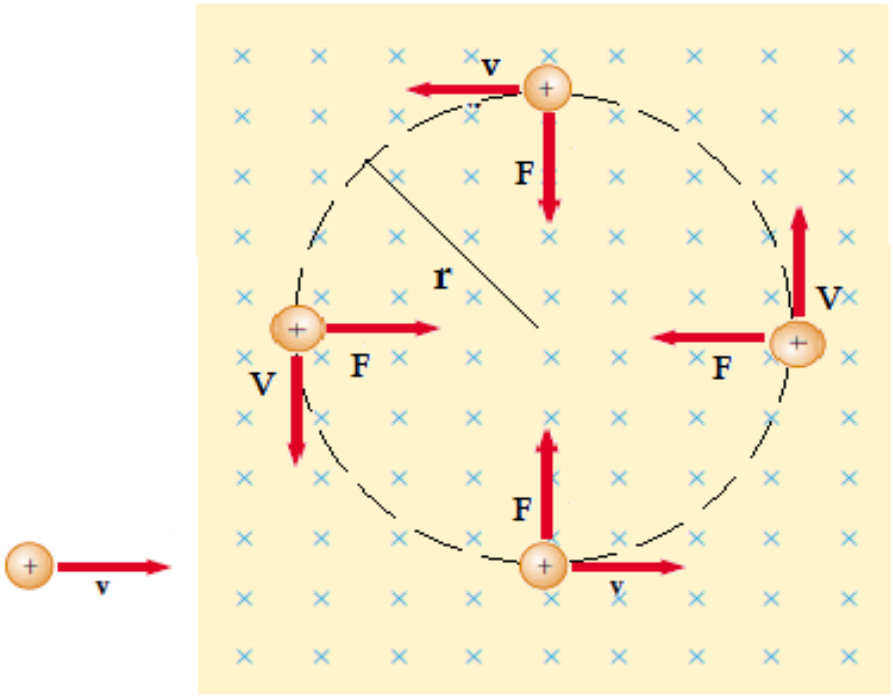
$v = \frac{E}{B}$

$$\frac{q}{m} = \frac{E^2}{2VB^2}$$

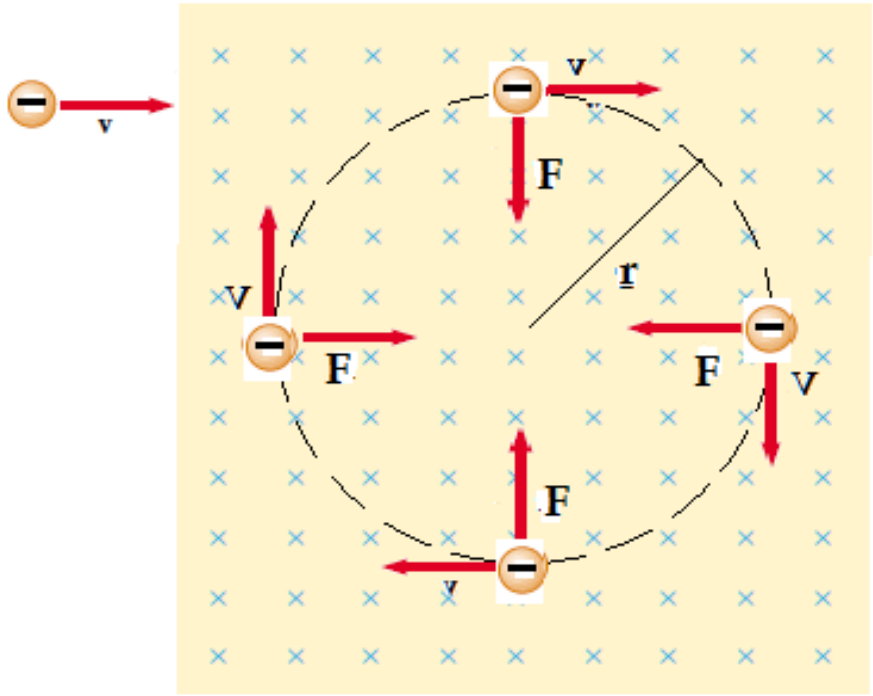
Circular motion of particles in magnetic fields

Suppose a positive charged particle is launched at right angle to the field acting into the page as shown below. From the right hand rule, you know that the force on a charged particle is always at right angles to the direction of its velocity.

- The force therefore acts as a centripetal force and so the particle follows a *circular path*.

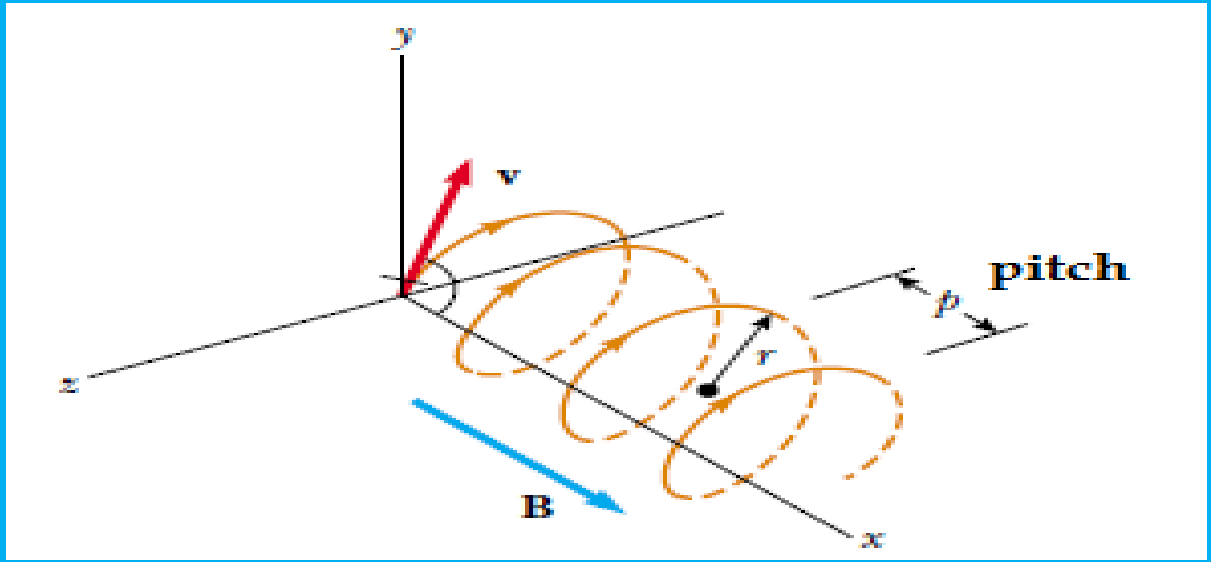


For Positive charge
Anti-clockwise circular path



For Negative Charge
Clockwise circular path

- If the charged particle moves parallel to the field the magnetic force is zero, thus the particle follows ***straight path***.
- If the charged particle moves at an angle θ of which is neither parallel nor perpendicular, has to velocity components, the perpendicular component makes the charged particles to have circular path; but the horizontal allow it to move along straight path, therefore the particle follows a ***helical path***.



For the perpendicular motion of particle in a field,

You know that $F = Bvq$ and $F = \frac{mv^2}{r}$ are equal.

Where q is the charge on the particle,

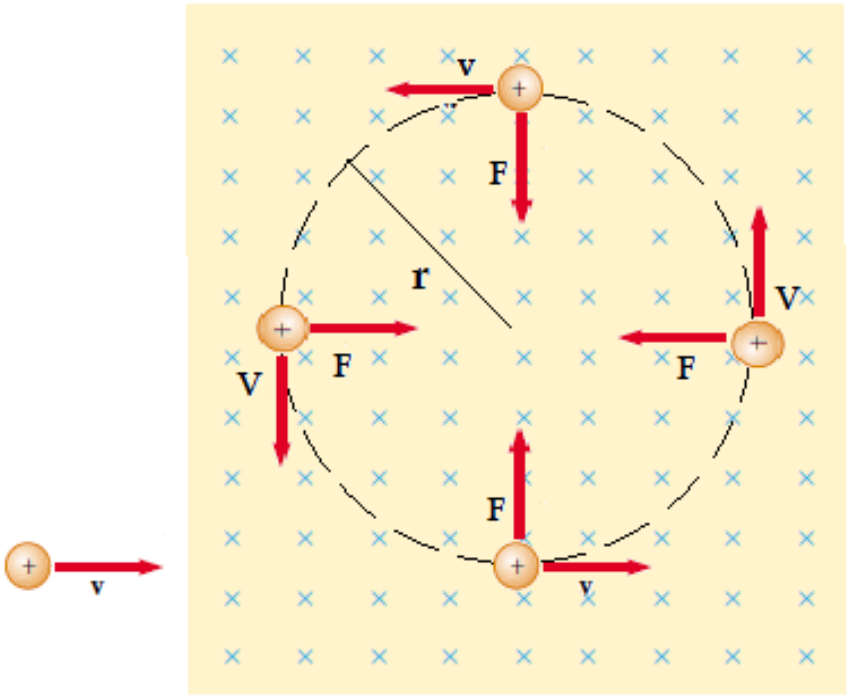
v is the velocity of the particle,

B is the strength of the magnetic field and

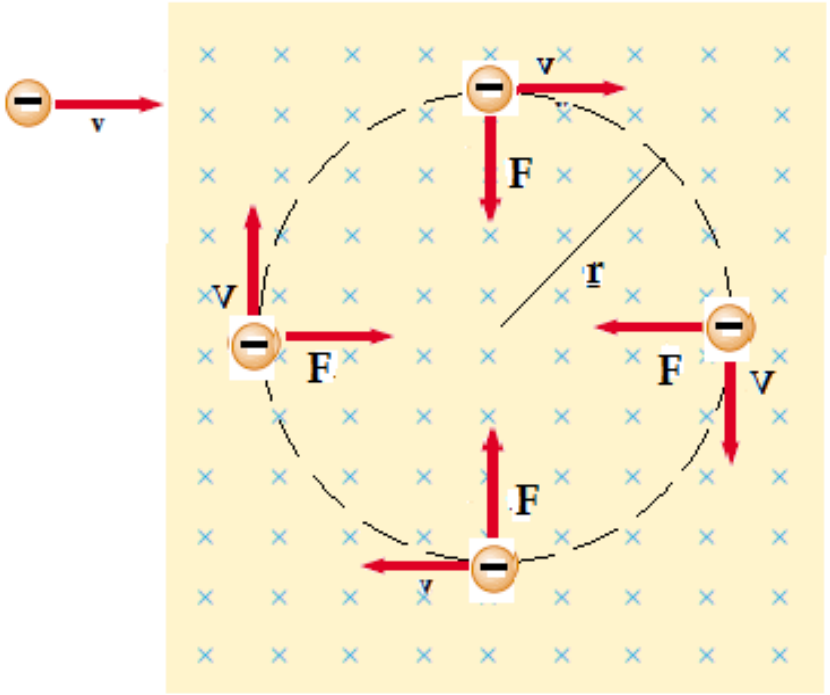
r is the radius of the circular path.

From this we can see that $\frac{mv^2}{r} = Bvq \Rightarrow \frac{mV}{Bq} = r$ this is the radius the circular path formed.

$$r = \frac{mV}{Bq}$$



For Positive charge
Anti-clockwise circular path



For Negative Charge
Clockwise circular path

We can also find the period, T, since $V = \frac{s}{t}$ *for* $s = r\theta$ $V = \frac{r\theta}{t}$

but $\theta = 2\pi$ *for complete path* and t=T

$$V = \frac{2\pi r}{T} \Rightarrow \quad T = \frac{2\pi r}{V} \qquad T = \frac{2\pi}{V}\left(\frac{mV}{Bq}\right) \rightarrow T = \frac{2\pi m}{Bq}$$

$$T = \frac{2\pi m}{Bq}$$

The frequency, f, is the inverse of the period so

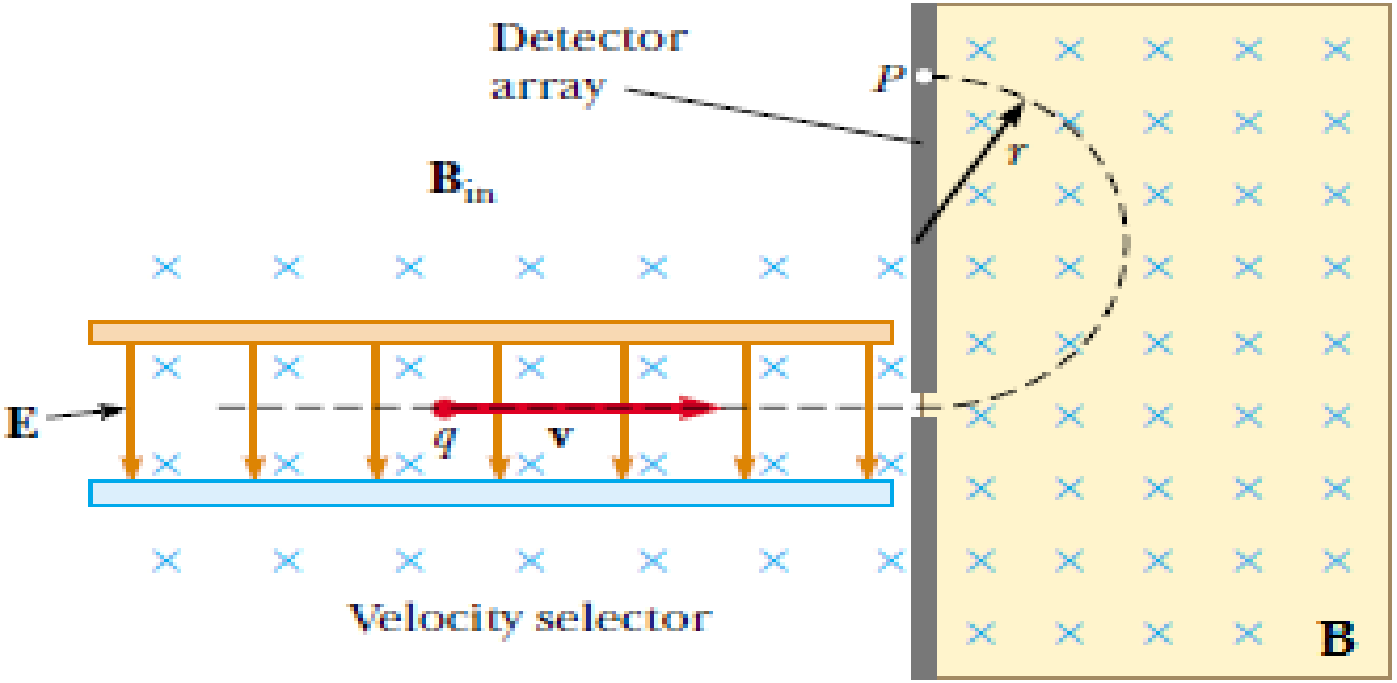
$$f = \frac{1}{T} \rightarrow f = \frac{1}{\frac{2\pi m}{Bq}} = \frac{Bq}{2\pi m} \Rightarrow f = \frac{Bq}{2\pi m}$$

$$\boldsymbol{f = \frac{Bq}{2\pi m}}$$

The angular velocity, ω, is 2πf, so $\omega = 2\pi f$ $\omega = 2\pi\left(\frac{Bq}{2\pi m}\right) = \frac{Bq}{m}$

$$\boldsymbol{\omega = \frac{Bq}{m}}$$

iii) The Mass Spectrometer



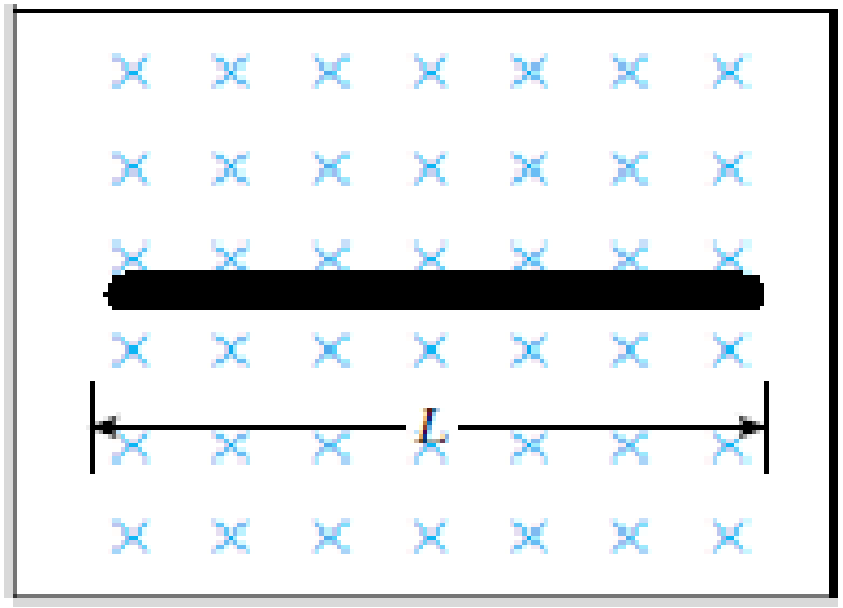
it is a device used to separate ions according to their mass-to-charge ratio./ used to determine the mass of different chemicals (isotopes of chemicals).

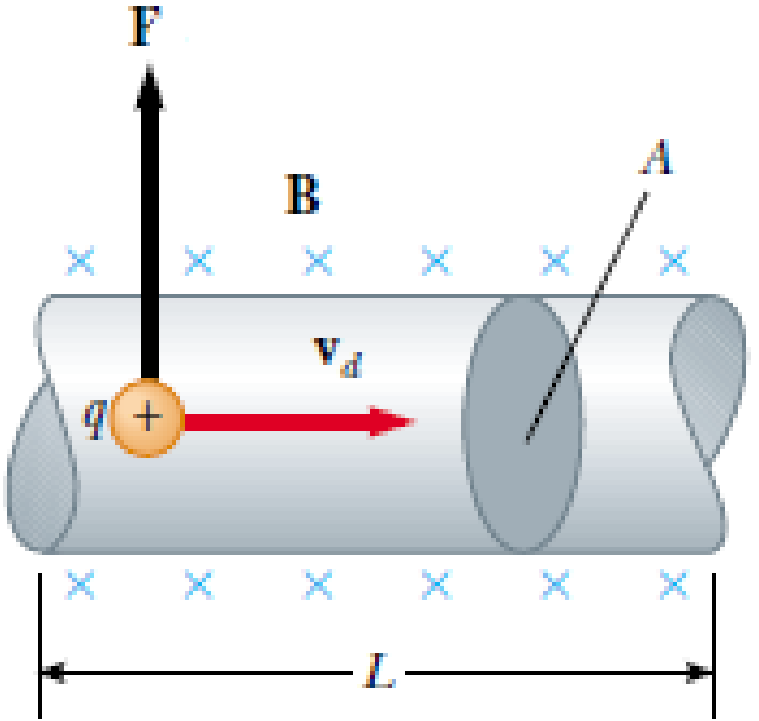
$$v = \sqrt{\frac{2qV}{m}}, \quad \& \quad \frac{Bqr}{m} = v$$

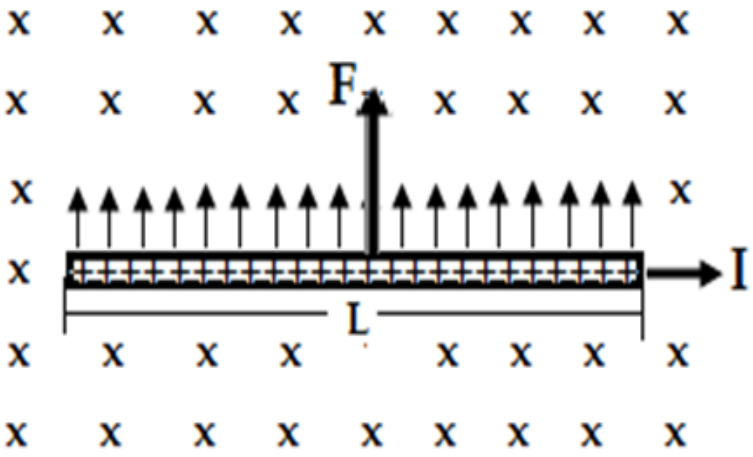
$$\frac{Bqr}{m} = \sqrt{\frac{2qV}{m}} \Rightarrow \left(\frac{Bqr}{m}\right)^2 = \frac{2qV}{m} \Rightarrow \frac{B^2q^2r^2}{m^2} = \frac{2qV}{m}$$

$$\mathbf{m = \frac{B^2q}{2V}(r^2)}$$

Magnetic Force Acting on a Current-Carrying Conductor







Force due to a point charge is given $F = q\vec{V}x\vec{B}$ for N number of charges and velocity (v) $v = \frac{L}{\Delta t}$

$F = Nq\frac{\vec{L}}{\Delta t}x\vec{B}$ Where N= total Charge, L= length of the wire

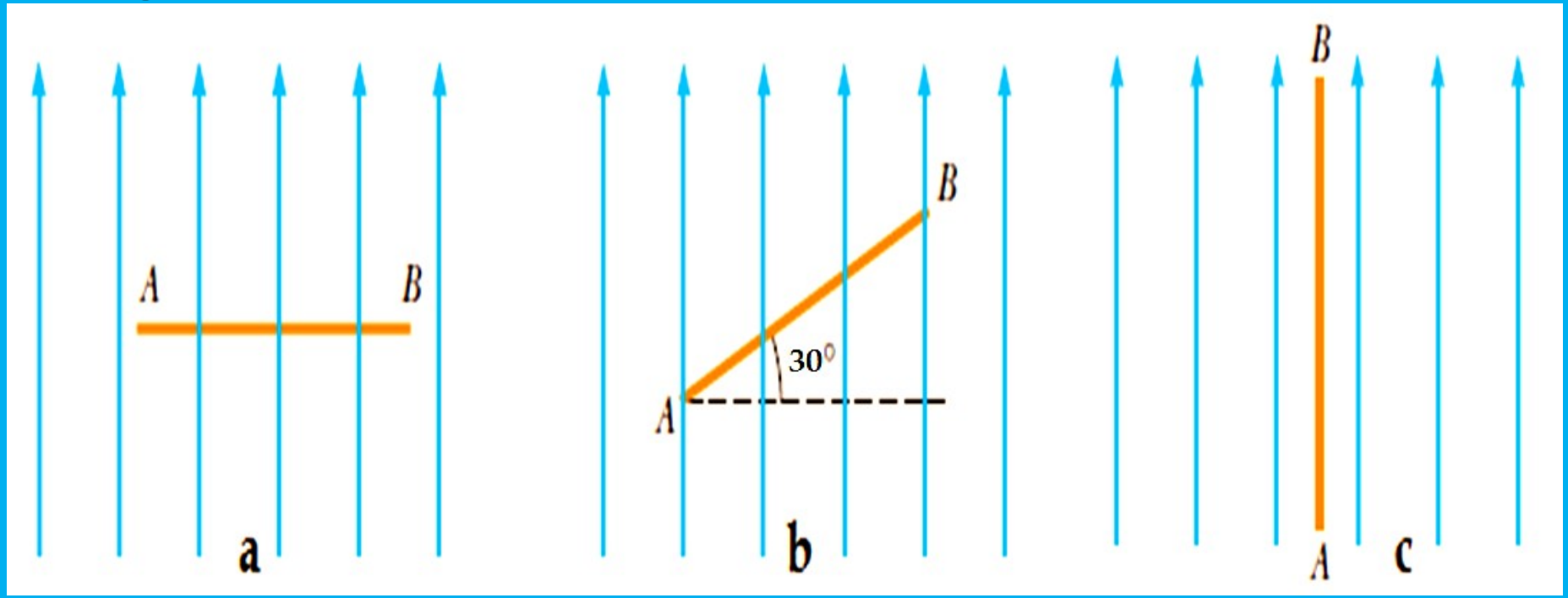
$\Rightarrow F = (N\frac{q}{\Delta t})\vec{L}x\vec{B} \Rightarrow F = I\vec{L}x\vec{B}$ Where I= current through the wire

$$F = ILB\,Sin\,\theta$$

$\Rightarrow$ But if $I \perp B \Rightarrow \theta = 90 \Rightarrow$ $F = BIL$ Maximum force

Exercise

What is the magnetic force on wires of length 6 cm carries a current of 10mA from (A) to (B) in a uniform magnetic field of 12mT oriented as shown below?



ANSWER

a)7.2 μ N

B) 3.6 μ N and

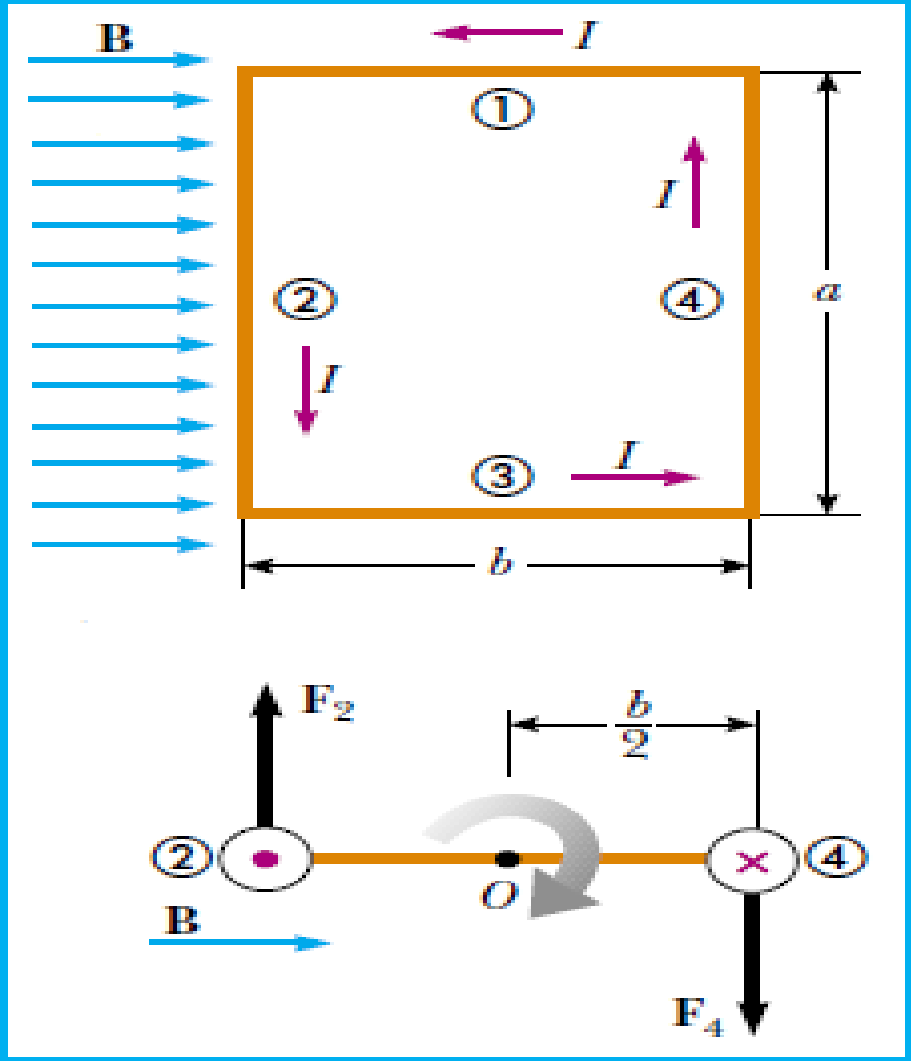
C) 0

Torque on a Current carrying Loop in a Uniform Magnetic Field

Consider a rectangular loop carrying a current I in the presence of a uniform magnetic field directed parallel to the right, as shown in Figure below.

* Let us see the magnetic force exerted on each segments!

- No magnetic forces act on sides 1 and 4 because these wires are parallel to the field. (why?)
- Maximum magnetic forces act on sides 2 and 4 because these sides are oriented perpendicular to the field.



The magnitude of the maximum torque in

using couple force system

$$\tau_{max} = F_2 \frac{b}{2} + F_4 \frac{b}{2} = 2(IaB)\frac{b}{2} = IabB$$

the area enclosed by the loop is $A=a\ b$, we can express the maximum torque as

$$\tau = IAB$$

$$\tau = IAB\sin\theta$$

$$\tau = I\vec{A}\ x\ \vec{B}$$

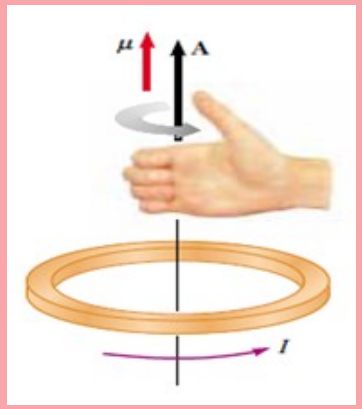
Where IA is known as Magnetic dipole moment(μ), $\vec{\mu} = I\vec{A}$ the direction is along the normal line to the area.

If we have N number of turns of loops the torque becomes

$$\tau = NIAB\sin\theta$$

$$\tau = N\mu B\sin\theta$$

$$\tau = N\vec{\mu}\ x\ \vec{B}$$



Exercise

A rectangular coil of dimensions 5.40 cm by 8.50 cm consists of 25 turns of wire and carries a current of 15.0 mA. A 0.350-T magnetic field is applied parallel to the plane of the loop.

(a) Calculate the magnitude of its magnetic dipole moment.

(b) What is the magnitude of the torque acting on the loop?

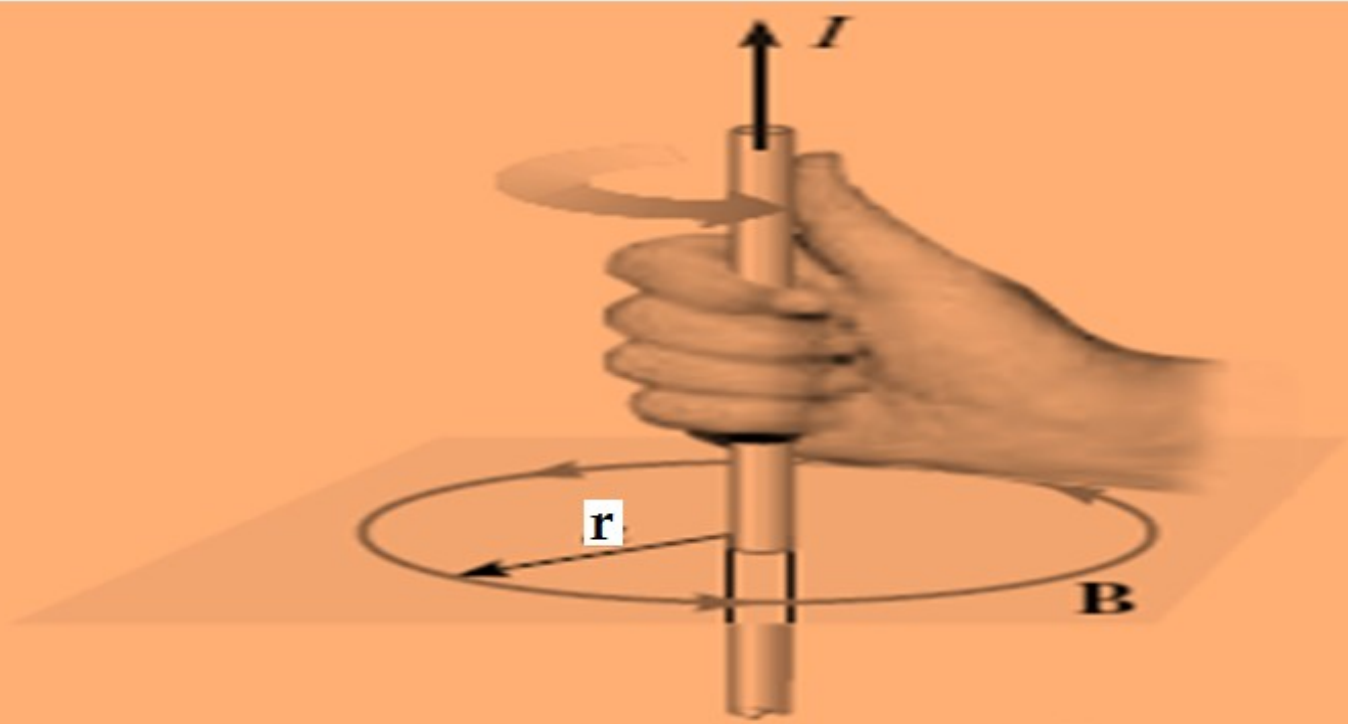
ANSWER

a) $\mu = IA \Rightarrow \mu = 6.9 \times 10^{-5}\ Am^2$

B) $\tau = N\mu B \Rightarrow \tau = 6.04 \times 10^{-4}\ Nm$

Magnetic field around current carrying conductors

A Danish Physicist ***Hans Oersted*** was the first to find relation between electricity and magnetism, he was able to produce magnetic field around the current carrying conductors.



The right-hand rul for determining the direction of the magnetic field surrounding a long, straight wire carrying a current.
Note that the magnetic field lines form circles around the wire.

NOTE:-

- Magnetic field around a current carrying conductor is directly proportional to the current through the wire.
  $B \sim I$
- Magnetic field has inversely proportional to separation distance from the charge.

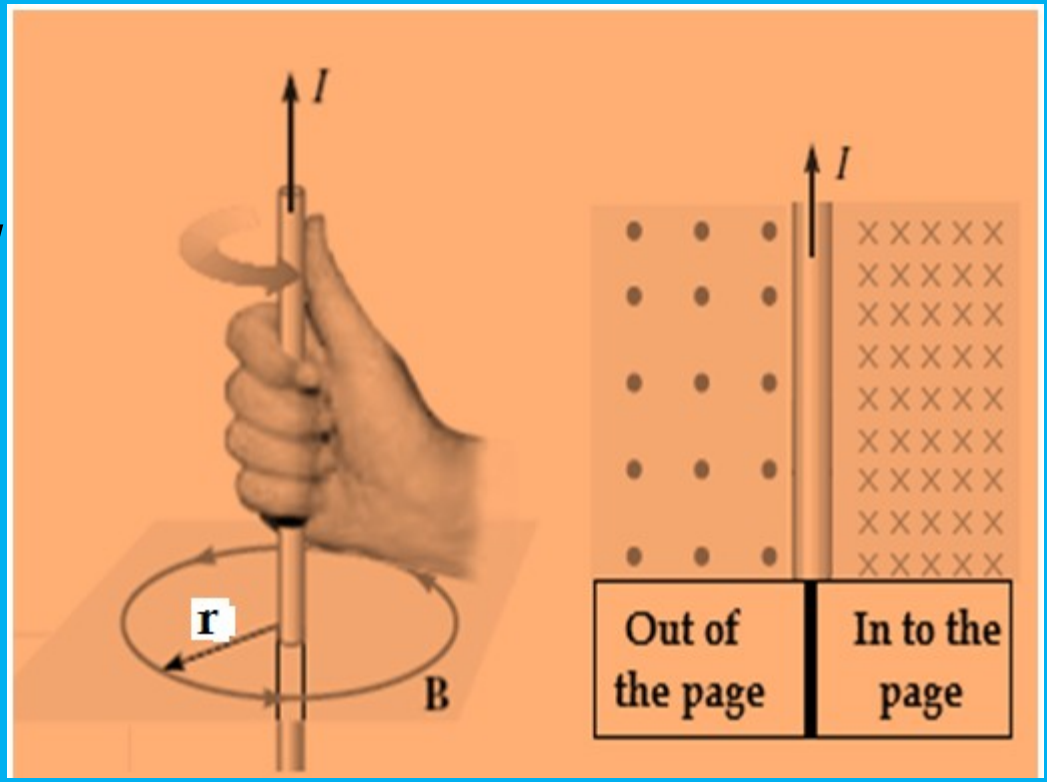
$$B \sim \frac{1}{r}$$

From the above two equations $B \sim \frac{I}{r} \Rightarrow B = k\frac{I}{r}$ where k is a proportional constant related to permeability of free space

$k = \frac{\mu_0}{2\pi}$ $\mu_0 = 4\pi x 10^{-7} \frac{T.m}{A}$, therefore $B = k\frac{I}{r} \Rightarrow B = \frac{\mu_0}{2\pi}\frac{I}{r}$

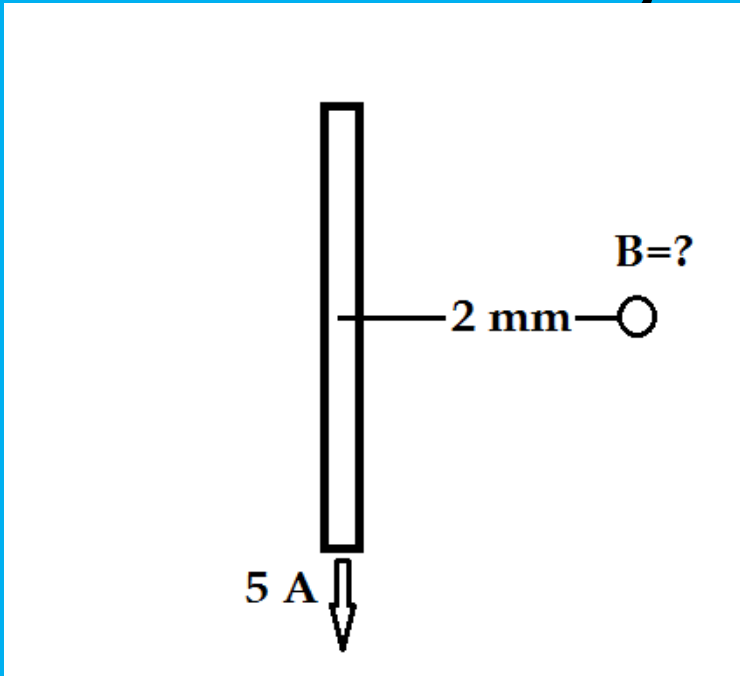
$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

- The direction of the magnetic field around the current carrying conductor forms concentric circle, and are determined by the right hand rule.
- It is also possible to represent it as follow



Exercise

What is the magnitude and direction of magnetic field at 2mm to the right from a wire which carries current of 5 A flows downward as shown below?



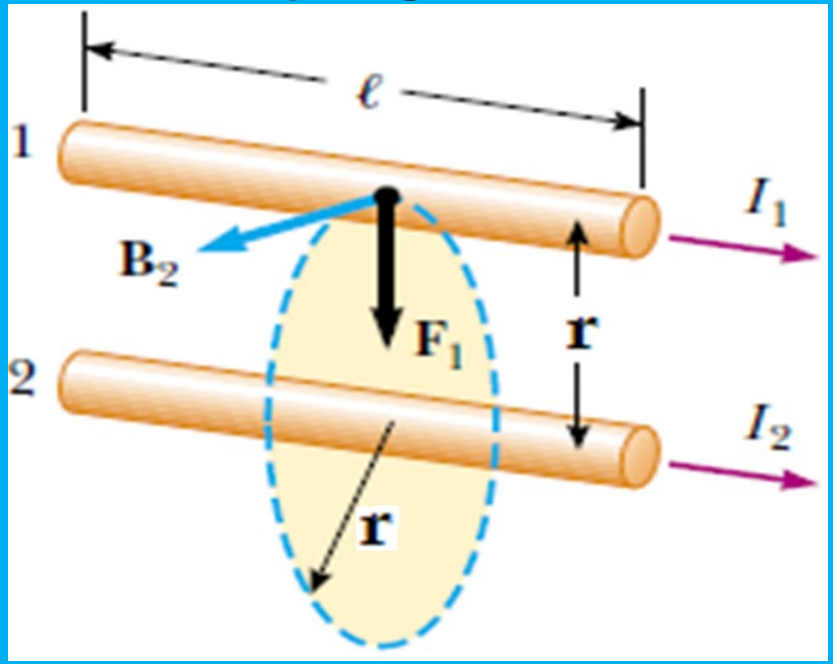
Answer

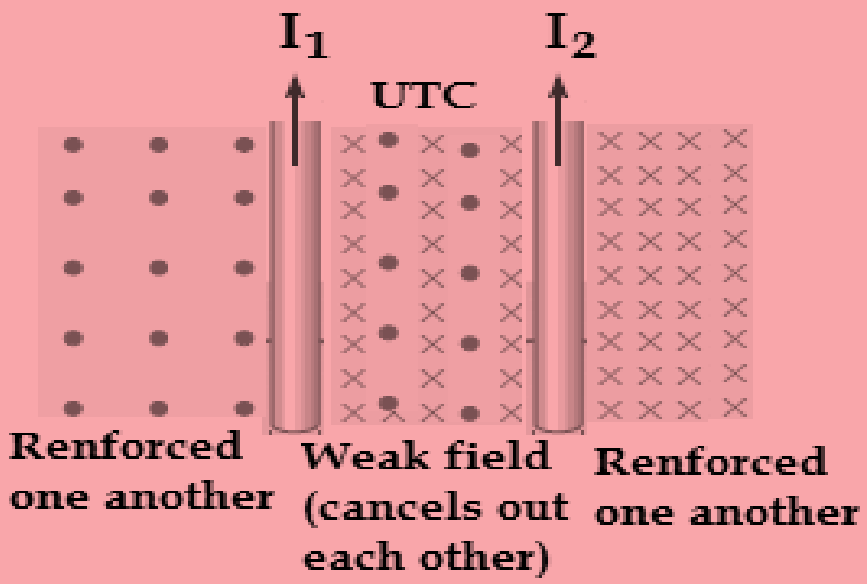
ANSWER

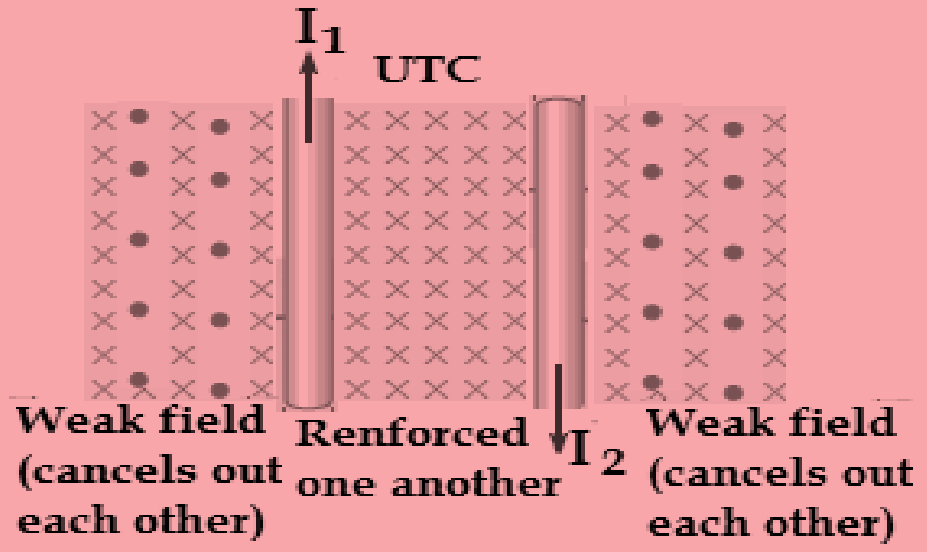
a) $B = 5 \times 10^{-5}\ T$

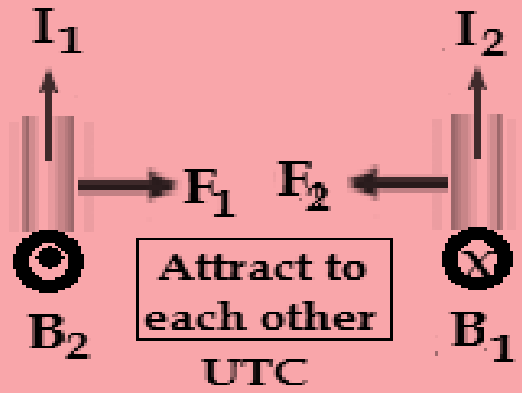
The Magnetic Force between Two Parallel current carrying Conductors

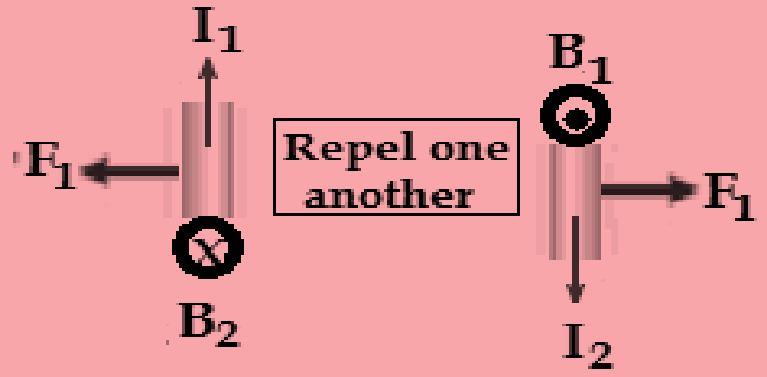
Consider two long, straight, parallel wires separated by a distance *r* and carrying currents I_1 and I_2 either in the same direction or in opposite, but here let us Take currents with the same direction as shown in the Figure below.




I1
I2
UTC
Renforced one another
Weak field (cancels out each other)
Renforced one another


I1
UTC
Weak field (cancels out each other)
Renforced one another
I 2
Weak field (cancels out each other)


I1
I2
F1
F2
Attract to each other
B2
B1
UTC


I1
B1
F1
Repel one another
F1
B2
I2

Note: - As two parallel wires carries the same current direction

- They have an attractive force.
- They have weak field in between the wires, but strong field at the two sides.

Note: - As two parallel wires carries the opposite current direction

- They have repulsive force.
- They have strong field in between the wires, but weak field at the two sides

The magnetic force on a length ℓ of wire1 is

$$F_1 = I_1 \ell B_2, \quad \text{But} \quad B_2 = \frac{\mu_0}{2\pi}\frac{I_2}{r}$$

$$F_1 = I_1 \ell \left(\frac{\mu_0}{2\pi}\frac{I_2}{r}\right) \Rightarrow \quad F_1 = \frac{\mu_0 I_1 I_2 \ell}{2\pi r} \Rightarrow$$

$$\frac{F_1}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

Because the magnitudes of the forces are the same on both wires, we denote the magnitude of the magnetic force between the wires as simply F whether the wires carries the current of having the same or opposite direction. We can rewrite this magnitude in terms of the force per unit length:

$$\frac{F}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

Example

Two parallel copper wires are 1 cm apart. Lightning sends a 10,000 A pulse of current along each conductor. What is the force per unit length on one conductor?

ANSWER

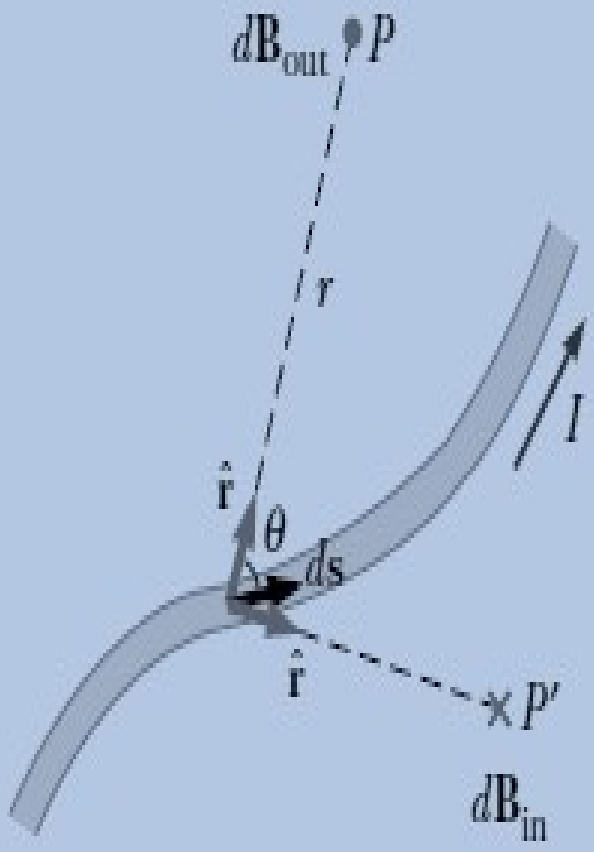
$$F/l = 2 \times 10^5 N/m$$

Magnetic field around Current carrying conductors

Magnetic field around any shaped Current carrying conductors can be determined by applying two basic rules and these are **Biot-Savart law** and **Ampere's law.**

i. The Biot–Savart Law

Shortly after Hans Oersted's discovery that a compass needle is deflected by a current-carrying conductor, J.B. Biot and F. Savart performed quantitative experiments on the force exerted by an electric current on a nearby magnet.



Biot- Savart Law

The magnetic field $d\mathbf{B}$ at a point P associated with a length element $d\mathbf{s}$ of a wire carrying a steady current I as shown in the figure.

- The vector $d\mathbf{B}$ is **perpendicular both** to $d\mathbf{s}$ (which points in the direction of the current) and to the unit vector $\hat{r}$ directed from $d\mathbf{s}$ toward P. $dB \perp Id\vec{s}\,x\,\hat{r}$
- The magnitude of $d\mathbf{B}$ is **inversely proportional to r^2**, where r is the distance from $d\mathbf{s}$ to P. $dB \sim \frac{1}{r^2}$
- The magnitude of $d\mathbf{B}$ is **proportional to the current** and to the magnitude ds of the length element $dB \sim Id\vec{s}\,x\,\hat{r}$
- The magnitude of $d\mathbf{B}$ is proportional to $\sin\theta$, where θ is the angle between the vectors $d\mathbf{s}$ and $\hat{r}$.

The mathematical expression known today as the Biot–Savart law:

$$dB = \frac{\mu_0}{4\pi}\frac{Id\vec{s}\ x\ \hat{r}}{r^2} \Rightarrow \int dB = \int \frac{\mu_0}{4\pi}\frac{Id\vec{s}\ x\ \hat{r}}{r^2} \rightarrow B = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{s}\ x\ \hat{r}}{r^2}$$

$\Rightarrow$ Since $d\vec{s}\ x\ \hat{r} = /\ \hat{r}//d\vec{s}/\ sin\ \theta = ds\ sin\ \theta$

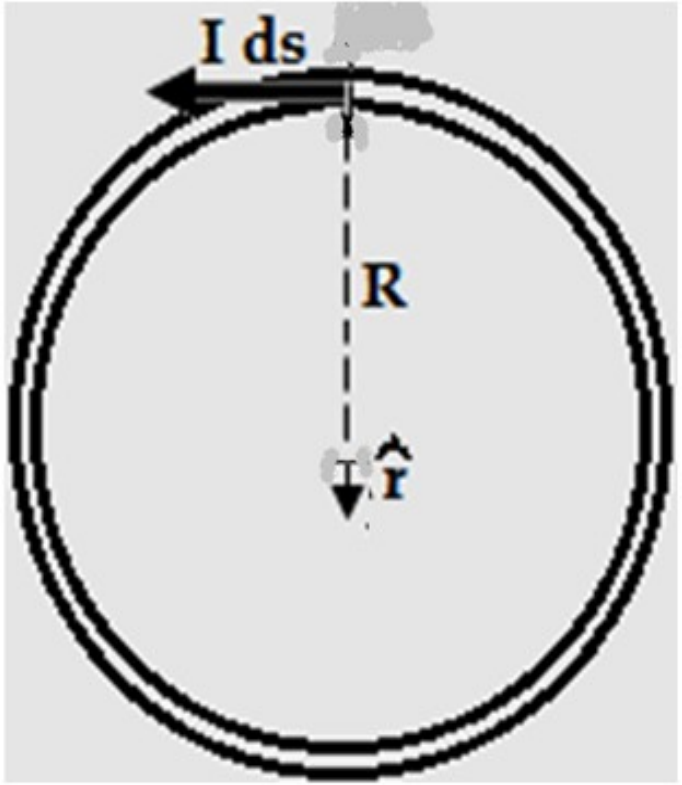
$$B = \frac{\mu_0 I}{4\pi}\int \frac{ds\ sin\ \theta}{r^2}$$

$$B = \frac{\mu_0 I}{4\pi}\int \frac{ds}{r^2}$$

Where θ is the angle between the vectors ds and $\hat{r}$ usually they are perpendicular and the current is constant. Thus, $\theta = 90^0$. $sin\ \theta = sin\ 90 = 1$

Exercise

Calculate the magnetic field at point o which is the center of a circular loop of radius (R) as shown below.



Solution

$B = \frac{\mu_0 I}{4\pi} \int \frac{ds \sin\theta}{r^2}$ Since ds and $\hat{r}$ are perpendicular

$(ds \sin\theta = ds)$

$B = \frac{\mu_0 I}{4\pi} \int \frac{ds}{r^2}$ but $ds = r d\theta \Rightarrow B = \frac{\mu_0 I}{4\pi} \int \frac{r d\theta}{r^2} \Rightarrow$

$B = \frac{\mu_0 I}{4\pi r} \int d\theta$ thus $\int d\theta = 2\pi$

$B = \frac{\mu_0 I}{4\pi r}(2\pi)$ r=R $B = \frac{\mu_0 I}{2R}$

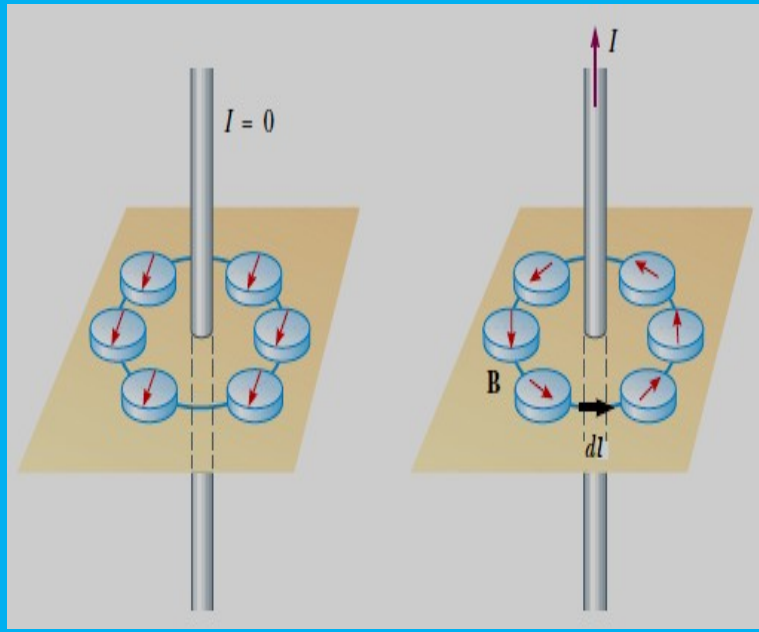
is magnetic field at the center of a ring of radius R.

ii. Ampere's Law

It is one of the well known law of magnetism proposed By Andre-Marie Ampere.

The rule has similar Application with Biot Savart law.

It can be stated as



"The sum of the products Bds over the closed path, is proportional to the flow of current"
the line integral of $B.ds$, is where $\oint ds = 2\pi r$ is the circumference of the circular path. Although this result was calculated for the special case of a circular path surrounding a wire, it holds for a closed path of *any* shape (an *amperian loop*) surrounding a current that exists in an unbroken circuit. The general case, known as Ampère's law, can be stated as follows:

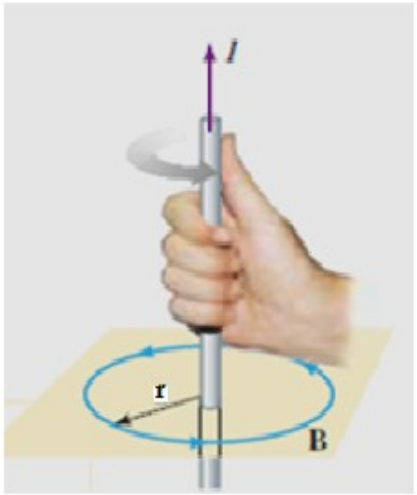
$$\oint B.ds = B\oint ds = (B)\,2\pi r = \left(\frac{\mu_0 I}{2\pi r}\right)(2\pi r) = \mu_0 I$$

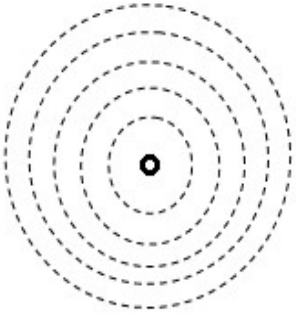
$$\oint B.ds = \mu_0 I \quad \text{--- Ampere's law}$$

Application of Ampere's law on current carrying materials

1. What is the magnetic field at a distance of r from a straight c.c.c. which carries a current of I?

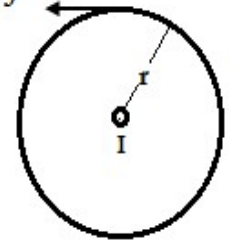






magnetic field lines forms concentric circle around a straight current carrying conductor. The direction is determined by RIGHT HAND RULE

$$\oint ds = 2\pi r$$



$$\oint B.ds = B \oint ds$$

$$(B)\,2\pi r = \left(\frac{\mu_0 I}{2\pi r}\right)(2\pi r)$$

$$\oint B.ds = \mu_0 I$$

AMPERE'S LAW

2. *Magnetic field inside a current carrying conductor*

Suppose a current carrying conductor of radius R carries a current of (I), what is the magnetic field at a distance of (r) where r<R (inside at section 2),

Solution

$\oint B.ds = \mu_0 I \Rightarrow B\oint ds = \mu_0 I \Rightarrow$ but $\oint ds = 2\pi r$

At r<R the Ampere's law is given as $B\oint ds = \mu_0 I'$

Where: - I' is the current enclosed in the portion at r<R. (Inside the wire).

Since the current density (J) remains constant in any part of the conductor of having steady current flow. Thus

$J = \frac{I}{A}$ and $J = \frac{I'}{A'}$ are equal i.e.

$\frac{I'}{A'} = \frac{I}{A} \Rightarrow I' = \frac{A'}{A} I$

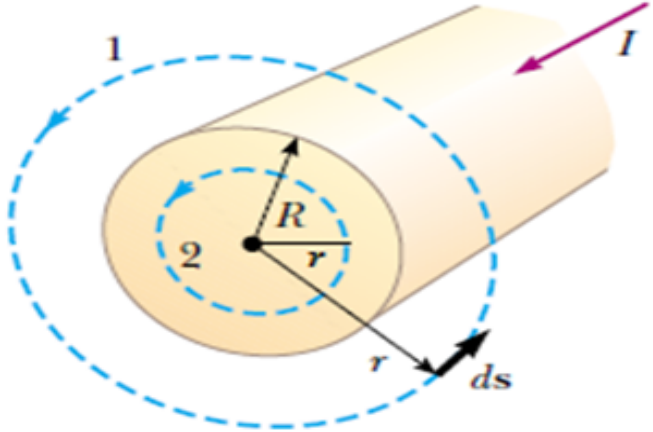
Where $A = \pi R^2$ and $A' = \pi r^2$

$I' = \frac{\pi r^2}{\pi R^2} I \Rightarrow I' = \frac{r^2}{R^2} I \Rightarrow$

$B(2\pi r) = \mu_0 I' = \mu_0 (\frac{r^2}{R^2} I)$

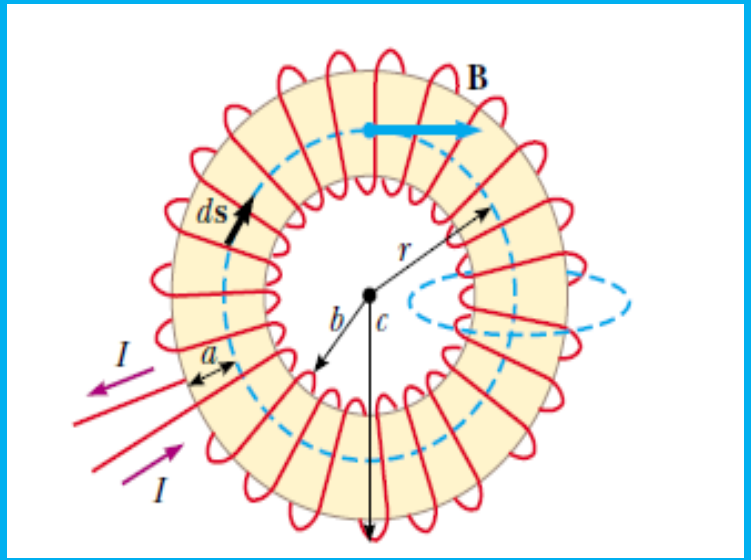
$B = \frac{\mu_0 r^2 I}{2\pi r R^2} = \frac{\mu_0 r I}{2\pi R^2} \Rightarrow$

$$B = \frac{\mu_0 I}{2\pi R^2} r$$



3. The Magnetic Field Created by a Toroid

For a toroid having *N* closely spaced turns of wire, calculate the magnetic field in the region occupied by the torus, a distance *r* from the center in the toroid.



Solution

$\oint B.ds = \mu_0 I \Rightarrow B\oint ds = \mu_0 I$ For N number of turns the equation is rearranged as

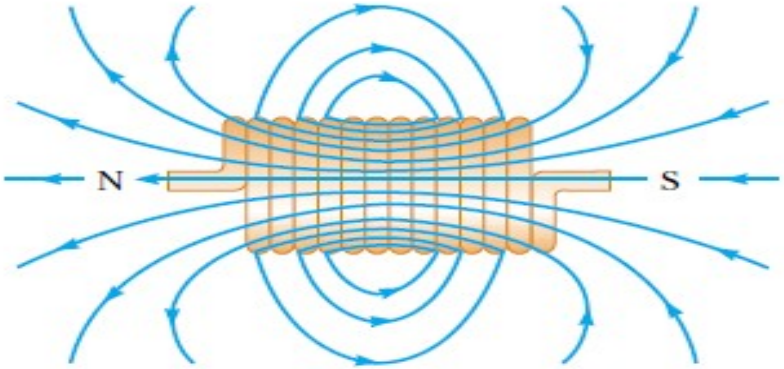
$$B = \frac{N\mu_0 I}{2\pi r}$$

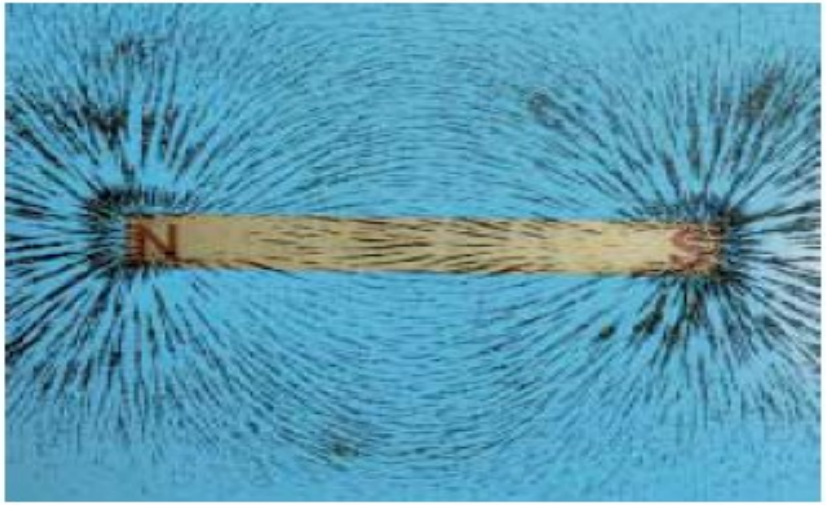
$B\oint ds = N\mu_0 I$ and $\oint ds = 2\pi r$

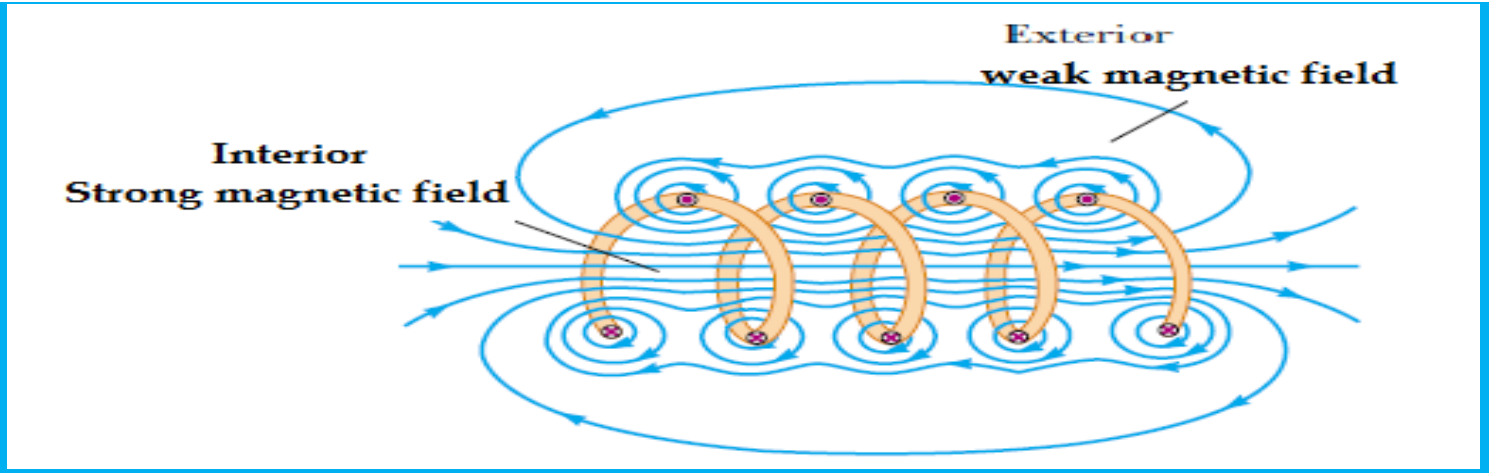
$B(2\pi r) = N\mu_0 I \Rightarrow B = \frac{\mu_0 NI}{2\pi r}$

4. The Magnetic Field inside of a Solenoid

A **solenoid** is a long wire wound *electromagnet* in the form of a helix. With this configuration, a reasonably uniform magnetic field can be produced in the space surrounded by the turns of wire-which we shall call the ***interior*** of the solenoid- when the solenoid carries a current. When the turns are closely spaced, each can be approximated as a circular loop, and the net magnetic field is the vector sum of the fields resulting from all the **turns**.

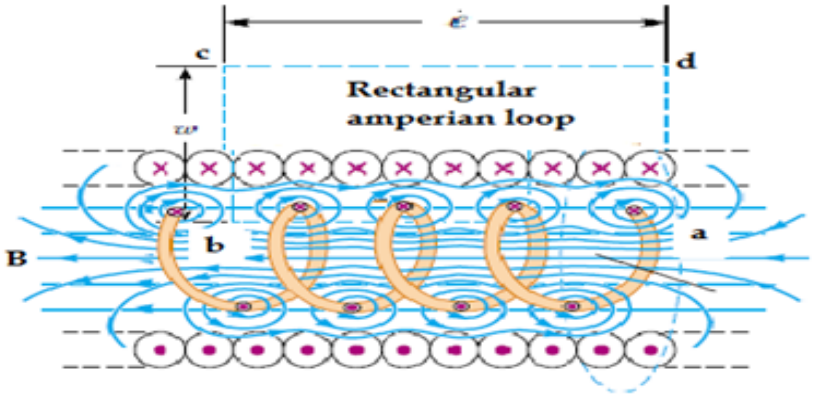

N
S


N
S


Exterior
weak magnetic field
Interior
Strong magnetic field

Magnetic field inside a solenoid of having N- number of turns, and current (I) can be determined as

Solution



$\oint B.ds = \mu_0 I$ For N number of turns of wound the law can be restated as $\oint B.ds = N\mu_0 I$

$$B.\oint ds = N\mu_0 I$$

$$\oint ds = \int_a^b ds + \int_b^c ds + \int_c^d ds + \int_d^a ds$$

$$B.(\int_a^b ds + \int_b^c ds + \int_c^d ds + \int_d^a ds) = N\mu_0 I$$

$$B.\int_a^b ds + B.\int_b^c ds + B.\int_c^d ds + B.\int_d^a ds = N\mu_0 I$$

For b to c and d to a B (magnetic field is perpendicular to ds thus $B.\int_b^c ds = 0$ and $B.\int_d^a ds = 0$ then c to d the field is parallel to ds but is exterior so that the magnetic field is very weak, $B.\int_c^d ds = 0$, therefore the only path left is between a and b

$B.\int_a^b ds = N\mu_0 I$ $B.\int_a^b ds = N\mu_0 I$ $B\int_a^b ds\cos\theta = N\mu_0 I$ B is parallel to the field to ds

$\theta = 0$ $\int_a^b ds = l$

$Bl\cos 0 = N\mu_0 I$ $B = \frac{N\mu_0 I}{l}$ $\Rightarrow B = \frac{N\mu_0 I}{l}$ The number of turns (N) per length (l) is given as $n = \frac{N}{l}$

$B = n\mu_0 I \Rightarrow$ $B = n\mu_0 I$

THANK YOU!