

Mathematics Grade 12

Unit 5

Summarized note

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INTRODUCTION TO INTEGRAL CALCULUS

In this unit we are going to see the following basic points:

- ✓ Indefinite integral
- ✓ Integration of some simple functions
- ✓ Techniques of integration
- ✓ Definite integral
- ✓ Applications of integration

5.1 THE CONCEPT OF INDEFINITE INTEGRAL

- The set of all anti derivative of function $f(x)$ is called indefinite integral of $f(x)$
- The indefinite integral of $f(x)$ is denoted by :

$\int f(x)dx$ and read as the integral of $f(x)$ with respect to variable x . where,

$\int$ is the integral sign

$f(x)$ is the integrand

dx is variable of integration is x

- If $F'(x) = f(x)$, then $\int f(x)dx = F(x) + C$, where C is any constant and it is called constant of integration

PROPERTIES OF INDEFINITE INTEGRAL

$$1. \int f'(x)dx = f(x) + C \text{ or } \int \frac{d}{dx} f(x)dx = f(x) + C$$

$$2. \frac{d}{dx} \int f(x)dx = f(x)$$

$$3. \int kf(x)dx = k \int f(x)dx$$

$$4. \int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$$

$$5. \int [f(x) - g(x)]dx = \int f(x)dx - \int g(x)dx$$

5.2 INTEGRATION OF SOME SIMPLE FUNCTIONS

1. INTEGRATION OF CONSTANT FUNCTION

$$a) \int 0dx = C$$

$$b) \int kdx = kx + C$$

Examples:

$$1. \int 2dx = 2x + C$$

$$2. \int dx = x + C$$

2. INTEGRATION OF POWER FUNCTION

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

For $n \neq -1$

Derivative of x^{n+1} is $(n+1)x^n$, hence

$$\int (n+1)x^n dx = x^{n+1} + C$$

$$(n+1) \int x^n dx = x^{n+1} + C \text{ (divide both sides by } n+1,$$

For $n \neq -1$)

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

EXAMPLES EVALUATE THE FOLLOWING INTEGRALS

1. $\int x^{-3} dx$

2. $\int x^{\frac{3}{2}} dx$

3. $\int 4x^3 dx$

SOLUTION:

$$1. \int x^{-3} dx = \frac{x^{-3+1}}{-3+1} + C = \frac{x^{-2}}{-2} + C = \frac{-1}{2x} + C$$

$$2. \int x^{\frac{3}{2}} dx = \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + C = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C = \frac{2}{5} x^{\frac{5}{2}} + C$$

$$3. \int 4x^3 dx = 4 \int x^3 dx = 4 \frac{x^{3+1}}{3+1} + C = x^4 + C$$

EXERCISE

EVALUATE THE FOLLOWING INTEGRAL

1. $\int x^{\frac{2}{5}} dx$

2. $\int \frac{4}{x^{1.5}} dx$

3. $\int x\sqrt{x} dx$

END OF LECTURE ONE

SELECTED SOLUTION FOR PREVIOUS LECTURE

$$\int x\sqrt{x}dx = \int x \cdot x^{\frac{1}{2}} dx = \int x^{\frac{3}{2}} dx = \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + C = \frac{2}{5}x^{\frac{5}{2}} + c$$

3. INTEGRATION OF A FUNCTION OF THE FORM

$$f(x) = (ax + b)^n$$

$$\int (ax + b)^n dx = \underline{\hspace{2cm}}$$

Let $u = ax + b$, then $\frac{du}{dx} = a$ and $\frac{du}{a} = dx$

$$\int (ax + b)^n dx = \int u^n \frac{du}{a}$$

$$= \frac{1}{a} \int u^n du$$

$$= \frac{1}{a} \frac{u^{n+1}}{n+1} + C$$

$$= \frac{1}{a} \frac{(ax+b)^{n+1}}{n+1} + C$$

EXAMPLES EVALUATE THE FOLLOWING INTEGRALS

1. $\int (3x - 2)^5 dx$

2. $\int (4 - 3x)^2 \sqrt{4 - 3x} dx$

SOLUTION:

$$1. \int (3x - 2)^5 dx = \frac{1}{3} \frac{(3x-2)^{5+1}}{5+1} + C = \frac{1}{18} (3x - 2)^6 + C$$

$$\begin{aligned} 2. \int (4 - 3x)^2 \sqrt{4 - 3x} dx &= \int (4 - 3x)^{\frac{5}{2}} dx = \frac{-1}{3} \frac{(4-3x)^{\frac{7}{2}}}{\frac{7}{2}} + C \\ &= \frac{-2}{21} (4 - 3x)^{\frac{7}{2}} + C \end{aligned}$$

4. INTEGRATION OF EXPONENTIAL FUNCTIONS

$$a) \int e^x dx = e^x + C$$

$$b) \int e^{ax+b} dx = \underline{\hspace{2cm}}$$

Let $u = ax + b$, then $\frac{du}{dx} = a$ and $\frac{du}{a} = dx$

$$\int e^{ax+b} dx = \int e^u \frac{du}{a} = \frac{1}{a} \int e^u du$$

$$= \frac{1}{a} e^u + C$$

$$= \frac{1}{a} e^{ax+b} + C$$

EXAMPLES:
EVALUATE THE FOLLOWING INTEGRAL

$$1. \int e^{2x+4} dx = \frac{1}{2} e^{2x+4} + C$$

$$2. \int e^{-x} dx = -e^{-x} + C$$

C, INTEGRATION OF THE FORM $f(x) = a^x$

$$\int a^x dx = \underline{\hspace{2cm}}$$

The derivative of a^x is equal to $a^x \ln a$

Hence, $\int a^x \ln a \, dx = a^x + C$

$$\ln a \int a^x dx = a^x + C$$

Dividing both sides by $\ln a$ we get:

$$\int a^x dx = \frac{a^x + C}{\ln a}$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

EXERCISE

EVALUATE THE FOLLOWING INTEGRALS

1. $\int 4^x dx$

2. $\int 2e^{4-2x} dx$

3. $\int \sqrt[3]{e^x} dx$